MECHANICS OF WEAK LAYER SNOW UNDER CYCLIC LOADING STUDIED BY FEM

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ABSTRACT: The mechanical behavior of snow weak layers is a crucial element in dry snow slab avalanche forecasting. Here, we use Finite Element Method (FEM) modeling to investigate the mechanics of layered snow under cyclic loading at a high acceleration rate with gradually increasing amplitudes of stresses leading to fracture. The provided 2-D model is based on the 'Cast3M' code. It treats the weak layer as an interface with variable constitutive behavior parameters (an approach which was initially developed for rock joints). The applicability of the proposed model is evaluated against the previous cold laboratory snow fracture experiments of Podolskiy et al. (2010) (shaking table tests studying the dynamic response of weak layer snow samples). Results: (i) show that joints with no tensional strength (as used in some previous studies) are not capable of reproducing the experiments; (ii) suggest that the Mohr-Coulomb failure criterion (controlled by tension strength and angle of friction) is sufficient to describe experiments; and (iii) highlight the complexity of strain and stress evolution within snow samples, and especially the significance of tension at the edges of the weak layer. Our simulations demonstrate that the analytical solution, based on Newton's second law (Nakamura et al., 2010; Podolskiy et al., 2010), may underestimate real shear stresses due to inertial effects. Hence, modeling allows refining the experimental data and provides reliable estimates of weak layer mechanical properties.

1. INTRODUCTION

The theory of linear elasticity and the first realistic images of snow microstructure have been known since the work of Robert Hooke (e.g., Hooke, 1665). Despite ignoring the complexity of snow structural properties and the limitations of the theory, snow under high strain-rates and at the macro-scale has been successfully modeled as an elastic homogeneous continuum. Since early efforts by Smith et al. (1971) and up to now, this approach, realized through the Finite Element Method (FEM), has being employed numerous times in various studies of snow stability on slopes (e.g., Mahajan et al., 2010; Gaume et al., 2012). Here we use a similar framework for investigating previous cold laboratory tests made by Podolskiy et al. (2010). In these experiments "sandwich" snow samples (two blocks of snow with a weak layer in the middle) were frozen to a shaking platform and oscillated with gradually growing accelerations until snow failure occurred. Earlier FEM numerical investigations treated snow weak layers by more than nine various mechanical approaches of different complexity and computational costs, ranging from an elastic layer with a lower Young's modulus than in neighboring layers to a collapsible layer with cohesive interface elements at its bottom and top (Wilson et al., 1999; Mahajan et al., 2010). In this study, by modeling multiple tests individually and considering weak layers as interfaces of zero thickness and zero volume, we test the applicability of the simple Mohr-Coulomb failure criterion for snow failure analysis. To the best of the author's knowledge, surprisingly, until now no modeling study has been performed on the validation of this classic constitutive law (e.g., Mellor, 1975) for dynamical loading of snow and, for example, a large unverified scatter of such basic properties as the angle of friction exists (from ~5° to ~80°; Mellor, 1975, and Nakamura et al., 2010, respectively). Despite the major shift of a focus of recent studies away from shear strength concept and towards studying the crack propagation process (e.g., Birkeland et al., 2009), it will be shown that the shear strength approach has reasonable predictive power, and that the Mohr-Coulomb law is sufficient and provides reliable estimates of weak layer properties. Another important insight, provided through modeling, is reexamination of the analytical approach that was previously used for interpretation of the discussed experimental method (Nakamura et al., 2010; Podolskiy et al., 2010), which remains to be verified.

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2. METHOD

A detailed description of the cold-laboratory experiments used for validation of the model can be found in Podolskiy et al. (2010). Here, we just note that snow samples with weak layers were subjected to shaking at the oscillating platform driven by an electric motor. At the instant that accelerations reached critical values, stresses produced by the inertia of the overlaying snow mass exceeded some critical value and caused snow break-up off the platform. The failure plane along the weak layer had an area of about 0.3 m by 0.2 m; the thickness of the upper block (with a density of ~200 kg m^{-3}) varied. Tests were repeated for different overlaying masses and inclinations (0°, 25° and 35°), thus providing varying conditions of normal/shear stresses and presenting an interesting ground for testing the Mohr-Coulomb failure criterion. Additionally we tested the strain-softening Mohr-Coulomb interfacial law (which is described in Gaume et al., 2012).

The model is implemented through open-source FE code Cast3M (Laborderie and Jeanvoine, 1994). For computational reasons we ignore the lower blocks and consider further only the upper fractured snow blocks. By breaking the 2-D geometry domain of samples into quadratic elements approximately 1 by 1 cm, and prescribing a Young's modulus and Poisson ratio (~ 1-1.5 MPa and 0.04, respectively) to blocks according to their density following Mellor (1975) and Teufelsbauer (2011), we introduce gravity and then start gradually increasing the frequency of horizontal oscillations (with an amplitude of 1.65 cm similar to experiments) of the boundary between a block and an interface. The imposed displacements (Fig. 1) produce inertia-induced stress oscillations within the blocks which increase with each cycle. We prescribe particular values of tensile strength, σ_c, and angle of friction, φ, to the interface and record the time of total failure within the joint, which is defined as the instant when all its nodes reach the failure criterion. By comparing modeled time-to-failure, t_m, with the time documented in experiments, t_e, we adjust σ_c and φ to minimize the difference. Hereafter the latter is described as Root Mean Square Error, or RMSE

$$\sqrt{\frac{\sum_{i=1}^{n}(t_{mi}-t_{ei})^2}{n}}.$$

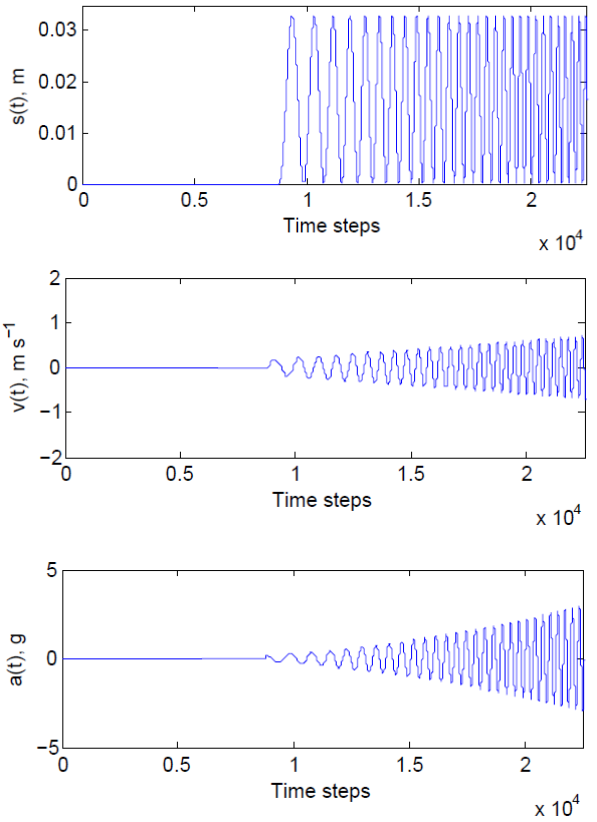

Figure 1: Examples of imposed displacements and their two derivatives (speed and acceleration).

This procedure is repeated for an ensemble of tests (n=19 in total) or later for a sample of this population in order to minimize overall cost function and reduce effects from unknown random errors or uncontrolled variations in experimental results. Through a search over the parameter space (σ_c: 0.5 – 3.0 kPa; φ: 30 - 60°), we attempt to find the global minima, which could be representative, or, in other words, optimal for the behavior of all experiments. Due to the dynamics of the problem and the fine time-steps, computational time for each simulation ranges between 24 and 744 hours, depending on its total duration (varying between 8 and 24 sec), size and mechanical properties. Accordingly, instead of covering our parameter space by all possible discrete combinations, after multiple sensitivity tests, we performed the search by first visiting extremes and then we follow cost function gradients intuitively by selecting a small representative sample of experiments (5 tests).

3. RESULTS (PRELIMINARY)

From the first simulations, we find out that the Mohr-Coulomb strain-softening law in the form it had being previously employed by Gaume et al. (2012) was not applicable for realistic reproduction of the tests. This stemmed from the tensile stress appearing at the edges of the joint after the start of oscillations. The law becomes undefined just after entering into zero-normal pressure conditions, since such a state corresponds to the opening of the joint. Obviously, tensional strength of the weak layer is needed for realistic representation of tests. Accordingly, all further discussion is based on modeling with the classic Mohr-Coulomb failure criterion, which allows simple control over both shear and tension strengths, through the angle of friction.

After several sensitivity tests, the shear and normal interface stiffnesses ($K_{s,n}$) were set to 10^8 N m^{-3} to reduce peak elastic displacements of the system to realistic values on the order of 10^{-3} m. An order of magnitude reduction of stiffnesses (to 10^7) resulted in larger displacements (~10^{-2} m) and slightly earlier failure than stiffer joints, though the difference is not comparable with the magnitude produced by changes in σ_c and φ. The imposed oscillations gave shear stresses with changing directions and produced strong oscillations of normal pressure at the edges of the interface (Fig. 2).

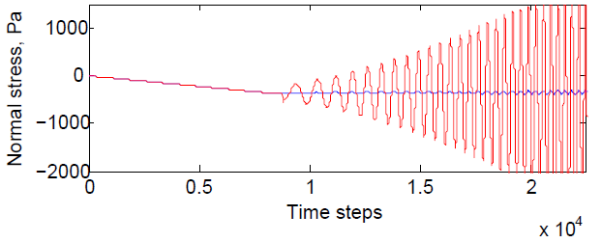

Figure 2: (a) Example of evolution of normal stresses in the middle and at the edge of the interface (blue and red curves, respectively).

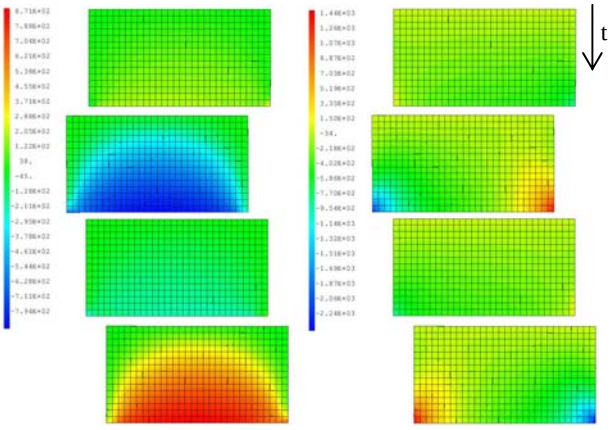

Figure 2: (b) Shear and normal stress concentrations within the block at different consequent phases of oscillations. Left side: shear, right side – normal pressure.

Overall mechanical conditions along the interface are dominated by shear stresses, which control the total failure, in all types of tests. Figure 3 shows the differences between analytical (see Nakamura et al., 2010) and FEM solutions. For the assumed parameters the FEM gives larger shear stresses (for about 20% in the middle of the joint).

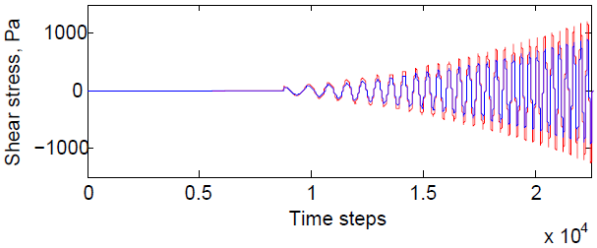

Figure 3: Example showing shear stress differences between simple analytical (for h=0.15, ρ=226 kg m^{-3}) and FEM (for the middle of the joint) solutions (blue and red curves, respectively).

For the considered range of parameters, time-to-failure is reproduced within ± 2.5 s precision for the majority of tests with only a few outliers larger than that (e.g., Fig. 4).

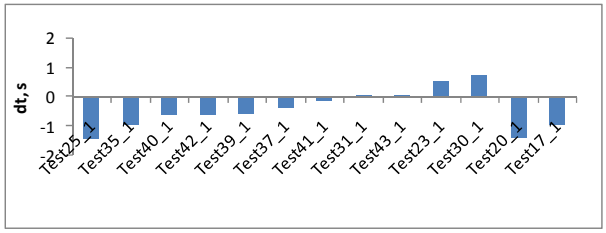

Figure 4: Difference between modeled and experimental time-to-failure for several tests for particular set of parameters (σ_c = 1.6 kPa and φ = 45°, K_s=K_n=10^7 N m^{-3}).

The sensitivity of sample's RMSE to the prescribed σ_c and φ is shown in Fig. 5. Computations were not completely finished at the time of submission, though at the moment it is possible to suggest that the global minima may not exist and could be presented by a few local minima (Fig. 5b). These conjunctions will be verified with ongoing tests. Another possibility, for example, is a limited range of experimental stress conditions, which may be insufficient for further clarification of φ. Nevertheless, constraints for the value of cohesion are already evident (~1.6-1.8 kPa) (Fig. 5b).

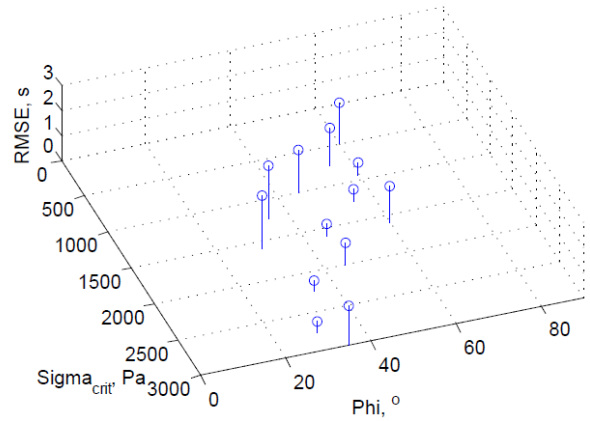

Figure 5: (a) Effects of σ_c and φ adjustments on time delay between modeled and experimental failures (RMSE, or cost function, for a sample of 5 tests).

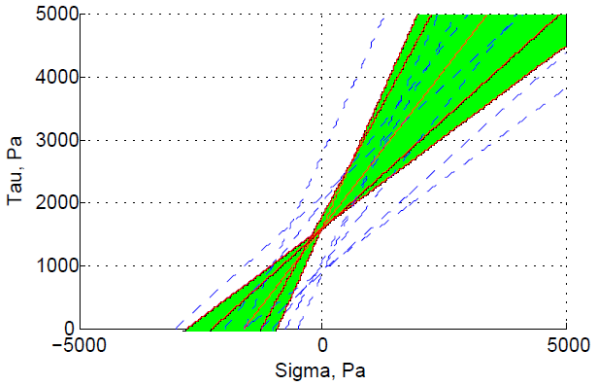

Figure 5: (b) Illustration of all tested pairs of σ_c and φ; red curves (with green shading) indicate the most successful simulations (lowest RMSE for a sample of 5 tests).

4. CONCLUSIONS (PRELIMINARY)

The application of Cast3M FEM code for reconstruction of experiments for the dynamic loading of snow was examined. Linear elasticity for snow blocks and the treatment of snow weak layers as interfaces following the Mohr-Coulomb failure criterion were validated by comparison with previous cold laboratory shaking platform tests. We find such a simplified approach to be sufficient for dynamic snow failure analysis.

One of the important points of the study is that the interrelated values of σ_c and φ could be narrowed down to certain limits. However, for the present moment it is hard to conclude whether the tensile strength of snow is equal or larger than its shear strength, at least for the tests under discussion. This is not clearly evident from multiple experimental measurements of snow shear and tensile strengths, and conclusions of several previous studies also differ (e.g., Mellor, 1975; Jamieson et al., 2001; Schweizer et al., 2004).

Another finding of the paper is that the interpretation of experiments through the previously used analytical solution is limited, due to substantial edge effects (originating from non-uniform normal pressure oscillations from compression into tension) and underestimated shear stresses along the failure plane caused by inertial effects.

The preliminary results of the paper are presented here in order to facilitate discussion during the

ISSW'12. Further work is in progress and will be published elsewhere.

5. ACKNOWLEDGEMENTS

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