Numerical modelling of liquid water movement in snow cover

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ABSTRACT: Liquid water movement in snow cover is an important aspect of wet snow metamorphism, and understanding it is vital for forecasting wet snow avalanches. Despite its importance, it has been over-simplified in the numerical snowpack model SNOWPACK, and the reproducibility of the water content profile is poor. In general, estimations of liquid water flux in porous material are based on hydraulic conductivity and suction, which several researchers have measured and parameterised. Using these parameterisations, we have developed a new water transport model that, given the initial values of water content, snow grain size, and density, and the boundary condition of the water supply, can simulate water movement in an unsaturated snowpack. Using this model, we simulated water movement between snow layers of different grain sizes. The results indicate that a water-saturated layer forms at the boundary between fine and underlying coarse rounded grains. This layer did not form under adverse layering conditions.

The new water transport model was incorporated into the larger SNOWPACK model, which subsequently produced an unstable water-saturated layer at the boundary between different grain sizes. The ability to reproduce these water movement processes accurately contributes to the improved prediction of wet snow avalanches.

KEYWORDS: Water transport model, capillary pressure, hydraulic conductivity, numerical snowpack model

1 INTRODUCTION

In recent years, numerical snowpack models have been developed to estimate the level of avalanche danger (Durand et al. 1999; Lehning et al. 2004, 2008). One of these models, SNOWPACK, has been used for snowy regions of Japan (Nishimura et al. 2005; Hirashima et al. 2008). The model was improved by implementing a dry snow metamorphism factor to aid the accurate prediction of dry snow avalanches (Hirashima et al. 2009). An improvement in wet snow avalanche prediction was not considered. Liquid water movement through snow and wet snow metamorphism affects wet snow avalanches. Therefore, understanding liquid water transport processes is important for the accurate prediction of wet snow avalanches. However, liquid water movement is often over-simplified in many numerical snowpack models. For example, liquid water is modelled percolating into lower layers in the snowpack when the total volumetric water content exceeds a threshold value (e.g., Bartelt and Lehning, 2002). In reality, liquid water flows in unsaturated conditions before this threshold is reached (Colbeck and Davidson, 1973) and water-saturated layers form along boundaries between layers of different grain sizes due to the capillary effect (Waldner et al., 2004). Therefore, unsaturated hydraulic conductivity and capillary pressure must be considered in order to predict water movement in snow accurately. This study developed a new water transport model incorporating these processes and integrated it into the SNOWPACK model.

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2 DEVELOPMENT OF THE WATER TRANSPORT MODEL

We developed a water transport model that considers capillary pressure and hydraulic conductivity. Since snow is a porous material, water flux (q) can be estimated using Darcy's law as a function of hydraulic conductivity (K) and suction (h)

$$q = K\left(\frac{dh}{dz} + 1\right) \quad (1)$$

where z is the snow height. Suction is proportional to capillary pressure.

Shimizu (1970) formulated saturated hydraulic conductivity based on the results of laboratory experiments using kerosene and it can be estimated as a function of grain size and snow density.

$$K_s = 0.077 r_g^{\,2} \frac{g}{\nu} \exp\left(-7.8 \frac{\rho_s}{10^3} \right) \quad (2)$$

Here, r_g is grain size (m), g the gravitational acceleration (m s^{-2}), ν the kinematic viscosity coefficient of water at 0 ℃, and ρ_s the snow density (kg m^{-3}). Colbeck and Davidson (1973) estimated the unsaturated hydraulic conductivity in snow (K) as being proportional to the cube of the effective water saturation (S*) multiplied by the saturated hydraulic conductivity (K_s) (Equations (3) and (4)),

$$K = K_s S^{*3} \quad (3)$$

$$S^* = \frac{S_w - S_{wi}}{S_{ws} - S_{wi}} \quad (4)$$

In Equation (4), S_w is weight water content, S_{ws} is the water content when liquid water has filled the snow pore spaces, and S_{wi} is the irreducible water saturation, which is permanently retained by capillary forces.

Suction (h) is proportional to the capillary pressure (Pc). Colbeck (1974) estimated capillary pressure using snow with an approximate grain size of 1 mm, and kerosene in place of water to make the retention curve (Equation (5)).

$$Pc = 43 S^{*-1} + 380 Nm^{-2} \quad (5)$$

$$h = \frac{Pc}{\rho_w g} \quad (6)$$

where ρ_w is the water density (kg m^{-3}). Using Equations (1) to (6), water transport can be simulated. Generally, however, the smaller the grain size, the larger the capillary pressure, but Equation (5) does not consider grain size. In Colbeck's (1974) experiment, dependency of grain size was not measured.

Sugie and Naruse (2000) measured the relationships between water content, capillary force, and unsaturated hydraulic conductivity for fine, intermediate, and coarse grains. The results were approximated as power functions (Equations (7) and (8)).

$$K = a S_w^{\,b} \quad (7)$$

$$h = c S_w^{\,d} \quad (8)$$

In Equations (7) and (8), a, b, c, and d are adjustment parameters used to estimate the measured moisture characteristics and the point diagram of unsaturated hydraulic conductivity. Table 1 lists the snow density, grain size, and parameters in Equations (7) and (8). The capillary pressure of fine grains is approximately three times higher than that of coarse grains with the same water content. Further, the unsaturated hydraulic conductivity of coarse grains is more sensitive to the water content than that of fine grains. Therefore, it is important to consider grain size to estimate water transport in snow accurately.

Table 1. Snow characteristics and parameters (Sugie and Naruse, 2000)

	fine	middle	coarse
density (kgm^{-3})	530	620	610
grain size (mm)	0.5	1.3	1.7
a	101	1.05	1.41
b	1.87	4.34	6.04
c	32.9	22	14.6
d	−0.39	−0.5	−0.56

Equation (8) estimates capillary pressure as a power function of the water content. Van Genuchten(1980) developed a more accurate expression in a soil science study (Equation (9))

$$S^* = \left[\frac{1}{1 + (\alpha h)^n} \right]^m \quad \text{where } m=1-1/n \quad (9)$$

In Equation (9), α and n are adjustment parameters used to estimate the measured moisture characteristics curve.

Van Genuchten (1980) combined Equation (9) with the theory of Mualem (1976) to estimate the unsaturated hydraulic conductivity (Equations (10) and (11)).

$$K_r = S^{*1/2} \left[1 - \left(1 - S^{*1/m} \right)^m \right]^2 \quad (10)$$

$$K = K_s \cdot K_r \quad (11)$$

Daanen (2004) estimated α and n for snow as a function of grain size based on the observations of Marsh (1991) (Equations (12) and (13)).

$$\alpha = 3 \times 10^4 r_g + 12 \quad (12)$$

$$n = 8 \times 10^2 r_g + 3 \quad (13)$$

Yamaguchi et al. (2009) measured the water retention curve and unsaturated water saturation using a method similar to Sugie and Naruse

(2000). However, the relationship between grain size and parameters α and n was not included in their calculations, so it cannot be incorporated into the new water transport model described here. Our water transport model was developed using Equations (1), (2), and (7) to (13) above.

3 SIMULATION RESULT

3.1 REPRODUCIBILITY OF THE WATER SATURATED LAYER

In Equation (7), the unsaturated hydraulic conductivity of fine grains is higher than that of coarse grains when the weight water content is less than 5%. When the weight water content exceeds 5%, the unsaturated hydraulic conductivity of fine grains is less than that of coarse grains. Therefore, Sugie and Naruse (2000) proposed the following hypothesis:

1. When the water flux is low, a water-saturated layer forms at the boundary between fine, rounded grains and underlying, coarse, rounded grains.

2. When the water flux is high, a water-saturated layer forms at the boundary between coarse, rounded grains and underlying, fine, rounded grains.

To assess this hypothesis, we simulated the water movement in two different snow layers. In the initial condition, the layer sequences were fine over coarse and coarse over fine, with layer thickness set at 20 cm (the total snow depth was 40 cm). The rate of water supply was set to 10.8 mm (low flux) and 108 mm (high flux) per hour. The simulation results are shown in Figure 1.

The results indicate that a water-saturated layer forms at the boundary between the fine grains and the underlying coarse grains, while it did not form under the opposite layering conditions, even with a high water flux.

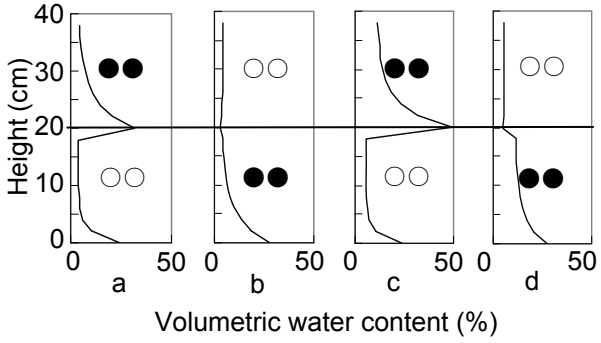


Figure 1. Profiles of the simulated water content: a and b, low water flux (10.8 mmh^{-1}); c and d, high water flux (108 mmh^{-1}).

This implies that the formation of a water-saturated layer at the boundary between coarse grains and underlying fine grains cannot be explained by hydraulic conductivity.

3.2 APPLICATION TO THE NUMERICAL SNOWPACK MODEL

The relationships between water content, suction and hydraulic conductivity depend on grain size and snow density, which must be considered when integrating the new water transport model into the SNOWPACK model. Sugie and Naruse (2000) did not characterise these relationships, so equation (9) to (13) developed by Daanen (2004), were used.

Water transport was simulated using both the improved and unimproved SNOWPACK models. In the initial condition, the layer sequence was fine (grain size 0.8 mm) over coarse (grain size 1.8 mm) with layer thickness set to 20 cm (total snow depth 40 cm). The surface and bottom temperatures were set to 0 ℃. Liquid precipitation was added at a rate of 2 mm per hour. With these boundary conditions, the SNOWPACK model was run for 48 hours.

The simulated water content and shear strength results from both the original and improved SNOWPACK model are illustrated in Figure 2. In the simulations, a water-saturated layer formed with the improved SNOWPACK model, but not with the original version of the model. The water-saturated layer acted as an

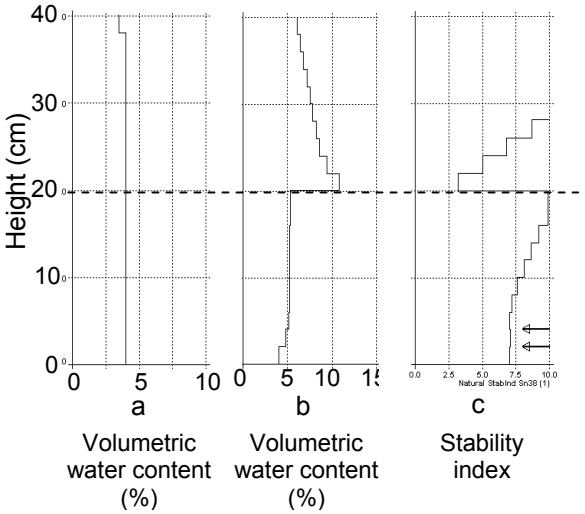


Figure 2. Simulated profiles before and after improving the SNOWPACK model by incorporating the water transport model. Layers (each 20 cm thick) of fine grains overlying coarse grains were set as the initial condition. The liquid precipitation rate was set to 2 mmh^{-1}. The simulation period was 48 hours: (a) Volumetric water content before improvement; (b) volumetric water content after improvement; (c) stability index after improvement.

unstable layer. Hirashima et al. (2008) included the effect of volumetric water content on shear strength in the SNOWPACK model based on the observed results of Yamanoi and Endo (2002), which cover a volumetric water content up to 8%. In their equation, water content and shear strength are negatively correlated. This reproducibility of an unstable water-saturated layer contributes significantly to the development of accurate predictions for wet snow avalanches.

4 CONCLUSION

A new water transport model was developed to consider capillary pressure and unsaturated hydraulic conductivity. The model simulated the formation of a water-saturated layer at the boundary between layers of fine snow grains overlying coarse snow grains and has been incorporated into the larger SNOWPACK model. The improved SNOWPACK model reproduces a water-saturated layer at the boundary of different snow layers. The water-saturated layer is unstable. To improve the accuracy of the formulae, capillary pressure and hydraulic conductivity were measured in a laboratory experiment (Yamaguchi et al. 2009, this issue). Future work will include integrating these results into the improved SNOWPACK model and validating the output.

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