

Modelling the spatial moments of SWE, taking into account changes in snow free areas

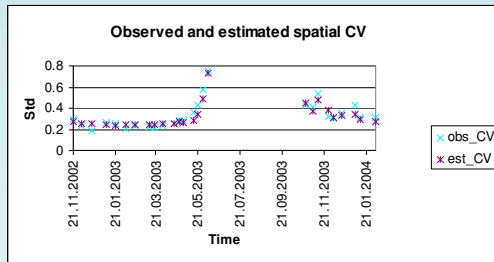
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Introduction

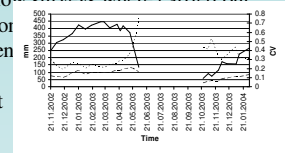
The representation of the spatial frequency distribution of snow water equivalent (SWE) in the rainfall-runoff model used operationally for flood forecasting purposes in Norway, the Swedish HBV model (Bergström, 1992), is static, in that the coefficient of variation of SWE is a fixed parameter of the model. Observations of SWE, show that this distribution changes continuously due to accumulation and ablation events. This study aims at developing a model that, from input of daily values of precipitation and temperature, estimates the spatial values of the mean and variance of SWE and model these parameters as summations of correlated gamma distributed variables. The evolution of snow free areas are taken into account in the spatial estimates. This study is a continuation of previous work where the spatial moments for completely snow covered areas were estimated (Skaugen, 2007).



$$f_{\alpha_0, \nu_0}(y) = \frac{1}{\Gamma(\nu_0)} \alpha_0^{\nu_0} y^{\nu_0-1} e^{-\alpha_0 y} \quad \alpha_0, \nu_0, y > 0$$

Data and Results

Snow surveys was carried out every second week during the accumulation season and every week during the snowmelt season. The site is at Norefjell, located 110 km northeast of Oslo, Norway, above the tree line at 1100 m.a.s.l. The route was fixed (by GPS) for the 2 km long snow course and snow depth was measured every 10 meters. Snow density was measured twice during the snow course. In total 26 snow surveys were carried out covering, in total, a complete accumulation and ablation. Spatial mean and standard deviation of SWE have been estimated for the Norefjell site. When the parameters, α_0 and ν_0 are tuned, the Nash Sutcliffe coefficient of efficiency, R^2 is found to be $R^2 = 0.89$. When α_0 and ν_0 are estimated from observed precipitation, data we obtain $R^2 = 0.87$.

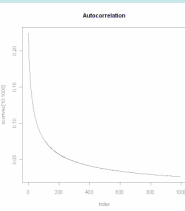


Observed data at Norefjell. Spatial mean (solid line), standard deviation (long dashed line) and CV (short dashed line)

It is seen from the figure of observed data that the spatial standard deviation of SWE increases also after the onset of melting. From the figures below we see that this feature is reproduced, although not to the same degree. The key factor to this behaviour, as it is modelled here, is the negative covariance between units of SWE (n) and units of melt (u). At the start of the melting season, when the spatial mean start to decrease, this variance contribution is not large enough to make an impact, i.e. variance continues to increase. As the melting season continues, the negative part starts to dominate, and variance also starts to decrease.

Method

The covariance matrix is investigated as means for estimating the spatial variance after accumulation and melting events. Further complexity is introduced when changes in snow covered area (SCA) is taken into account. For a single accumulation event, the mean and variance is straightforwardly estimated as: $E(z'(t)) = \sum_{i=1}^n E(y_i) = n \frac{\nu_0}{\alpha_0}$ and $Var(z'(t)) = n \frac{\nu_0}{\alpha_0^2} + n(n-1)c(n) \frac{\nu_0}{\alpha_0^2}$ where α_0 and ν_0 are parameters of the gamma distribution and estimated by the method proposed by Skaugen and Andersen (2010). The correlation function $c(n)$ is a decaying function of the number of summations, n .



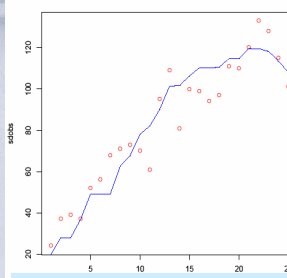
Correlation function as a function of number of summations.

For **accumulation** events (nn elements are added) $E(z'(t)) = SCA(nn) \frac{\nu_0}{\alpha_0} + (1-SCA)nn \frac{\nu_0}{\alpha_0}$ the covariance matrix only contains positive elements. $Var(z'(t)) = SCA_{1-1}^2 Var(z'(t))_{SCA-1} + (1-SCA_{1-1})^2 Var(z'(t))_{1-SCA-1}$

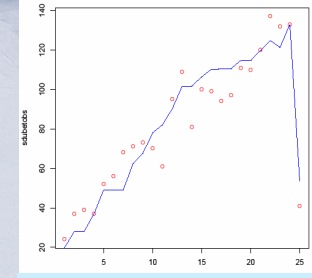
For **melting** events (u elements melt), we have to establish the conditional moments (i.e. conditioned on snow), which are estimated taken into account the reduction in SCA. The covariance matrix has negative elements. Melt is negatively correlated to SWE (i.e. it melts the most where we have the least SWE).

$$E(z'(t)) = (n \frac{\nu_0}{\alpha_0} - u \frac{\nu_0}{\alpha_0}) / SCA$$

$$Var(z'(t)) = \frac{\nu_0^2}{\alpha_0^2} - 2u' n \frac{\nu_0}{\alpha_0} c_{mt}(u') + u' \frac{\nu_0}{\alpha_0^2} + u'(u'-1) \frac{\nu_0}{\alpha_0^2} c(u')$$



Simulated and observed conditional standard deviation



Simulated and observed non-conditional standard deviation

Conclusions

- The spatial moments of SWE are simulated to a high degree of precision using information from observed precipitation series.
- The changing spatial coverage of snow is taken into account.
- The algorithm may be implemented into rainfall-runoff models, reducing the

References

- Bergström, S., 1992. The HBV model—its structure and applications, SMHI Hydrology, RH no.4, Norrköping, 35 pp.
- Skaugen, T. 2007. Modelling the spatial variability of snow water equivalent at the catchment scale. *Hydrol. Earth System Sci.*, **11**, 1543-1550.
- Skaugen, T. and Andersen, J., 2010. Simulated precipitation fields with variance-consistent interpolation. *Hydrol. Sci. J.* **55**(5), 676-686.

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