BENDING STRESS MODEL FOR SNOW-BURIED TREES: HYSTERESIS EFFECTS OF CROWN SNOW AND ACCUMULATED SNOW LOADING

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ABSTRACT: Climate change is projected to increase wet snowfall in regions that continue to receive snow under rising temperatures. Wet snow has a higher water content and density than dry snow, so it accumulates on tree crowns more readily and increases the crown snow load. Snow-buried trees, the understory species and the saplings of canopy species buried within the snowpack, experience greater mechanical strain and higher mortality than snow-exposed trees. Because they are the next generation of the canopy and the understory, their survival directly determines forest regeneration. However, no structural-mechanics-based bending stress model has been developed specifically for snow-buried trees. We developed a bending stress model that accounts for the load hysteresis arising from the sequential application of a crown snow load and a snowpack pressure load. The crown snow load, applied as a concentrated tip load, is the snow that adheres to the crown; the snowpack pressure load, applied as a distributed load, is the force the surrounding snowpack exerts on the buried trunk, including both the weight of the snow and the pressure from its settlement (termed the accumulated snow loading in the title). At each step the trunk form deformed by the previous step was carried over, and the bending stress was calculated from the curvature change using large deformation theory. The model was validated against field measurements from 12 snow-buried Japanese beech, and the agreement was close. Simulations gave two key findings. First, the snowpack pressure load, not the crown snow load, is the direct driver of the bending stress that can break a trunk. Second, an increased crown snow load amplifies the bending stress during the snowpack pressure phase, acting as an indirect but significant risk factor. These results indicate that future increases in the crown snow load driven by wetter snow will indirectly but substantially increase the bending stress in snow-buried trees, with implications for trunk-breakage risk and forest regeneration.

KEYWORDS: biomechanics; trunk deformation; large deformation theory; simulation; wet snowfall

1. INTRODUCTION

Climate change is expected to increase wet snowfall in regions that continue to receive snow as temperatures rise. Wet snow has a higher water content and adheres to tree crowns more readily than dry snow, so warming increases the snow that accumulates on a crown. In regions that already receive heavy snow, warming does not greatly reduce the total snowfall, but it makes the snow wetter, and heavy wet snowfall is projected to increase, including along the Japan Sea coast of Japan (Kawase et al., 2016; Ohba and Sugimoto, 2020). At our study region, the proportion of wet snow in early winter increased over 2011 to 2019 (Brandt et al., 2020). How this shift toward wetter snow affects trees needs to be evaluated quantitatively.

Most work on snow damage has dealt with mature canopy trees that keep their crowns above the snow all winter (Peltola et al., 1999; Päätalo et al., 1999). Young trees in the understory and subcanopy have received far less attention, although their situation is very different. In heavy-snow regions of Japan the seasonal snowpack can reach 3 to 4 m, yet broadleaf forests grow there, and their young trees are buried within the snowpack for several months each winter (Seki et al., 2005; Miyashita et al., 2020). These snow-buried trees include the saplings of canopy species, and whether they survive the buried stage determines which individuals reach the canopy and shapes forest regeneration (Mori and Komiyama, 2008; Mori et al., 2008).

A trunk breaks when its bending stress exceeds the bending strength of the wood (Kato and Nakatani, 2000; Spatz and Bruechert, 2000). Treating a trunk as a continuous elastic body allows the bending stress to be computed along it, and this approach has been used to assess snow-damage risk (Peltola et al., 1999; Gaffrey and Kniemeyer, 2002). These models, however, were built for trees tall enough to keep their crown above the snow, and cannot describe the load on a trunk that becomes buried.

On a buried trunk, snow acts in two successive phases, which we treat as two load components

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(Yamanoi, 2006). The first is the crown snow load, the snow that adheres to the branches and the crown in early winter; it accumulates at the top of the plant and bends the trunk from its tip. The second is the snowpack pressure load, the force that the surrounding snowpack exerts on the trunk once the plant is buried. The snowpack pressure load is more than the weight of the snow resting on and around the trunk: it also includes the pressure that develops as the snowpack settles and densifies, together with creep and glide, and it builds up gradually over the season (Miyashita et al., 2020). This snowpack pressure load is what the title of this paper refers to as the accumulated snow loading. The bending stress depends not only on the applied load but also on the trunk form at the moment of loading, and this form changes as the trunk deforms, so the loading carries a memory of its history (Gaffrey and Kniemeyer, 2002).

Here we develop a structural-mechanical bending stress model for snow-buried trees that incorporates this load history, or hysteresis, arising from the sequential crown snow and snowpack pressure loading. We validate the model against field measurements on 12 Japanese beech (*Fagus crenata*) and use it to examine how the crown snow load affects the bending stress produced by the later snowpack pressure load.

2. METHODS

2.1 Bending stress model

The trunk was treated as a continuous elastic body and, following large deformation theory, represented as a series of connected elastic elements, each 0.1 m long. For one element, the change in curvature ΔK (m^{-1}) produced by a bending moment M (N m) is

$$\Delta K = M / (E\, I) \qquad (1)$$

and the resulting bending stress σ (Pa) is

$$\sigma = E\, y\, \Delta K \qquad (2)$$

where E is the modulus of elasticity (Pa), I is the second moment of area (m^4), and y is the distance from the neutral axis to the outer edge of the cross-section (m). The crown snow load was modelled as a concentrated load P (N) at the trunk tip, and the snowpack pressure load as a distributed load q along the trunk. Each load was applied in small increments, and after each increment the trunk form was updated so that each increment acted on the form left by the previous one. The crown snow phase was computed first, and the deformed form at its end was carried over as the initial form of the snowpack pressure phase. This carry-over, within each phase and between the two phases, embeds the loading history, or hysteresis, in the trunk form (Fig. 1). The full incremental formulation follows Ishii et al. (2022).

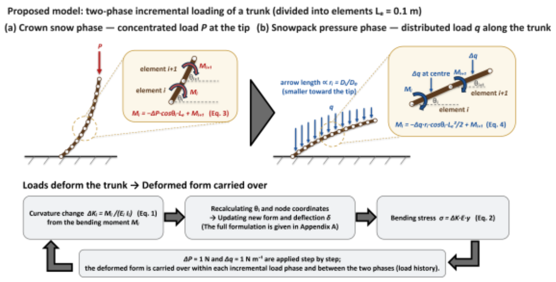

Figure 1: Concept of the bending stress model for a snow-buried trunk. The trunk is divided into connected elastic elements and loaded in two sequential phases: (a) a concentrated crown snow load P at the tip, and (b) a distributed snowpack pressure load q along the trunk. Each load is applied incrementally, and the deformed form is carried over within and between the phases (the loading history).

2.2 Field measurements and validation

The study site was a cool-temperate beech forest in Yamagata, Japan (790 m a.s.l.), where the maximum snow depth reached 4.65 m in the 2024–2025 winter. We selected 12 field-grown *Fagus crenata* (tree height 3.3 to 6.6 m). For each tree we measured the change in trunk form between the snow-covered and summer seasons (the trunk deflection and the inclination angle), the trunk geometry, and the modulus of elasticity, and we described each trunk form with a fitted function.

To validate the model, we compared the modelled trunk deflection and inclination angle with the measured values using the RMSE-to-standard-deviation ratio (RSR), where a lower RSR indicates closer agreement (Moriasi et al., 2015). For each tree we found the combination of P and q that minimised the RSR, and we used the mean of these best-fit loads across the 12 trees (P = 50.42 N, q = 1125 N) as a reference load. The agreement was further assessed by reduced major axis (RMA) regression (Smith, 2009).

2.3 Numerical experiments

Using the measured trees, we varied the crown snow load P (0 to 150 N) and the snowpack pressure load q (0 to 3000 N), and computed the bending stress along the trunk. We took the largest value along the trunk as the maximum bending stress σ_{max}. We averaged the maximum bending moment and the maximum bending stress across the 12 trees and mapped them against P and q. To show the process clearly, we also present the bending stress distribution along the trunk of one representative tree, first during the crown snow phase and then during the snowpack pressure phase, for a range of crown snow loads (P = 0 to 150 N) applied beforehand.

3. RESULTS

The modelled trunk deflection and inclination angle agreed well with the field measurements. For the deflection, the reduced major axis (RMA) regression of modelled on measured values had a slope of 1.06 and r = 0.85; for the trunk lean (tan θ), the slope was 1.00 and r = 1.00. The model therefore reproduced the deformation of a buried trunk through the snow season.

Across the range of loads, both the maximum bending moment and the maximum bending stress, averaged across the 12 trees, increased with the crown snow load P and the snowpack pressure load q, and more steeply with q (Fig. 2). At the reference load the mean maximum bending stress was about 85 MPa, and it rose to about 120 MPa at the largest loads.

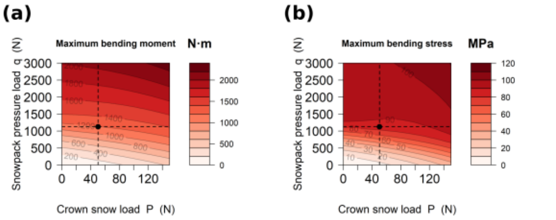

Figure 2: Maximum (a) bending moment (N m) and (b) bending stress (MPa) in the trunk, averaged across the 12 trees, as functions of the crown snow load P (N) and the snowpack pressure load q (N). The dashed lines and the dot mark the reference load (P = 50.42 N, q = 1125 N).

During the crown snow phase, the bending stress stayed small along the whole trunk, reaching at most about 18 MPa even at the largest crown snow load (P = 150 N; Fig. 3a). This is far below the green bending strength of *Fagus crenata*, 57.9 MPa (Miyashita et al., 2020), consistent with the field observation that buried beech rarely break under crown snow alone (Homma, 1997).

During the snowpack pressure phase the bending stress was much larger, and it was concentrated in the lower trunk, peaking about 1 m above the base rather than at the base or the tip (Fig. 3b). The snowpack pressure load was therefore the main source of bending stress. A crown snow load applied beforehand raised the whole stress distribution: at the same snowpack pressure load, the peak bending stress increased from about 40 MPa with no crown snow (P = 0 N) to about 75 MPa with the largest crown snow load (P = 150 N). A crown snow load that by itself added little stress thus amplified the stress produced by the later snowpack pressure.

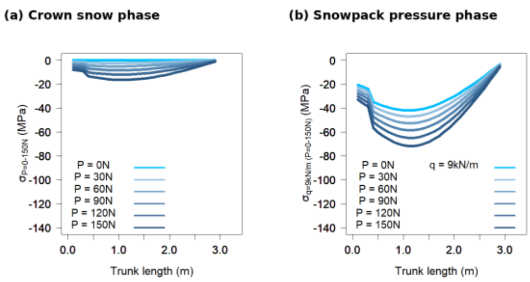

Figure 3: Bending stress distribution along the trunk of one representative tree, shown for crown snow loads of P = 0 to 150 N (light to dark). (a) During the crown snow phase, the stress stays small along the whole trunk. (b) During the snowpack pressure phase (at a snowpack pressure load q = 9 kN m^{-1}, about 900 N in the load units of Fig. 2), the stress is much larger and peaks about 1 m above the base; a heavier crown snow load applied beforehand raises the whole distribution, showing how the crown snow amplifies the snowpack-pressure-driven bending stress.

4. DISCUSSION

Two points emerge. First, the crown snow load bent the trunk but produced little bending stress of its own. Second, the snowpack pressure load produced the bending stress, but the amount depended on the trunk form it acted on. Because the crown snow phase deforms the trunk before the snowpack pressure phase, the crown snow load changes the form on which the later snowpack pressure load acts.

This links the two phases through the trunk form. A heavier crown snow load leaves the trunk more leaning, and the snowpack pressure load then acts on this more leaning form and produces a higher bending stress (Fig. 3b), even though the crown snow load adds little stress directly. The crown snow load thus raises the snowpack-pressure-driven bending stress indirectly, by reshaping the trunk, through the between-phase hysteresis of the loading.

This route matters under a warming climate. Where snow is retained, the total snowfall is expected to change little, but the shift toward wetter snow, which sticks to crowns more readily, is expected to increase the crown snow load (Lehtonen et al., 2016; Ohba and Sugimoto, 2020). A heavier crown snow load could therefore raise the bending stress that buried trees carry during the snowpack pressure phase, and so increase their risk of trunk breakage, even where the snowpack itself does not deepen. This added risk would be missed if only the direct stress from crown snow were considered.

Because snow-buried trees are the next generation of the canopy, a higher breakage risk at this stage would affect which individuals reach the canopy and shape forest regeneration (Mori et al., 2008). The present model treats the trunk as elastic and applies each load as a steady increase, so it does not represent gradual failure or the within-season changes in snow load; coupling it with a weather-driven snow-load model and with a model of gradual failure are natural next steps. Even so, the model shows a clear mechanism: through the trunk form it leaves behind, an increase in crown snow driven by wetter snow indirectly amplifies the snowpack-pressure-driven bending stress in snow-buried trees.

ACKNOWLEDGEMENTS

We thank Ms S. Ogawa, Ms S. Shoji, Mr F. Shoji, Mr J. Yamashita, and Mr K. Terashima for their help with the demanding snow-excavation fieldwork. This work was supported by the Sasakawa Scientific Research Grant from The Japan Science Society (Grant Number 2025-5045).

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