NEW INSIGHTS INTO SNOW AVALANCHE DYNAMICS AND ENTRAINMENT MECHANISMS FROM DEPTH-RESOLVED PARTICLE-BASED SIMULATIONS

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ABSTRACT: In mountainous regions, snow avalanches pose significant threats to both populations and infrastructure. As the flow progresses, it can accumulate additional mass by entraining bed material along its path, thereby amplifying its destructive potential. Volume increases of over tenfold the initial volume have been documented from avalanche observation and measurements. Entrainment is usually accounted for in depth-averaged models through a source term in the mass conservation equation and simplified entrainment criteria. However, quantitative assessment of erosion/entrainment rates with various flow types and bed material properties has seldomly been explored under well-controlled conditions. In this work, we investigate avalanche dynamics as well as erosion/entrainment mechanisms on the basis of depth-resolved particle-based simulations. Through such a modeling approach, processes like frontal ploughing, basal abrasion as well as erosion-deposition waves naturally emerge. Our findings highlight that, the interaction of the flow with the initially static bed material may have multiple and diverse effects on the flow behavior. First, the bed material can be eroded, but only partially entrained in the flow. Second, while the flow mobility is generally reduced by the presence of the bed, it may be enhanced in some cases due to the mechanical weakening of the bed. Finally, we propose a relationship linking the entrainment rate to a dimensionless parameter that compares the flow-induced stress to the bed shear resistance. This work contributes to a better understanding of the dynamics of snow avalanches and entrainment process and can lead to a refinement of depth-averaged formulations for practical purposes. Future research should consider factors such as rate-dependent snow rheology, air pore pressure and bed fluidization mechanisms to further refine our understanding and predictive capabilities in avalanche modeling.

Keywords: avalanche dynamics, erosion, entrainment, modeling, Material Point Method, Discrete Element Method.

1. INTRODUCTION

Mass movements grow in size by entraining bed material along their path, significantly increasing their destructive potential. Snow avalanches, for instance, can expand more than 10 times their initial volume due to snow cover entrainment (Sovilla et al., 2006). The propensity for entrainment depends on flow dynamics and snow cover properties. In particular, field measurements indicate that snow temperature and cohesion influence flow dynamics and runout distances (Köhler et al., 2018a; Vera Valero et al., 2015; Köhler et al., 2018b). Observations show that avalanches with higher mass entrainment may exhibit greater kinetic energy and longer runout distances (Sovilla et al., 2006). Due to its importance in avalanche dynamics, various entrainment formulations have therefore been proposed for depth-averaged numerical models (Norem and Schieldrop, 1991; Naaim et al., 2004; Issler and Pastor Pérez, 2011; Eglit and Demidov, 2005). However, most operational models do not account for entrainment or do not explicitly consider snow properties in affecting entrainment and flow dynamics (Nishimura et al., 2021).

Due to climate change, the importance of cohesion in snow avalanche dynamics is becoming increasingly significant, leading to a rise in wet avalanches and flow regime transitions (Eckert et al., 2024; Castebrunet et al., 2014; Naaim et al., 2016). Cohesion may also play a significant role in the entrainment processes (Li et al., 2022).

Similarly, in cohesionless granular flows (such as cold dry snow avalanches), entrainment may increase flow mobility and runout distance, influenced by erodible layer depth and slope angle (Mangeney et al., 2007; Mangeney et al., 2010; Viroulet et al., 2019; Edwards et al., 2021). Furthermore, entrainment of fresh and weak snow covers may possibly be aided by pore air pressure generation (Gauer and Issler, 2004; Louge et al., 2011; Issler, 2017) and fail-

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ure of weak snow layers (Gaume et al., 2019; Köhler et al., 2018). However, the influence of snow cover properties on these trends remains unclear, particularly in characterizing entrainment mechanisms and rates.

The aforementioned entrainment processes may also appear concomitantly with mass reduction through deposition in the flow tail (Sovilla et al., 2010). All these processes are significantly affected by the mechanical properties of the snow cover: Frontal ploughing occurs with low-strength snow covers (dry snow) or in cases with large flow heights (wet snow), while stronger beds favor basal abrasion with lower entrainment rates (Sovilla et al., 2006; Issler, 2014; Ligneau et al., 2024b). Unlike field experiments and depth-averaged models, 3D depth-resolved simulations offer detailed insights into entrainment processes (Gaume et al., 2023) by explicitly considering the mechanical properties of the snow avalanche and snow cover. These simulations allow us to quantify interactions between flow rheology, snow cover properties, and avalanche mobility, providing a comprehensive understanding through repeated simulations with varied parameters. In this contribution, we describe recent numerical experiments aimed at better understanding the erosion and entrainment processes, their magnitude and influence on snow avalanche mobility.

2. METHODS

In this paper, we describe and discuss results obtained from two different depth-resolved numerical methods: i) the Material Point Method (MPM) and ii) the Discrete Element Method (DEM). Details about the MPM model can be found in Gaume et al., 2019; Li et al., 2021; Kyburz et al., 2024. Details about the DEM model can be found in Ligneau et al., 2022; Ligneau et al., 2024b. We recall here the main characteristics of these models:

2.1 MPM

MPM is an Eulerian-Lagrangian particle-based numerical technique for solving partial differential equations in continuum mechanics. Introduced by Sulsky and colleagues in 1994 (Sulsky et al., 1994) as an extension of the Particle-In-Cell method, MPM has primarily been developed and applied in the geomechanics and graphics communities. It effectively addresses problems involving large deformations, collisions, and fractures, and naturally handles free-surface flows, which require special treatment in mesh-based methods like the finite volume or finite element method. Recently, MPM has gained increasing attention in snow and avalanche science. Our modeling framework is based on finite strain elastoplasticity and employs a constitutive snow model derived from critical state soil mechanics (Schofield and Wroth, 1968). Specifically, we use the Cohesive Cam Clay model (Gaume et al., 2018), which incorporates elastic parameters (Young's modulus and Poisson's ratio) and three parameters defining the yield surface (failure envelope): cohesion β, friction M, and compressive strength p_0. Additionally, it includes a parameter ξ for characterizing the hardening law.

We present here MPM results obtained using i) an idealized concave topography with a release zone and a finite-length erodible bed and ii) real topography fully covered with an erodible snow cover. More details about the setup can be found in Li et al., 2022 and Kyburz et al., 2024.

2.2 DEM

DEM is a powerful tool for studying the mechanics of granular matter in complex systems. DEM simulations act as non-invasive numerical experiments, allowing independent variation of parameters to evaluate their impact on quantities difficult to measure experimentally, such as the magnitude and orientation of contact forces. When optimized, DEM can simulate interactions among millions of particles, providing detailed insights into how complexity arises from the collective behavior of individual particles governed by simple contact laws. In snow modeling, DEM has been used for various applications, including snowflake fragmentation (Comola et al., 2017), wind-driven snow transport (Comola et al., 2019), and snow failure (Hagenmuller et al., 2015; Mede et al., 2018; Gaume et al., 2017). It has also been employed to simulate avalanche release processes, such as crack propagation (Gaume et al., 2015; Bobillier et al., 2021), and the impact of snow avalanches on obstacles (Kyburz et al., 2020; Kyburz et al., 2022a; Kyburz et al., 2022b), typically using a bond contact model to account for cohesion. This method is particularly well-suited for simulating very porous assemblies like snow.

We present here DEM results obtained using a standard parallel-bond contact model combined with an aggregation force threshold to allow for new bonds to be formed during the simulation. A generic topography consisting of three main sections is used: (A) a slope with a constant angle θ, which includes the release zone and the main flowing zone; (B) a parabolic slope that smoothly transitions from the first section to the run-out zone; (C) a flat run-out zone. The whole slope is covered with a snow cover of height h_{bed} representing the erodible bed. The avalanche starts with the release of a finite portion of the slope located in the upper part.

3. RESULTS

3.1 MPM simulations

In the idealized concave slope setup, the mechanical properties of both the release zone and the erodible snowpack were varied. Figure 1a illustrates the erosion patterns for different mechanical properties of

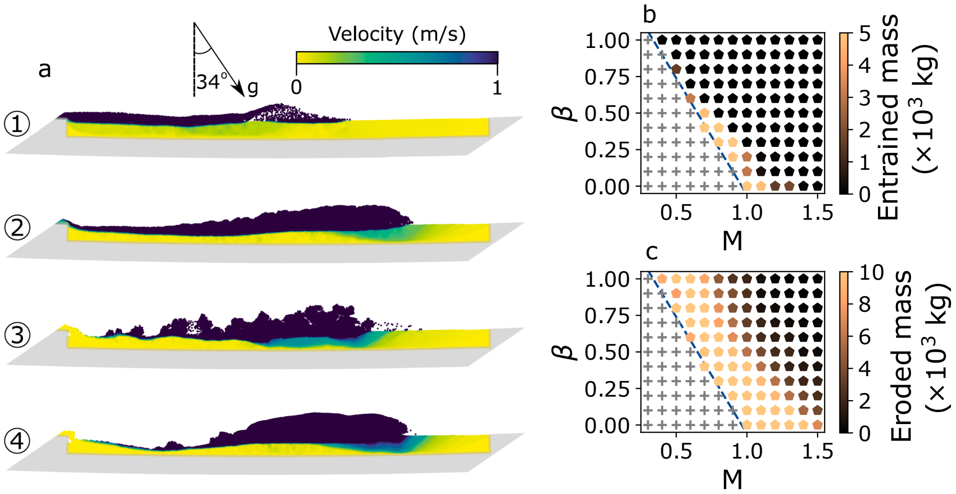

Figure 1: (a) Frontal erosion and basal abrasion observed in the simulated scenarios. The average slope angle of the erodible zone is 34°. (b) Effects of cohesion β and friction M on the (a) entrained and (b) eroded mass in scenario 2. Reprinted after Li et al., 2022.

the released snow while maintaining constant properties for the erodible bed which are chosen to simulate a metastable snowpack (see below and Li et al., 2022 for parameter values used). Scenario 1 depicts a cohesionless released snow. Scenarios 1 to 3 represent cases with brittle snow, where the compressive strength, cohesion, and friction of the released snow increase from Scenario 1 to Scenario 3 within a realistic range (Mellor, 1974). Scenario 4 is a distinct case featuring more ductile snow in the release zone, which allows for greater plastic compaction compared to the previous scenarios.

In the case of cohesionless released snow (Scenario 1), we observe a highly dilute front characterized by multiple surges and waves, which are induced by the interaction between the cohesionless flow and the cohesive bed. As the cohesion and friction of the released snow increase (Scenario 2), the front becomes denser. With a significantly higher initial strength of the released snow (Scenario 3), we observe fragmentation and granulation mechanisms. In the Scenario 4, involving a more ductile and compressible released snow, a plug flow develops. Despite the variations in snow properties, all scenarios exhibit similar erosion patterns, including both frontal ploughing and basal abrasion. Scenarios 2 and 4 result in greater snow entrainment, likely because these flows develop higher flow heights and, consequently, exert greater stresses on the bed, leading to more efficient breakdown and entrainment of the bed material into the flow.

To examine how bed properties influence erosion and entrainment propensity in dense snow avalanches, we selected release Scenario 2, and simulated varying bed cohesion and friction values. For the analysis, we distinguish between erosion and entrainment based on the definition proposed by Gauer and Issler, 2004. Basically, snow particle within the erodible bed is considered eroded if its displacement from the initial position exceeds 0.5 meters. This threshold of 0.5 meters is based on the grid size of 0.1 meters used in the MPM simulation. Although variations in this threshold (such as 0.4 meters or 0.6 meters) affect erosion rates, they do not alter the overall trends and conclusions. In addition to the displacement criterion, a snow particle is classified as entrained if its final position is outside the boundaries of the erodible bed. Conversely, a particle initially within the avalanche is classified as deposited if it remains inside the erodible bed with a velocity less than 0.1 meters per second.

MPM simulations with varying bed cohesion and friction reveal three distinct regimes:

- Unstable Regime: Occurs when cohesion and/or friction are too low, leading to premature flow of the bed before the incoming avalanche arrives.
- Stable Regime: Characterized by minimal erosion and entrainment and significant deposition when cohesion and/or friction values are high.
- Metastable Regime: Features potential erosion and entrainment (and possibly deposition), where the bed is in a transitional state and can respond to incoming avalanches with varying degrees of bed entrainment.

The snow cover can undergo significant erosion but may only be partially entrained into the flow. In scenarios of maximum erosion, the mass of eroded snow is typically about twice the mass of the entrained snow. Note that this ratio may depend on the specific setup used and dimensionality of the problem. Simulations indicate that entrainment at and near the avalanche front is more pronounced than basal abrasion, consistent with findings from field experiments (Sovilla et al., 2006).

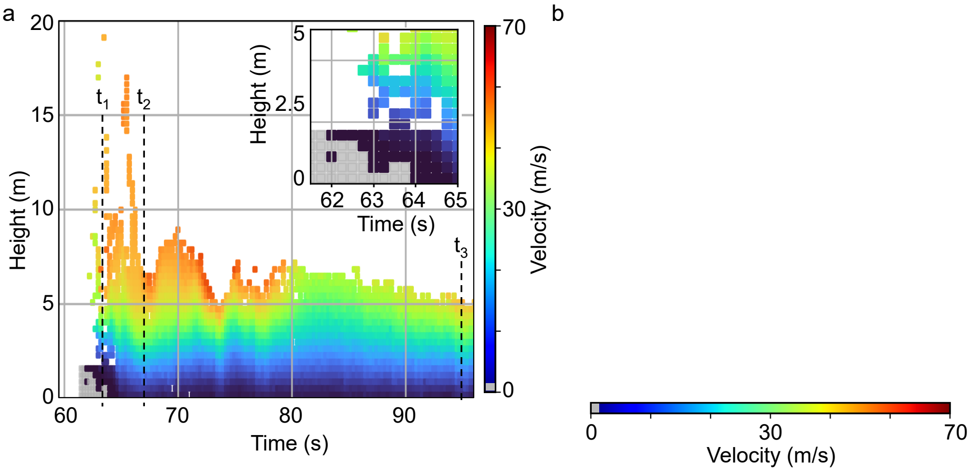

Figure 2: MPM simulations of the Salezer avalanche, in Davos. Analysis of the simulated avalanche front flow behavior at a fixed location. (a) Temporal evolution of flow velocity at a specific location, plotted against flow height. The inset provides a close-up view of the data at the flow front. (b) Visualization of the avalanche front at the location where the velocity data in panel (a) was extracted. Reprinted after Kyburz et al., 2024.

Based on the analysis of momentum changes between the inlet and outlet of the erodible zone, these MPM simulations provide insights into how snow entrainment influences avalanche mobility. Our findings indicate that the presence of an erodible layer can result in either positive or negative momentum changes, corresponding to increased or decreased avalanche mobility, respectively. Two competing processes govern this behavior: the entrainment of snow requires the avalanche to fracture the snow cover and accelerate the resulting fragments to match the avalanche's velocity, which consumes energy and can lead to a momentum loss. Conversely, the avalanche can gain momentum through the conversion of the potential energy of the entrained snow into kinetic energy and/or because of small bed friction values if shearing is concentrated in weakened just-entrained snow. Our simulations demonstrate that only scenarios with very low bed cohesion and friction within the metastable regime result in an overall increase in momentum. In contrast, cases with high bed cohesion and friction often lead to the arrest of the avalanche within the erodible layer. This outcome highlights a limitation of the MPM numerical method, where particles interact through the background grid rather than through direct contact as in DEM, potentially leading to an unphysical "sticky" behavior.

The same depth-resolved modeling approach was subsequently applied to complex topography in 2D, using the thalweg slope of the Swiss avalanche test site at Vallée de la Sionne (Li et al., 2024), and in 3D to replicate the 2019 Salezer avalanche in Davos (Kyburz et al., 2024) and the 2019 Flüela Wisshorn rock-snow avalanche (Cicoira et al., 2022). Unlike previous mesoscale analyses with a finite erodible zone, these studies covered the entire slope topography with a metastable erodible snowpack. This approach naturally gave rise to complex and intermittent flow dynamics, including secondary releases, roll waves, erosion-deposition waves, and slope-normal dispersive effects (Bagnold, 1954). Some of these effects are illustrated in Fig. 2 and discussed in Kyburz et al., 2024. Specifically, slope-normal velocities exceeding ±5 m/s are observed particularly near the avalanche front, with these bursts closely correlated to local topographical variations. Specifically, we observe that numerous clusters of snow particles are ejected from the dense basal layer and remain suspended above the dense flow for several seconds, despite the absence of simulated turbulent interactions between the snow particles and air. We speculate that these flow structures, often referred to as mesoscale coherent structures (Sovilla et al., 2018) are triggered by sudden changes in the topography of the gully, where the avalanche, carrying high kinetic energy, interacts with the terrain.

3.2 DEM simulations

In view of gaining a micro-mechanical perspective on entrainment mechanisms and incidentally addressing the MPM-related stickiness issue evidenced above, the discrete element method (DEM) is used to simulate entrainment on a generic topography.
First, two different entrainment processes emerged in the simulations: frontal ploughing and basal erosion (Fig. 3). This was quantified by means of the distance *d* between the flow front and the location where the entire bed is eroded (Fig. 3b). Ploughing corresponds to positive values of *d* and is typically associated with very low bed bond strength values. Basal abrasion is associated with negative *d* values and occurs mainly for highly cohesive snowpacks. In addition, the entrainment velocity was quantified and was shown (Figs. 3c and 3d) to be in the range 0.5 m/s (basal abrasion, high cohesion) to 3 m/s (frontal

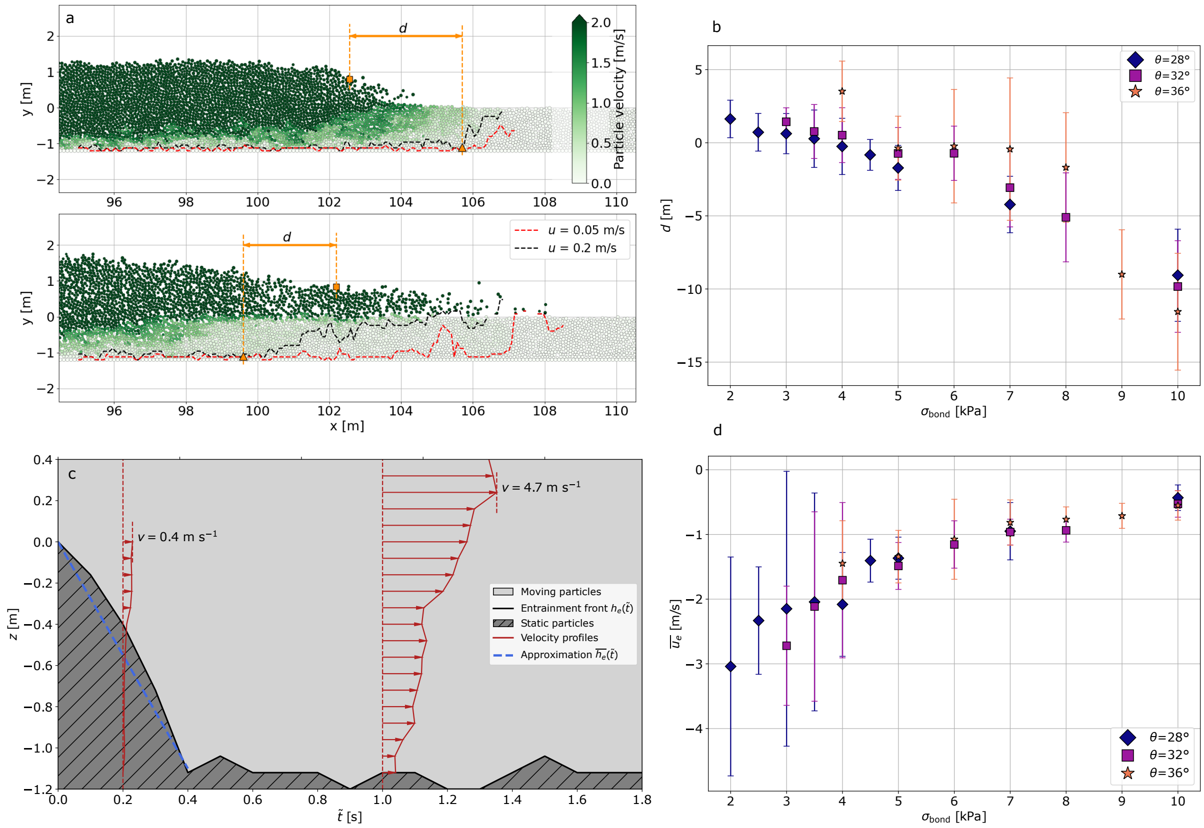

Figure 3: DEM simulations in cases with ploughing and basal abrasion. (a) Detail of the particles at the flow front during steady state. The panels depicts two case with two different entrainment processes: ploughing (top) and abrasion (bottom). The colormap represents particle velocities u (up to 2 m/s). The red dashed line shows the limit where u=0.05 m/s and the black dashed line indicates the entrainment limit u=0.2 m/s. The orange squares and triangles display the positions of the flow front and the position where the whole depth of the erodible bed is entrained, respectively. (b) Horizontal distance d between the flow front and the position where the whole depth of the erodible bed is entrained versus bond strength, for different slope angles. (c) Location of the entrainment front (black line) in time, separating the entrained particles from the static particles. The x-axis corresponds to the time after the first layer of particles is entrained. Two velocity profiles are shown for $\tilde{t}$ = 0.2 s and 1.0 s, and their maximum velocity in the visible range is annotated. The approximation $\bar{h}_e(\tilde{t})$ of the height of the entrainment front, used to compute the entrainment velocity $\bar{u}_e$, is shown with the dashed blue line. This figure is inspired by a figure in Issler and Pérez, 2011. (d) Entrainment velocity $\bar{u}_e$ versus bond strength, for different slope angles. Reprinted after Ligneau et al., 2024b.

ploughing, low cohesion) which is in line with indirect observations made from avalanche back-analysis (Issler and Pérez, 2011; Gauer and Issler, 2004; Sovilla et al., 2006; Issler et al., 2020).

Furthermore, a sensitivity analysis was performed to determine the mechanical properties of the snow leading to a sustained granular flow: in line with the results from MPM simulations, three regimes were highlighted: unstable snowpack, metastable snowpack with significant entrainment, and stable snowpack (without the stickiness issue hindering the avalanche mobility). In addition, different flow characteristics were analyzed and compared, including the avalanche front velocity and the entrainment rate (Fig. 4). This is performed for different erodible bed heights (Fig. 4b), different bond strength values in the erodible bed (Fig. 4c), different aggregation thresholds (not shown) and multiple combinations of these parameters. The simulations show that cohesion and the ability to form new bonds significantly affect avalanche mobility. In general, cohesion negatively affects avalanche mobility with lower front velocities, runout and entrained mass. In addition, cases in which aggregation is switched on show significantly less mobility compared to cases in which only bond fragmentation is allowed. With full depth entrainment, we show that the entrainment rate is directly proportional to the front velocity and erodible bed height. Cases with low snowpack cohesion (representative of dry snow) and/or large erodible bed height show an accelerating flow. In contrast, cases with large values of cohesion (representative of wet snow) and/or low erodible bed height show a deceleration of the avalanche.

4. DISCUSSION AND CONCLUSIONS

Based on the presented results, depth-resolved particle-based simulations like MPM and DEM demonstrate significant potential for enhancing our understanding of erosion and entrainment mechanisms, as well as for informing the development of

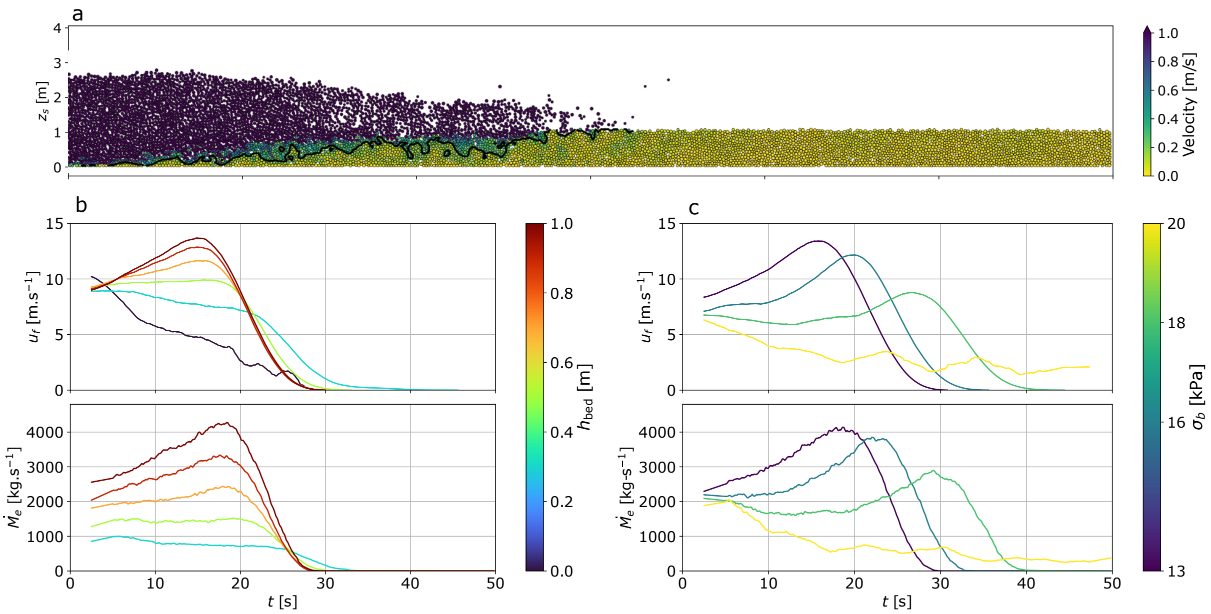

Figure 4: DEM simulations over idealized topography. (a) Particle velocity with color scale based on a maximum velocity of 1 m/s. (b) Time evolution of the front velocity u_f and entrainment rate $\dot{M}_e$ for different erodible heights h_{bed} and (c) for different erodible bed bond strength values. Reprinted after Ligneau et al., 2024a

accurate theoretical entrainment models for depth-averaged approaches.

Regarding MPM simulations, given the level of physical detail captured in the results, particularly the transient flow structures at the avalanche front, we conclude that the model demonstrates significant potential for both back analyses and prediction of snow avalanches and other types of geophysical mass flows. It is particularly well-suited for identifying critical impact pressure peaks caused by these transient flow structures (Kohler et al., 2024). Additionally, the model offers valuable insights into dynamic flow features that are challenging to measure directly in the field. As climate change alters the frequency and characteristics of snow avalanches, physics-based modeling approaches like the MPM model presented here will become increasingly important for hazard assessment. While models calibrated with historical data remain valuable, they may struggle to accurately reflect the evolving physical processes driven by a warming climate (Kyburz et al., 2024). On the other hand, while models like DEM may not match MPM in computational efficiency for large-scale simulations, DEM offers unparalleled micro-mechanical insights into complex processes, emerging from simple contact laws and the collective behavior of bonded and unbonded particles.

Yet, further research is clearly needed to consolidate and generalize these findings but also to transfer them to real snow avalanches in which many properties can be inter-related and can vary through the snowpack, along the flow path due to processes which are not taken into account in these simple simulations such as frictional heating, formation of lubricated layers, temperature variations along the flow path, etc.

In MPM, the stickiness issue hinders direct comparison with DEM analyses, particularly for strong beds. More broadly, varying setups and mechanical properties across different studies complicate direct comparisons and the generalization of results. This can lead to potential misinterpretations. For instance, Figure 1a suggests that the fastest flow (Scenario 1) results in the lowest entrainment, which seemingly contradicts the DEM simulations that indicate a proportional relationship between entrainment rate and flow velocity. However, it's important to consider several key points: first, the linear relationship $\dot{M}_e \approx \rho_{\rm bed} u_f h_{\rm bed} w$ suggested by DEM simulations is valid only when the entire erodible bed is entrained ($w = 1$ m is the flow width). In contrast, MPM simulations in Figure 1a show only partial bed entrainment. In the case of partial entrainment, (Ligneau et al., 2024a) also showed the strong influence of the flow induced stress τ and bed shear strength τ_p. Second, Scenario 1 in the MPM simulations (Fig. 1a) also features the lowest flow height, which would naturally result in a lower τ-value and thus lower entrainment rate. Third, in the case of dry snow avalanches—potentially analogous to Scenario 1 in the MPM simulations—the flow front is highly dilute, which can also negatively influence entrainment.

Future research should focus on investigating the discrepancy between the entrainment relationship suggested by DEM simulations—consistent with some previously proposed models (Eglit, 1983; Sovilla et al., 2007; Christen et al., 2010)—and theoretical tangential jump entrainment models, which propose that the entrainment rate is proportional to the difference between flow-induced shear stress τ and bed shear strength τ_p but inversely proportional to flow velocity (Issler et al., 2024). Although this could tentatively be explained in the case of a fast inertial flow in which $\tau \sim \rho u^2 \gg \tau_p$, this deserves further clarification. Additionally, the interaction between slope-normal flow characteristics, entrainment, and wave phenomena warrants deeper exploration. Future studies should also examine the role of bed porosity in 3D, assess the impact of interstitial air on flow mobility using methods such as CFD-DEM or CFD-MPM modeling (Vicari et al., 2024), and explore the concept of lubrication in slow wet snow avalanches and its effect on entrainment and avalanche mobility.

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