

MERGING DECISION STRATEGIES FOR BACKCOUNTRY SKIERS IN AVALANCHE TERRAIN USING FUZZY LOGIC AND IMPRECISE PROBABILITY THEORY

Christian Pfeifer^{1,*}, Thomas Fetz²

¹Department of Statistics, University of Innsbruck, Innsbruck, Austria

²Institute of Basic Sciences in Engineering Science Unit for Engineering Mathematics, University of Innsbruck, Innsbruck, Austria

ABSTRACT: This paper presents two methods within the fuzzy or subjective probability framework in order to merge different decision strategies (particularly: Snowcard, Munter and our own empirical approach).

Keywords: Avalanche danger, decision strategy, fuzzy.

1. INTRODUCTION

In the Alps, most fatal snow avalanche accidents are caused by skiers or snowboarders. As a consequence several decision strategies for backcountry skiers have been established in the last two decades in order to prevent avalanche accidents or avalanche fatalities. The most important among them are: Munter's method of reduction, Snowcard method (DAV), Stop or Go (OeAV) and an empirically driven decision strategy (Plattner (2001), Pfeifer (2009)) where the fundamental input variables are x = "danger level of avalanche", y = "incline of the slope". However, the skier's or expert's perception of danger level, incline/aspect of the slope and the strategies itself are more or less subjective and imprecise (or fuzzy which for instance was taken into account in a recent paper using a rule-based fuzzy approach (Pourraz et al. (2017))). Our goal is to put these strategies finally together embedding them in an imprecise probabilistic framework.

In a first approach we apply a rule-based fuzzification to each original decision strategy similar to Pfeifer and Fetz (2015). With the aid of a training data set as a result of different strategies we calculate an all over fuzzy system using the adaptive Neuro-Fuzzy algorithm ANFIS (Jang (1993)). In a second approach we use limit state functions $g(x,y)$ which are less or equal to zero in cases where backcountry skiers trigger an avalanche and positive otherwise.

2. RULE-BASED FUZZY APPROACH

If we consider the Snowcard approach, see the e.g. the case with unfavorable exposition in Figure 1,

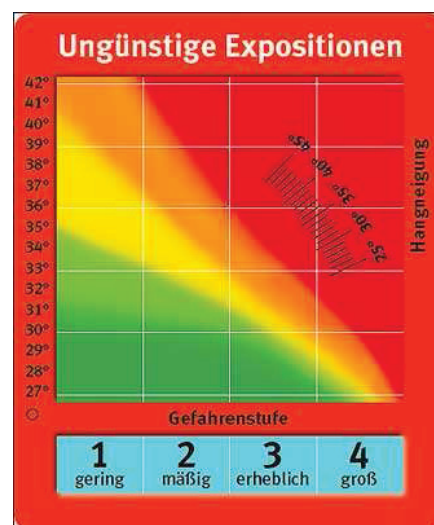


Figure 1: Snowcard North

the decision strategies could be seen as decision Matrix with green (go) yellow (be cautious) and red (stop) areas.

At first, we define within the fuzzy framework triangular membership functions for danger level (low (1), moderate (2), considerable(3), high (4); extreme (5) not considered) – which could be read,

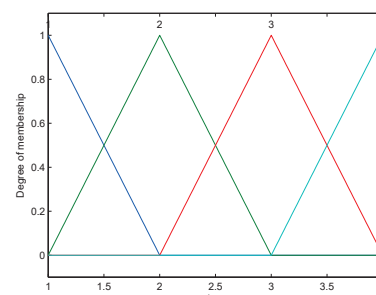


Figure 2: Membership functions LWS

for example, in case of "Considerable" as sure if the

*Corresponding author address:
Christian Pfeifer, Department of Statistics, University of Innsbruck,
6020 Innsbruck Austria;
email: christian.pfeifer2@chello.at

value is equal to 3, but it could also be able for some probability that "Considerable" is smaller than 3 (e.g. 2.5) due to skier's misinterpretation or underestimation – and trapezoidal membership functions for incline of the slope (flat (< 30), moderate (30-33), steep (33-36), very steep(36-39), extreme (> 39))

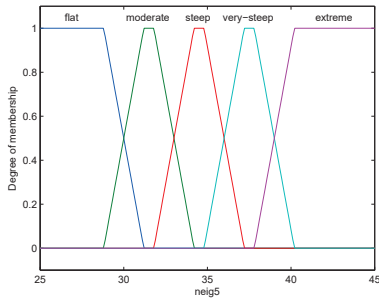


Figure 3: Membership functions Neig

In a next step we do a fuzzification with fuzzification rules such as:

**If Neig5 is flat and lws is 1 (low)
then decision is Go!**

with the help of fuzzy toolbox available in the numerical computing package MatLab using the Sugeno fuzzy model or Sugeno fuzzy inference system (step 1: fuzzify inputs; step 2: apply fuzzy operations, step 3: apply implication method). About 20 rules are defined in case of Snowcard, unfavorable exposition.

We further carry on to defuzzify the fuzzy system resulting in the surface plot such as:

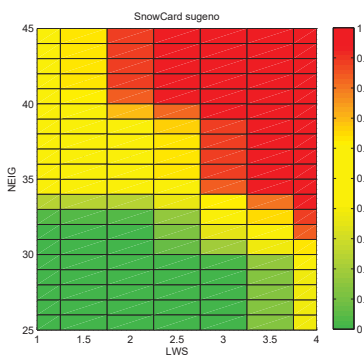


Figure 4: Surface Snocard

2.1. Merging the different decision strategies

After fuzzifying (and defuzzifying) further decision strategies (Plattner (2001), Pfeifer (2009)) we generate a training data set as a result of the different decision strategies for the Neuro-Fuzzy algorithm

ANFIS (Jang (1993)) calculating an all over fuzzy system.

The result of defuzzification in our case is (see Figure 5):

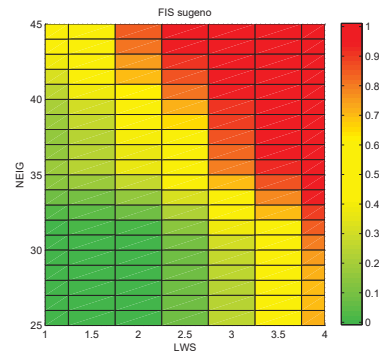


Figure 5: Surface Merging

3. THE LIMIT STATE APPROACH

We denote $g(x = (x_1, x_2))$ as limit state function (x_1 danger level and x_2 incline of slope, for instance) or $h(x, z)$ as parameterized limit state function according to an additional parameter z (Fetz (2012)). Values of $g(x)$, $h(x, z)$ mean failure if $g(x)$, $h(x, z) \leq 0$!

Furthermore, the probability of failure is:

$$p_f = \int_{\mathcal{X}} \int_{\mathcal{Z}} \underbrace{\chi(h(x, z) \leq 0)}_{\text{indicator function}} f^{Z|x}(z) d(z) f^X(x) d(x)$$

$$= \int_{\mathcal{X}} \underbrace{\int_{\mathcal{Z}} \chi(h(x, z) \leq 0) dP_{Z|x}(z)}_{=q(x)} \overbrace{dP_X(x)}^{\text{prob. measure}}$$

where χ denotes the indicator function, $f^{Z|x}(z)$, $dP_{Z|x}(z)$ the conditional probability measure or density relating to z and $f^X(x)$, $dP_X(x)$ the probability measure or density relating to x .

In our case we consider the set Q

$$Q = \left\{ \begin{array}{c} q_S \\ \text{SnowCard} \end{array}, \begin{array}{c} q_M \\ \text{Munter} \end{array}, \begin{array}{c} q_E \\ \text{empirical} \end{array} \right\}$$

of imprecise failure regions ('unsafe regions'), where q_S , q_M and q_E is referring to the Snowcard, Munter and empirical approach,

and the set of probability measures $\mathcal{M}_X$

Using this sets of probability measures and imprecise failure regions, we are finally able to calculate specific probabilities according to:

$$\begin{aligned}\bar{p}_f &= \sup_{\substack{P_X \in \mathcal{M}_X \\ q \in Q}} \int_{\mathcal{X}} q(x) dP_X(x) \\ &= \sup_{P_X \in \mathcal{M}_X} \int_{\mathcal{X}} \bar{q}(x) dP_X(x)\end{aligned}$$

where $\bar{q}$ denotes the upper envelope of the set Q of uncertain failure regions; $\bar{q}$ can be seen as membership function of the union of the fuzzy sets described by all $q \in Q$.

3.1. Example

Uncertainty of $x = (x_1, x_2)$ is modelled by a set A such as (see Figure 6):

- level of danger ‘moderate’ means $x_1 \in [1.5, 2.5]$
- incline of slope $\leq 34^\circ$ means $x_2 \in [0^\circ, 34^\circ]$

which results in a set of Dirac probability measures

$$\begin{aligned}\mathcal{M}_X &= \{\delta_a : a \in A\} \\ A &= [1.5, 2.5] \times [0, 34]\end{aligned}$$

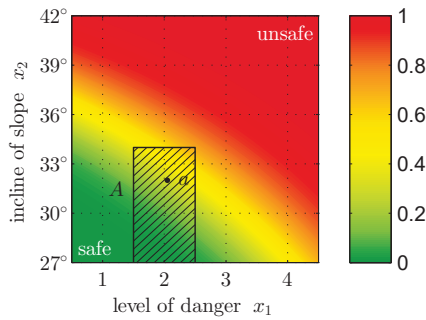


Figure 6: Example limit state approach

Further calculation leads to:

$$\begin{aligned}\bar{p}_f &= \sup_{P_X \in \mathcal{M}_X} \int_{\mathcal{X}} q(x) dP_X(x) = \max_{a \in A} \int_{\mathcal{X}} q(x) d\delta_a(x) \\ &= \max_{a \in A} q(a) = 0.72\end{aligned}$$

REFERENCES

- Fetz, T. (2012). Modelling uncertainties in limit state functions. *Int. J. of Approximate Reasoning*, 53(1), 1–23.
- Jang, J.-S.R. (1993). ANFIS: adaptive-network-based fuzzy inference system. *IEEE Transactions on Systems, Man and Cybernetics*. 23(3).
- Pfeifer, C. (2009). On probabilities of avalanches triggered by alpine skiers. An empirically driven decision strategy for backcountry skiers based on these probabilities. *Natural Hazards*, 48(3), 425–438.
- Pfeifer, C. and Fetz, T. (2015). Decision strategies for backcountry skiers in avalanche terrain using fuzzy logic and imprecise probability theory. *AUSHUN15 Joint Austrian-Hungarian Mathematical Conference Győr, Hungary, AUG 25–27*.

- Plattner, P (2001). Werner Munter's Tafelrunde. Strategische Lawinenkunde auf dem Prüfstand. *Berg&Steigen* 4/2001
- Pourraz, F. Verjus, H. and Mauris, G. (2017). Visual Analytics for Aiding Decisions of Ski Touring Itinerary in a Risky Snow Avalanche Environment. *IEEE International Conference of Computational Intelligence and Virtual Environments for Measurement Systems and Applications Annecy, France, JUN 28–28*.