BIVARIATE SPATIAL MODELING OF SNOW DEPTH AND SNOW WATER EQUIVALENT EXTREMES IN AUSTRIA

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ABSTRACT: In many applications like hydrological studies focusing on potential runoff, or climate studies investigating water availability, a spatial representation of snow depth (HS) and snow water equivalent (SWE) and their extremes is of great interest. In Austria, although many locations with long-term HS measurements exist, SWE observations are very rare and mostly short. To provide a spatial extreme value model for SWE in Austria, firstly SWE is modeled stationwise from long-term HS observations, employing a newly developed snow layer model, which derives SWE solely from daily HS, without any other meteorological input. Secondly, with a model selection procedure suitable covariates are selected, for modeling the margins of the GEV distribution. Among mean SWE and topographical parameters, a hidden property of the snowpack, namely the time difference between the occurrence dates of the maximum SWE and HS within a winter season is used. Then, different bivariate max-stable processes (Hüsler-Reiss, Extremal-Gaussian, Extremal-t) are fitted to Austrian HS and modeled SWE data. As expected, the bivariate Extremal-t max-stable process remains as the most suitable model, allowing for the estimation of conditional return levels and the use in risk analysis due to its spatial extremal dependence structure. First validation results consolidate the bivariate approach against a smooth model and a univariate Extremal-t max-stable model.

Keywords: Bivariate model, spatial extremes, snow depth, snow water equivalent, conditional return level, snow load, max-stable process.

1. INTRODUCTION

Along with snow depth (HS), snow water equivalent (SWE), the amount of water contained in the snowpack, or it's weight, is one of crucial snow properties used e.g. by Hydrologists for water supply forecasting or in construction business for the estimation of snow load on roofs. As the assessment of extreme flood events or the fundamental idea in worldwide building codes rely on the concept of SWE values with a certain exceedance probability, a spatial representation of SWE (or snow load) extremes is of great benefit.

Relying on Extreme value theory (Coles, 2001), extremes at locations with reasonable data series are relatively easy to estimate locally. The assessment of return levels for an arbitrary point then relies on a more or less notional interpolation of the locally estimated extreme values to the location of interest. An intuitive way to bring local estimates of extremes into space would be a spatial interpolation. Unfortunately, this approach has some disadvantages, as uncertainties may be hard to assess and quantiles for more complex (joint) events cannot be mapped at all (Blanchet and Lehning, 2010). Despite those constraints kriging variants were used to interpolate snow load extremes for hazard mapping in Canada (Hong and Ye, 2014) and China (Mo et al., 2016). As an improvement, Blanchet and Lehning (2010) suggested a direct estimation of a spatially smooth generalized extreme value (GEV) distribution, called *smooth spatial modeling*. Unfortunately, smooth modeling does not provide any spatial dependence of the extremes. To naturally account for spatial dependencies, *max-stable processes* can be used (de Haan, 1984). With max-stable processes, the margins and their spatial dependence can be modeled simultaneously but independently. In a few studys, max-stable processes have been used to model extremes of snowfall (Blanchet and Davison, 2011; Gaume et al., 2013; Nicolet et al., 2015). In all of them more flexible max-stable processes, like Brown-Resnick (Brown and Resnick, 1977) or Etremal-t (Opitz, 2013) achieved more accurate return levels. For a better understanding and an improvement in application, Genton et al. (2015) used *multivariate* max-stable processes, to investigate daily maximum wind speed and wind gust simultaneously.

As HS and SWE are highly correlated, spatial modeling of extreme SWE would be strongly supported

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by the information buried in long HS series. The aim of this study is therefore to investigate spatial extremes of SWE and HS by means of multivariate max-stable processes.

2. METHODOLOGY

Unfortunately long-term measurements of SWE needed for extreme value analysis are very scarce in Austria, while, on the contrary, many long-term observations of daily HS exist. Therefore, firstly we use a newly developed semi-empirical snow density layer model to derive historical SWE solely from daily HS measurements (Winkler and Schellander, 2018). After that step, daily HS and SWE data series are available for a large number of stations in Austria. Secondly, by means of an AIC-based (Akaike, 1974) model selection procedure, marginal models for the GEV parameters μ, σ and ξ are selected out of some reasonable covariates. Then, different multivariate max-stable processes using the fixed marginal models from the previous step are fitted to Austrian HS and modeled SWE data. Fitting was accomplished in R (R Development Core Team, 2008) using the unpublished package "SpatialExtremesZAMG" (Gstöhl, 2017).

3. RESULTS

3.1. Data and Model Selection

In this study 362 Austrian stations of ZAMG and the Hydrological Service each comprising 43 common years between 1970 and 2012 with daily HS and SWE values were used for model fitting. The variables longitude, latitude, mean SWE were used as covariates to fit the max-stable models. In addition, a hidden relation between the occurrence times of the maximum HS and the maximum SWE at a location within the same season was used (*dday* henceforth). As the most difficult GEV parameter for modeling is the shape parameter ξ, additional variables, describing its spatial characteristics would be beneficial as covariate. Typically, the SWE maximum occurs within one month *after* HS reached its maximum value, although at few locations (lowlands, wet snow) it can be vice versa. The shape parameter significantly decreases from positive to negative values, with increasing difference between the SWE and HS maxima (Figure 1).

Using dday as a covariate as well adds significant value to the models of the GEV parameters, in that not only the altitudinal correlation is satisfied, but also the typical time difference between the occurrence of maximum SWE and HS is pertained. Model selection based on AIC then leads to the following marginal GEV models for SWE (note that very similar models are selected for HS):

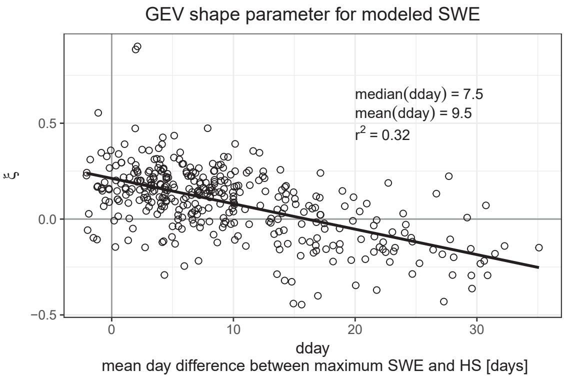

Figure 1: Typically the SWE maximum occurs up to one month later than the HS maximum. At locations with Frechet type distributions ($\xi > 0$), the SWE and HS maxima occur within only a few days.

$$
\begin{aligned}
\mu &\sim \text{latitude} + \overline{\text{SWE}} \\
\sigma &\sim \text{latitude} + \text{altitude} + \overline{\text{SWE}} + \overline{\text{dday}} + \mu \\
\xi &\sim \text{longitude} + \text{altitude} + \overline{\text{SWE}} + \overline{\text{dday}} + \mu + \sigma
\end{aligned} \quad (1)
$$

3.2. Bivariate Spatial Modeling

After bivariate fitting with SWE in one and HS in the other component using a composite maximum-likelihood procedure, the Extremal-t max-stable process (Opitz, 2013) in its bivariate form with a Matèrn covariance function (Gneiting et al., 2010) remained as the one with the smallest composite likelihood information criterion (CLIC, Davison and Gholamrezaee, 2012) among the max-stable processes of Hüsler-Reiss (Hüsler and Reiss, 1989) and Extremal-Gaussian (Schlather, 2002).

The fitted bivariate model can then be used to compute return levels of SWE conditioned on HS on the same location, or return levels of SWE (HS) at one location, conditioned on SWE (HS) at another location. The q-year conditional return level of Z_i at location $a_k \in \mathcal{A}$ given variable Z_j at location $a_l \in \mathcal{A}$ is defined as the threshold B_{ik} such that the conditional probability that $Z_i(a_k)$ exceeds this threhsold is $1/q$, given that $Z_j(a_l)$ is within the interval $\left(C_{jl}^{(1)}, C_{jl}^{(2)}\right]$:

$$\mathbb{P}\left[Z_i(a_k) > B_{ik} \mid Z_j(a_l) \in \left(C_{jl}^{(1)}, C_{jl}^{(2)}\right]\right] = \frac{1}{q}, \quad (2)$$

where $(i, j, k, l) \in \mathcal{D}$ with

$$\mathcal{D} = \Big\{(i, j, k, l) \in \mathbb{N}^4 \; : \left.\begin{array}{l} i \in \{1, 2\}, j \neq i \text{ and } k, l \in \{1, \ldots, K\} \\ i \in \{1, 2\}, j = i \text{ and } l \neq k \text{ for } k, l \in \{1, \ldots, K\} \end{array}\right\}$$

and $\mathcal{K} = |\mathcal{A}|$. This complex notation simply means, that conditional return levels can be computed either for the same variable $i = j$ but for different locations

$l \neq k$, or for different variables $i \neq j$ at the same location $l = k$.

3.3. Conditional Return Levels

A typical question arising during a winter season is, how the actual snow load at an arbitrary location compares to the 50-year return level, foreseen in the Austrian snow load regulations (ÖNORM B 1991-1-3:2012, 2012). In an ideal case, one could measure the actual snow load at a location of interest, and compare it with return levels derived from a GEV distribution fitted to yearly maxima of SWE at that location. However, SWE observations are almost unavailable in Austria, as already pointed out in Section 2. To answer that question, the fitted Extremal-t model of Section 3.2 together with equation (2) can be used. We are interested in the q-year return level of SWE (which is essentially snow load) at location k (e.g. St.Anton). If we have an actual measurement of SWE at another location l (e.g. Seefeld), we can use that as condition.
Table 1 shows the covariables necessary for the marginal GEV models (1) that are to be used with the fitted Extremal-t model. $\overline{\text{dday}}$ and $\overline{\text{SWE}}$ were derived from the climate version of the snow model SNOWGRID (Olefs et al., 2013). Taking an actual observation of $\text{SWE}_{\text{Seefeld}} = 231$ mm at location Seefeld (Tyrol, Austria) from 24.1.2018 as condition, the 50-year return value at St. Anton (Tyrol, Austria, roughly 70 km to the Southwest of Seefeld) then is $\text{SWE}^{50}_{\text{St.Anton}} = 536$ mm. A measurement on the same day in St. Anton yields a value of 339 mm (63%), showing that snow loads in St. Anton were not that extreme. Note that the currently valid Austrian snow load code issues a value of 541 mm for the 50-year return level of SWE in St. Anton.

Table 1: Covariables used in equation (2) for St. Anton and conditional location Seefeld.

Location	lon	lat	alt	$\overline{\text{dday}}$	$\overline{\text{SWE}}$
Seefeld	11.20	47.34	1190	13	171
St. Anton	10.27	47.13	1302	12	233

Although a detailed verification of the model performance is outstanding at this stage of the study, qualitative comparisons have been made between a smooth model, a univariate and the bivariate Extremal-t max-stable model of Section 3.2. Results show that the bivariate model is able to outperform both the smooth and the univariate max-stable model in terms of return levels of SWE at some locations that were not used for model fitting. Figure 2 compares SWE return levels computed locally from a 14-year dataseries of SWE measurements at station Felbertauern in Tyrol, Austria with different modeling approaches. Using the same marginal GEV models (1), a smooth model, a univariate Extremal-t max-stable process and the bivariate Extremal-t max-stable model described here were fitted to the data outlined in Section 3.1. It can be seen, that already at small return periods above 5 years the bivariate model, which was conditioned on HS observations at the same locations where SWE is available, follows the local estimation, whereas the other approaches overestimate the quantiles.

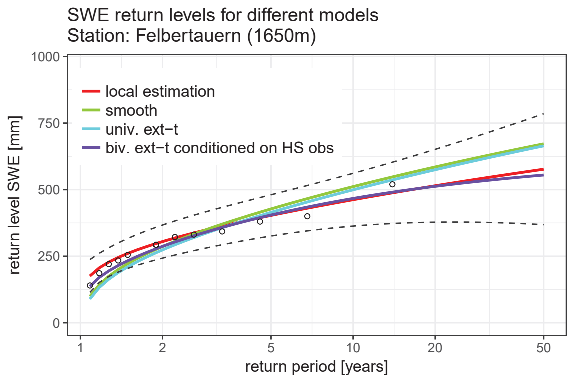

Figure 2: SWE return levels at station Felbertauern modeled with different approaches. Colors refer to different modeling approaches. Black dots depict observations, dashed lines 95% confidence intervals for the locally estimated return values.

CONCLUSIONS

A methodology is presented, which allows to derive extreme values of SWE at any arbitrary point, despite the fact, that no usable SWE measurements are available in Austria. This is achieved by the following workflow: (1) a newly developed snow layer model is used, to derive daily SWE values solely from HS measurements, without the use of supplemental meteorological forcing. (2) With an AIC-based model selection procedure, different covariates are probed for suitability. This leads to covariables longitude, latitude, altitude, $\overline{\text{SWE}}$ and the mean time difference between the occurrence dates of HS and SWE within a winter season $\overline{\text{dday}}$ as explaing variables for the GEV parameters location, scale and shape. (3) Using those marginal models the bivariate max-stable processes Hüsler-Reiss, Extremal-Gaussian and Extremal-t are fitted to Austrian HS and modeled SWE data series. The bivariate Extremal-t max-stable process using a Matèrn covariance function remains as the best suited model.
Especially the variable $\overline{\text{dday}}$, describing the mean time difference between the occurrence times of the maximum HS and SWE values within a season at one location seems to be beneficial for the marginal models of the scale and shape parameter of the GEV.
As there do not exist many useful (long-term)

SWE data-series in Austria, the presented approach could be useful for hydrological studies focusing on potential runoff from snow covered areas, climate studies investigating water availability and any application relying on extreme values of SWE or HS. The spatial dependence of the extremes which is excplicitly modeled with the bivariate max-stable process, can be used in risk analysis where e.g. joint exceedance probabilities are of interest. This could also be achieved by fitting a univariate max-stable process to modeled SWE values (see Section 2). But first validations suggest, that the bivariate max-stable model leads to better fits than the univariate model.

The bivariate approach makes it possible to compute quantiles of one variable conditioned on the other variable at the same location, or the same variable at another location. A practical application could be the estimation of snow depth return times at selected locations conditioned on the exceedance of a certain snow depth at a measurement site. This scenario could be of particular interest e.g. for the risk analysis of roads, railways or telegraph lines.

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