Multivariate parameter optimization for operational application of extended kinetic theory in simulation software

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ABSTRACT: State of the art avalanche simulation software is used for avalanche prediction and hazard mapping. Amongst other factors, the choice of an appropriate rheological model is of major importance. Here, we apply a rheological model based on kinetic theory, which unifies two different flow types; (i) the rapid motion of granular material, based on a statistical description of collisions between the particles and (ii) slow motions by incorporating the critical state theory. The involved model parameters strongly influence the simulation results. They are optimized with the help of an objective method, comparing different simulation results to documented avalanche events. This multivariate optimization approach incorporates variables such as runout length, velocity, affected area and volume growth.
Simulation results with good overall accordance to the observations can be identified, when applying the optimization to a single avalanche event. However, a comparison of multiple events shows, that the optimal parameter sets for single events can hardly be applied to a wide range of avalanches. Therefore, the optimization method is adapted for the combined optimization of multiple avalanches and the resulting parameter distributions are evaluated. By performing simulations with the obtained optimal parameter set and the optimal parameter sets from the single analysis for respective avalanches, the prediction accuracy can be evaluated.

Key words: snow avalanche, computational avalanche dynamics, simulation optimization, design event.

1. INTRODUCTION

Avalanche simulation software is a commonly employed tool for avalanche prediction and hazard mapping. The underlying process model, which consists of an appropriate friction relation as well as a reasonable description of entrainment, and its implementation are important for the reliability of simulation results.

In this work the practical application of a rheological model from the field of extended kinetic theory (Vescovi et al., 2013) to the depth averaged process model SamosAT DFA (**S**now **A**valanche **MO**delling and **S**imulation - **A**dvanced **T**echnology, Zwinger et al., 2003; Sampl and Zwinger, 2004; Sampl, 2007) is shown. The derived frictional relation (Rauter et al., 2016) unifies two different flow types; (i) the rapid motion of granular material, based on a statistical description of collisions between the particles and (ii) slow motions by incorporating the critical state theory.

An optimal parameter setting for the used rheological model can be identified for a single avalanche event by back calculation, using a objective optimization framework (Fischer et al., 2015). This framework is build on the comparison of simulations with varying parameters, respectively their results to different documented avalanche characteristics and a statistical evaluation of good simulation runs and their underlying parameters. Arising uncertainties in the simulation and optimization procedures (simplification of the process model, numerical uncertainties, observational errors) can be evaluated by consideration of parameters distributions.

The used framework is extended to the combined optimization of five avalanche events, in order to obtain optimal parameters, which are potentially suitable to a wide range of avalanches. The associated uncertainties, which arise in the estimation of optimal parameters is investigated and their effect on the forward simulation is observed.

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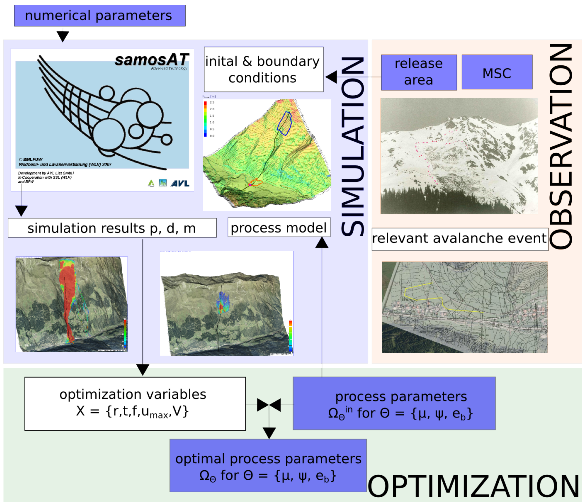

Figure 1: The optimization framework consists of three parts: observation, simulation and optimization. Data gathering and homogeneous processing is done in the observation box. The simulation box contains the simulation software (SamosAT) and its necessary input (process model, numerical parameters, initial and boundary conditions). The simulation results are used for the optimization, based on the comparison of documentation and simulation data and a statistical determination of optimized parameter distributions Ω_Θ for the process model.

2. OPTIMIZATION FRAMEWORK

The optimization framework in this work consists of three parts (see figure 1). First, reference data for the investigated avalanches has to be collected and reviewed in a homogeneous way. This data serves as simulation input as well as for the comparison to the simulation results. Second, a simulation set up for selected reference avalanches is determined. This includes the definition of initial and boundary conditions and the choice of the process model. Then, 10 000 Monte Carlo simulations with varying process parameters are performed. Third, the optimization of the varied process parameters is realised. Therefore the simulation results are transformed in an avalanche path dependent coordinate system and simulated avalanche characteristics, that can be compared to observed ones, are determined. Simulation runs with good accordance to the observation are identified and their underlying process parameters are statistically evaluated. Using this objective optimization algorithm, adjusted parameter distributions and further on optimal parameter combinations for the investigated process model are determined.

Optimization variables

A central concept of the used optimization framework is the usage of the optimization variables. They represent different characteristics, that can be accessed through both, observational data and simulation results. In this work, five optimization variables $X = \{r, t, f, u_{\max}, V\}$ are investigated and included in the optimization process to consider different avalanche characteristics. Quality and quantity of observational data is often limited and inhomogeneous. Therefore observational variables and their associated uncertainty are denoted by $\hat{X} \pm \sigma_{\hat{X}}$, whereas simulation variables by plain X.

2.1. Observation

The observational data serves as input for the simulation and is used to evaluate the quality of multiple simulation runs. In this work multiple avalanche events are considered in the optimization process. The five investigated avalanches are Alpenlahner avalanche (Bad Bleiberg, carinthia), Ganderwiesen avalanche (St. Anton, Tyrol), Heiligenblut avalanche (Lienz, East Tyrol), Lubitzgraben avalanche (Mallnitz, Carinthia), Trins avalanche (Gschnitz valley, Tyrol).

For the simulation input mainly the knowledge of release areas, potential entrainment areas and related snow cover distributions is necessary. The release areas are delineated by evaluating documented release scenarios, but also considering potential release areas according to guidelines and models (Maggioni and Gruber, 2003; Veitinger et al., 2015). The snow depth distribution is based on estimations of extreme snow depths (Leichtfried, 2010), which get projected onto the mountain slope (see **m**ountain **s**now **c**over, Fischer et al., 2015). This allows to assume a smooth and consistent snow cover distribution for release and entrainment conditions.

Optimization variables: The observational variables are based on chronicle data. If no field data is available, empirical laws may provide valuable data for the optimization variables. The affected area $\hat{A}_{\text{aff}}$ outlines multiple run out limits, i.e. run out delineations or points from single events get combined to an enfolding area. So the documented run out $\hat{r}$ is represented as the furthest point of the affected area $\hat{A}_{\text{aff}}$ in the avalanche path dependent coordinate system. The maximal velocity of the avalanche is approximated using the empirical estimate $\hat{u}_{\max} \approx 0.6\sqrt{g\,\Delta z}$ (McClung and Schaerer, 2006). The documented deposition volume is assumed by summarizing the initially released snow volume and the approximated entrainment volume (Sovilla et al., 2006, 2007) following $\hat{V}_{\text{dep}} = V_{\text{rel}} + 0.5\,h_{\text{msc}}\,A_{\text{ent}}$. The observed avalanche characteristics for the five investigated avalanches are summarized in table 3.

2.2. Simulation

For each of the investigated avalanches in the presented framework a simulation set up (consisting of

		Alpenlahner	Ganderwiesen	Heiligenblut	Lubitzgraben	Trins
$\hat{r}$	(m)	2384	3215	2736	1781	3203
$\hat{A}_{\text{aff}}$	(m^2)	85089	120266	63872	51696	53889
$\hat{u}_{\max}$	(m s^{-1})	61.96	69.98	62.75	56.32	64.34
$\hat{V}_{\text{dep}}$	(m^3)	388225	774390	355653	428370	366769

Table 1: Observational variables $\hat{X}$ for avalanches: $\hat{r}$ - runout, $\hat{A}_{\text{aff}}$ - affected area, $\hat{u}_{\max}$ - maximal velocity, $\hat{V}_{\text{dep}}$ - deposition volume. Related uncertainties $\sigma_{\hat{X}}$ are $\sigma_{\hat{r}} = 25$ m, $\sigma_{\hat{A}} = 0.15\,\hat{A}_{\text{aff}}$, $\sigma_{\hat{u}} = 10\,\text{m}\,\text{s}^{-1}$, $\sigma_{\hat{V}} = 0.25\,\hat{V}_{\text{ent}}$.

simulation input, process model and simulation output) is defined and 10 000 simulations with varying process parameters are performed.

The simulation input comprises initial conditions, such as the definition of release and entrainment areas and their respective snow depth distributions, and boundary conditions (e.g the digital terrain model).

The used process model unifies the description of frictional behaviour and entrainment and is specified by the governing equations, respectively their implementation in the simulation software (Sampl, 2007). In this work a simplified expression form of the friction relation is used (Rauter et al., 2016):

$$\tau^{(b)} = \mu\,\sigma^{(b)} + \bar{\rho}\,\psi\left(\frac{\bar{u}}{h}\right)^2, \tag{1}$$

where μ represents the dry friction coefficient and ψ accounts for dynamic stresses. This covers the basic features of the extended kinetic theory (collisional based kinetic theory at low volume fractions and the so called critical state theory from soil mechanics at high volume fractions, Vescovi et al., 2013). Equation (1) is similar to classic phenomenological friction relations like the Voellmy friction relation (Voellmy, 1955). The entrainment process, that is approximated by a simple law for erosive entrainment with the erosion energy parameter e_{b} (Fischer et al., 2015):

$$\dot{q} = \frac{\tau^{(b)}}{e_{\text{b}}}\,\|\bar{\mathbf{u}}\|\,. \tag{2}$$

Similar definitions can be found in the literature (see Christen et al., 2010).

The unknown process parameters can be summarized in a set $\Theta = \{\mu,\, \psi,\, e_{\text{b}}\}$. In the used probabilistic simulation set up 10 000 Monte Carlo combinations of Θ are performed. This means that the respective parameters get varied randomly within defined interval bounds. These are determined by the physically relevant parameter space, values found in the literature and previous studies or experimental results and are $\mu \in [0.1, 0.6]$, $\psi \in [0.001, 0.010]\,\text{m}^2$ and $e_{\text{b}} \in [0, 20000]\,\text{J}\,\text{kg}^{-1}$. The resulting input parameter distributions, which are distributed uniformly, are summarized in $\Omega_{\Theta}^{\text{in}}$ (see figure 1).

Optimization variables: The main results of the SamosAT simulation software are the temporal evolution of the state variables flow depth or velocity or their conversion to dynamic pressures. The used optimization approach uses the maximum values over the time, the so called dynamic peak pressures $p\,(x, y)$, and transforms them in an avalanche path dependent coordinate system (Fischer, 2013). By introducing a pressure threshold $p_{\lim} = 1$ kPa, a boundary of the impact zone of the simulated avalanche can be identified and further on interpreted as simulated affected area A_{aff}. The furthest point in the avalanche path dependent coordinate system where the pressure threshold is exceeded ($p > p_{\lim}$) marks the run out r. The matched affected area (true) t summarizes the area, where the simulated affected area coincides with the documented affected area ($A_{\text{aff}} \cap \hat{A}_{\text{aff}}$). The exceeded affected area (false) f are those areas, where the simulation result exceeds the documented affected area ($A_{\text{aff}} \setminus \hat{A}_{\text{aff}}$). Maximal velocity $u_{\max}$ and total volume V_{dep} are direct results of the simulation.

2.3. Optimization

The optimization approach, which is used in this work to calculate adjusted parameter distributions, is based on an informal statistical approach (Fischer et al., 2015). An arbitrary function is introduced to quantify the correspondence between observation and simulation, without explicitly considering model uncertainties (McMillan and Clark, 2009). A drawback of this method is that the resulting parameter distributions represent an estimate of total uncertainties and can therefore not be assigned explicitly to model or input uncertainties, which is possible with formal Bayesian approaches (Ancey, 2005; Eckert et al., 2007, 2008; Gauer et al., 2009; Hellweger et al., 2016).

The employed function, which determines the correspondence is a normalized, Gaussian function $\mathcal{N}$ with mean $\hat{X}$ and variance $\sigma_{\hat{X}}^2$. With this definition the accordance is bounded in the interval $\alpha_X \in [0, 1]$, where 1 means optimal accordance and 0 denotes no accordance for the respective optimization variable $X = \{r, t, f, p, u_{\max}, V\}$. The uncertainty of the documentation variable determines the tolerance and scale of the used function. So for each simulation run a final accordance α is determined by a

	μ	$\sigma_{\mu,50\,\%}$	ψ	$\sigma_{\psi,50\,\%}$	e_{b}	$\sigma_{e_{\mathrm{b}},50\,\%}$
Single analysis						
Alpenlahner avalanche	0.305	± 0.017	0.0070	± 0.0018	15797	± 2577
Ganderwiesen avalanche	0.254	± 0.015	0.0058	± 0.0026	6785	± 2080
Heiligenblut avalanche	0.286	± 0.021	0.0052	± 0.0021	7604	± 3329
Lubitzgraben avalanche	0.315	± 0.022	0.0072	± 0.0018	11134	± 3845
Trins avalanche	0.296	± 0.011	0.0059	± 0.0022	14583	± 3373
Mean	0.291	± 0.017	0.0062	± 0.0021	11180	± 3041
Combi analysis						
	0.292	± 0.021	0.0063	± 0.0022	10543	± 3884

Table 2: Mean values and interquartile ranges of the optimal parameter distributions, separated for single analysis and combi analysis. The full distributions are shown in figure 2.

multiplication of the single α_X

$$\alpha = \prod_X \alpha_X \,, \tag{3}$$

which for a good simulation tends towards $\alpha \rightarrow 1$. Simulations, which do not agree with the documentation in single or multiple regards are marked by a low accordance $\alpha \rightarrow 0$.

The objective of the single analysis is a statistical parameter optimization of 10 000 simulations with variable process parameters $(\mu, \psi, e_{\mathrm{b}})$ for a single avalanche path (Fischer et al., 2015) by identifying simulations, which match the documentation best and investigating the underlying parameter distributions. Using the optimization variables $X = \{r, t, f, u_{\max}, V\}$, the simulations are ranked by maximising the degree of simulation-observation-correspondence α. A statistical evaluation of the best simulation runs and their corresponding process parameters is performed.

The aim of the combi analysis is the statistical parameter optimization for the combination of multiple avalanche paths. Therefore the same simulation set up as for the single analysis is used and a fixed number of best simulations per avalanche event are identified. Their underlying parameters are identified and combined optimal parameter distributions are calculated. They can be analysed and characteristic values are obtained (e.g. median values or quantiles).

3. RESULTS

In this section the results of the optimization framework are shown. The optimal parameter distributions for the single analysis and the combi analysis are displayed and the related uncertainties are highlighted. In a further step, simulations of the same avalanche events, which were already used in the optimization framework, with the optimized parameter combinations are performed. Thus the effects of parameter uncertainties on forward simulation purposes are emphasised.

3.1. *Optimized parameter distributions Ω_Θ for $\Theta = \{\mu,\ \psi,\ e_{\mathrm{b}}\}$*

In the single analysis a statistical parameter optimization of 10 000 simulations with variable process parameters $(\mu, \psi, e_{\mathrm{b}})$ for every single avalanche path is performed. After calculating the optimization variables and furthermore the simulation-observation accordance, it is possible to identify the best $5\,\% \rightarrow 500$ simulations. A statistical evaluation of the best simulation runs and their corresponding process parameters is performed. The resulting parameter distributions (see 2) and their characteristic values (e.g. median and quartiles) are summarized in table 2.2. The median is used for each avalanche path for forward simulation.

Based on the best simulations of the single analysis, the optimal parameter distribution of the combi analysis can be evaluated. The fixed number of $5\,\% \times 10\,000 = 500$ best simulations from each avalanche event are identified and their parameter combinations combined. The resulting distribution of $5 \times 5\,\% \times 10\,000 = 2\,500$ parameter combinations is displayed in figure 2. The resulting parameter distributions can be analysed and used for forward simulation (e.g. median values of distributions, table 2.2).

In figure 2 the optimized parameter distributions for the single analysis for each avalanche event and for the combi analysis are shown for the parameters μ, ψ and e_b. The gray area summarizes the full $5\,\%$ best parameters, the blue ranges represent the $25\,\%$- and $75\,\%$-percentile and the red line shows the median. The medians and the interquartile ranges $\sigma_{\Theta,50\,\%} = \Theta_{75\,\%} - \Theta_{25\,\%}$ are summarized in table 2.2. For μ relatively clear peaks can be found, whereas for ψ no significant trend is observable. This indicates, that the influence of the Coulomb friction parameter μ to the observational variables is higher compared to the negligible influence of ψ. For the entrainment parameter e_{b}, remarkable differences between the different avalanche events can be observed. The ranges of $\sigma_{\Theta,50\,\%}$ indicate the spreading in the distributions of the optimal parameters and can be interpreted as related uncertainties. The

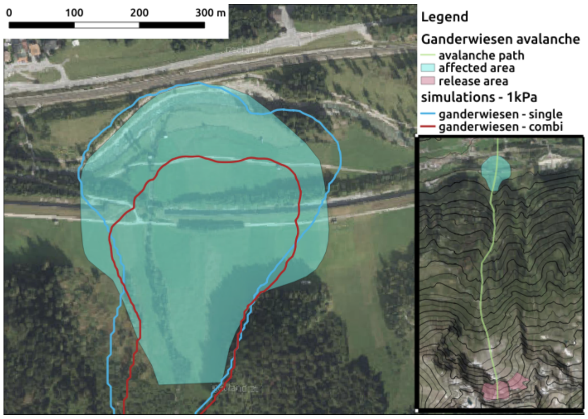

Figure 3: Forward simulation of the Ganderwiesen avalanche. The small box in the right bottom shows an overview of the Ganderwiesen avalanche with its release area (red polygon), affected area (azure polygon) and the avalanche path (green line). The left box is a zoom in the deposition area of the avalanche. The red (combi analysis) and blue (single analysis) lines shows the 1 kPa bounds of the simulations with optimized parameters.

mean value out of the single analysis shows less spreading than the combi analysis for each variable ($\sigma_{\mu,50\,\%} = 0.017$ versus $\sigma_{\mu,50\,\%} = 0.021, \ldots$), which indicates a rise of uncertainty for the combi analysis.

3.2. Forward simulation

For each of the avalanches two simulations with different process parameters are performed: the respective optimal parameter setting from the single optimization and the combined optimal parameters from the combi analysis (median values, see table 2.2). The simulation results are processed in the avalanche path dependent coordinate system and the optimization variables are determined and summarized in table 3.

In figure 3 the results of the forward simulations for the Ganderwiesen avalanche are shown. The azure polygon represents the documented affected area, whereas the red and blue lines show the 1 kPa bounds of the simulations with optimized parameters. It can be observed that the simulation with the optimal parameters from the single analysis (blue) shows a good accordance to the documented affected area and leads to a good runout estimation ($\varepsilon_r = 5\,\mathrm{m}$), whereas the simulation after combi analysis fails to predict the observed runout length ($\varepsilon_r = 105\,\mathrm{m}$) and also the lateral spreading is not reproduced well.

Note that other optimization variables like velocity or avalanche volume are not visible in this figure, but have to be taken into account to make a meaningful judgement about the quality of a simulation. Therefore the absolute errors $\varepsilon_r = |\,\hat{r} - r\,|, \varepsilon_u = |\,\hat{u}_{\max} - u_{\max}\,|$ and $\varepsilon_g = |\,\hat{g} - g\,|$ are introduced to get an idea about the uncertainties of the different optimal parameter settings regarding different documentation aspects (runout r, velocity u and volume growth $g = V_{\mathrm{dep}}\,/\,V_{\mathrm{rel}}$).

Table 3 summarizes these optimization variables and the respective errors for the five investigated avalanches. Runouts are underestimated and overestimated for the single analysis, but also for the combi analysis. However the mean error of simulations with optimal single analysis parameters are smaller than with optimal combi analysis parameters ($\pm\,23\,\mathrm{m}$ versus $\pm\,43\,\mathrm{m}$). The velocities are underestimated systematically ($\approx\,19\,\mathrm{m/s}$) for both optimization approaches, compared to the empirical estimate of the velocity. However, when comparing the simulated velocities for both parameter sets for each avalanche, small differences can be observed. The volume growth is generally underestimated, which can be attributed to the high estimate of possible entrainment due to the definition of possible entrainment areas A_{ent} and the approach for the snow cover distribution h_{msc}. However, values for the volume growth are in a reasonable range (Sovilla et al., 2006, 2007).

4. SUMMARY AND CONCLUSIONS

In this work a multivariate optimization method for process parameters, incorporating different observational variables to cover various avalanche characteristics, was applied to the operational avalanche simulation software SamosAT for five documented avalanche events.

It was shown, that for each avalanche event good parameter combinations were found in order to reproduce the respective observations in different regards quite good, e.g. the mean absolute error of the runout of the five investigated avalanches was $\varepsilon_r = 23\,\mathrm{m}$. When comparing the parameter distributions of the single analysis to the parameter of the combined analysis, it could be observed, that not surprisingly with the amount of considered avalanche events in the optimization, the uncertainty in the optimized parameter distributions has gone up. This can be addressed to the differences in the optimal parameter settings and their related uncertainties for the different paths.

Forward simulations with the median values of the optimized parameter distributions were performed and the main avalanche characteristics for the respective events were evaluated. A main focus was laid on the uncertainties of the forward simulations to the documentation. But also the differences between the optimization cases (single and combi analysis) were observed. It could be seen that runouts were underestimated and overestimated for both optimization cases, but the mean absolute error was smaller at simulations with optimal single analysis parameters than with optimal combi analysis parameters ($\pm\,23\,\mathrm{m}$ versus $\pm\,43\,\mathrm{m}$). This coincides with

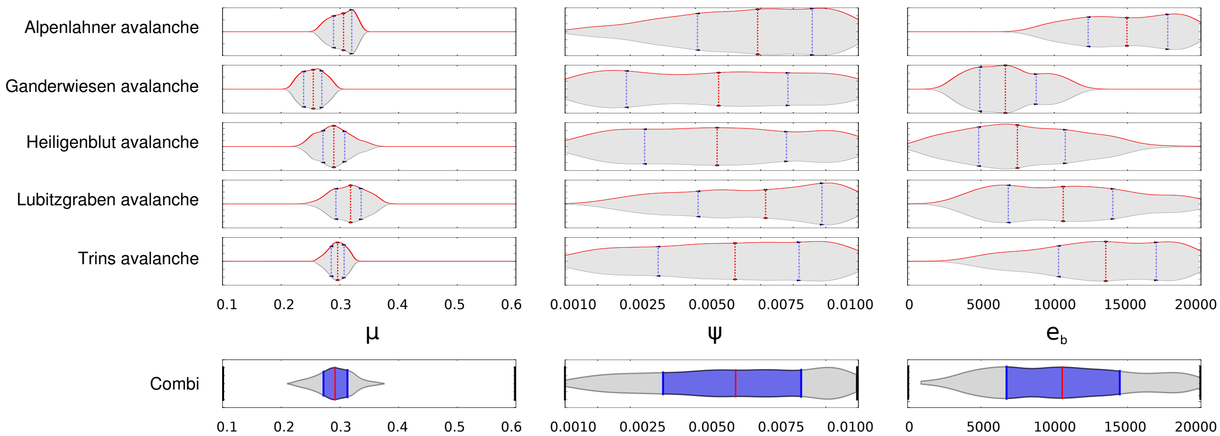

Figure 2: Optimized parameter distributions μ, ψ and e_b for both optimization approaches: the respective single analysis for each avalanche event and the combi analysis. The shown violin plots represent approximations of histograms of the best 500 simulations for the single analysis, respectively 2 500 simulations for the combi analysis. The gray areas summarize the full range of best parameters, the blue ranges represent the 25 %- and 75 %-percentile (lower and upper quartile) and the red lines shows the median values.

		r		ε_r	
	Docu	Single	Combi	Single	Combi
Alpenlahner avalanche	2 384	2 342	2 368	42	16
Ganderwiesen avalanche	3 215	3 220	3 110	5	105
Heiligenblut avalanche	2 736	2 764	2 681	28	55
Lubitzgraben avalanche	1 781	1 756	1 796	25	15
Trins avalanche	3 203	3 219	3 225	16	22
Mean	2 664		*Mean*	23	43
				$\approx$ 1 %	$\approx$ 2 %

		$u_{\max}$		ε_u	
	Docu	Single	Combi	Single	Combi
Alpenlahner avalanche	62	52	53	10	9
Ganderwiesen avalanche	70	53	50	16	19
Heiligenblut avalanche	63	30	29	33	34
Lubitzgraben avalanche	56	49	51	7	5
Trins avalanche	64	38	38	26	27
Mean	63		*Mean*	19	19
				$\approx$ 30 %	$\approx$ 30 %

		$g = V_{\text{dep}} / V_{\text{rel}}$		ε_g	
	Docu	Single	Combi	Single	Combi
Alpenlahner avalanche	1.78	1.59	1.86	0.19	0.08
Ganderwiesen avalanche	3.11	2.59	2.02	0.52	1.09
Heiligenblut avalanche	4.22	2.10	1.64	2.12	2.58
Lubitzgraben avalanche	2.94	1.79	1.84	1.15	1.10
Trins avalanche	1.48	1.42	1.52	0.06	0.04
Mean	2.70		*Mean*	0.81	0.98
				$\approx$ 30 %	$\approx$ 36 %

Table 3: Optimization variables (runout r, velocity u and volume growth) from simulations with the respective optimal parameter setting from the single optimization and the combined optimal parameters from the combi analysis. Additionally the absolute errors $\varepsilon_r = |\hat{r} - r|$, $\varepsilon_u = |\hat{u}_{\max} - u_{\max}|$ and $\varepsilon_g = |\hat{g} - g|$ are included.

the related uncertainties in the parameter estimation. Velocities and the volume growth were underestimated systematically compared to the introduced empirical laws, which on the other hand might also be subject to over- or underestimation. However due to lack of observational data of extreme events they serve as valuable estimates.

It could also be highlighted that it is important to consider the uncertainties of process parameters, when making forward simulations of avalanche events with a single optimal parameter combination. As a consequence, a forward simulation concept, which is based on a probabilistic simulation, following optimal parameter distributions, could be developed.

ACKNOWLEDGEMENT

This research work was carried out with the financial support of the project CoSiCaCV (BMLFUW - Federal Ministry of Agriculture, Forestry, Environment and Water Management, WLV - Austrian Service for Torrent and Avalanche Control). We thank Peter Sampl (AVL LIST GmbH) for his support and cooperation regarding SamosAT. We would further like to thank Elisabeth Hainzer for her contribution to the compilation of the used reference data.

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