ON HOW THE TENSILE STRENGTH OF THE SLAB AFFECTS CRACK PROPAGATION PROPENSITY

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ABSTRACT: The release of a dry-snow slab avalanche is preceded by a series of fractures. The two key processes involved are (1) failure initiation within the weak layer underlying the slab, probably due to damage accumulation by enhanced deformation, and (2) crack propagation, once the initial failure has reached a critical size – leading to the detachment of the slab. The focus on crack propagation has clearly improved our understanding of avalanche formation. The propagation saw test (PST) is the only in-situ fracture mechanical test and allows testing the crack propagation propensity. When conducting such tests, we observe that cracks sometimes only propagate a short distance, not to the end of the column, and stop at a crack through the slab. Furthermore, even if the crack propagates to the end of the column, conditions for slope failure, in other words slab avalanche release, are not always present on adjacent slopes. We calculate the tensile stress with a simple cantilever beam model and with finite element simulations. Results are compared to numerous PSTs from Canada to explore the relevance of slab properties, in particular the tensile strength, for crack propagation. Model results suggest that the slab needs a certain density in order to have sufficient strength and not fail in tension during the initial states of crack propagation. Analyzing the field data showed that the characteristics of the slab were significantly different in those PSTs where slab fractures (SF) were observed from those where the crack propagated to the end of the column (END) or arrested without slab fracture (ARR). The test result SF was associated with thin, soft and low density slabs. Whereas soft slabs provide ample deformation energy, they are prone to slab fractures. On the other hand, stiff slabs may require large crack lengths, until propagation starts – if at all, but due to their high tensile strength, propagation may be extensive. Obviously, there is an optimal range of slab stiffness allowing crack onset as well as crack propagation – the tensile strength of the slab might well be the limiting factor.

KEYWORDS: avalanche release, snow stability evaluation, avalanche forecasting, fracture

1. INTRODUCTION

Dry-snow slab avalanches release by a series of fractures. Once an initial failure in the weak layer underlying the slab has reached a critical size, a self-propagating crack within the weak layer leads to the detachment of the slab. The focus on fracture mechanical properties of snow (Sigrist, 2006) has clearly improved our understanding of avalanche formation over the last decade. It corroborated the view that failure initiation and crack propagation are the key processes to be considered in dry-snow slab avalanche formation (Schweizer et al., 2003). Furthermore, a true in-situ fracture mechanical test was developed: the propagation saw test (PST) (Gauthier and Jamieson, 2006; Sigrist and Schweizer, 2007). Gauthier and Jamieson (2008) showed that the critical cut length, a measure for crack propagation propensity, and in general snow instability, is related to the probability of avalanche triggering. The critical cut length, i.e. the length the slab is undercut by a snow saw until a running crack starts, integrates the properties of the weak layer (specific fracture energy) and of the slab layers (load and modulus) (Sigrist and Schweizer, 2007). Recent work has shown that all these properties can now be estimated from a penetration resistance profile acquired with the snow micro-penetrometer (SMP) (Reuter et al., 2013) or from particle tracking analysis of PSTs (van Herwijnen et al., 2014).

When performing a PST, the critical cut length as well as the type of fracture (test result or arrest condition) are recorded. As mentioned above, the

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critical cut length refers to the length when a self-propagating crack starts. However, the crack does not always run to the end of the column, but fracture arrest is observed, i.e. the crack comes to a halt before the end of the column. When the crack does not propagate to the end of the column, a tensile crack across the slab is sometimes observed – denoted as slab fracture.

Ross and Jamieson (2008) studied how snowpack characteristics, in particular weak layer depth and slab hardness affect crack propagation propensity. They analyzed the test results of 365 PSTs and found that PSTs with shallow, soft slabs usually resulted in slab fracture results.

Gauthier and Jamieson (2010) were among the first to focus on fracture arrest and suggested that the interaction or competition between the weak layer and slab fractures may determine the arrest condition. They coined the term sustainability, a property that should represent the capacity of the slab to transmit the information from one element to the next – based on the idea that a crack propagates in discrete steps as subsequent elements fail. Therefore, they postulated that a propagation criterion based solely on fracture energy would miss some crucial information – and suggested to consider the capacity of the slab to sustain and transmit the driving energy. However, it remained elusive what property of snow should be considered.

There are few field studies on the tensile strength of snow, probably because the tensile fracture at the crown of a slab avalanche is a secondary step in the slab release process. However, Mears (1998) found that with the cantilever beam test, also presented by Sterbenz (1998), the tensile strength of the slab can reliably be estimated. He repeatedly tested the same layer and found that the initial strength (typically < 1 kPa) increased within a couple of days to several kPa.

We hypothesize that during a PST a running crack can arrest due to (1) changes in weak layer specific fracture energy: the weak layer becomes stronger so that the strain energy released by the slab is no longer sufficient for fracture the weak layer, (2) changes in slab properties, so that the released strain energy is no longer sufficient for propagation, or (3) slab fracture so that crack propagation in the weak layer is no longer possible. We assume that slab fracture occurs due to low tensile strength of slab.

Our aim is therefore to evaluate whether the tensile strength of the slab limits crack propagation. We will consider the simple case of a cantilever beam to calculate the maximum tensile stress, study the stress distribution in a PST experiment using the finite element method, and finally compare our results to a large dataset of PST results and concurrent snow stratigraphy characteristics.

2. METHODS AND DATA

2.1 Tensile stress in cantilever beam

The tensile stress in a propagation saw test develops due to the bending of the undercut part of the slab. This configuration can be approximated by considering the maximal stress in a cantilever beam. For a horizontal, unsupported cantilever beam subject to a uniformly distributed load q per length [N/m] (thickness h, width b, length l) the maximal tensile stress $\sigma_{\max}(z)$ due to bending is

$$\sigma_{max}\left(z=\frac{h}{2}\right) = \frac{3\rho g l^2}{h}\,. \qquad (1)$$

Even after the onset of the running crack, i.e. during dynamic crack propagation, the bending and hence the tensile stress will increase until the slab touches the substratum again (Heierli, 2008). If we make assumptions for the modulus and the amount of collapse, the maximal unsupported length l_t can be calculated. The maximal downward slope-normal displacement at the free end of a cantilever beam is (McClung and Borstad, 2012):

$$\Delta y = \frac{3}{2}\,\frac{q}{E'}\left(\frac{l}{h}\right)^3 l \qquad (2)$$

where E' is the effective modulus. Hence the length l_t where the slab makes contact with the substratum again is given by:

$$l_t^4 = \frac{2}{3}\,\frac{E'h^3}{\rho g h}\,\Delta y\,. \qquad (3)$$

2.2 Finite element modeling

To use a more realistic geometry (i.e. a PST column) and account for layering, we used a finite element model of the PST as originally presented by Sigrist and Schweizer (2007). The refined model was built in ANSYS Workbench and includes a pre-meshed crack in the weak layer that has its own material properties (Stettler, 2014).

The 2-dimensional FE model of the PST was 1.2 m long, slab thickness was 0.3 m, and slope angle 0°. The elastic modulus changed with density according to the relation provided by (Scapozza, 2004), whereas Poisson's ratio was kept constant (0.25).

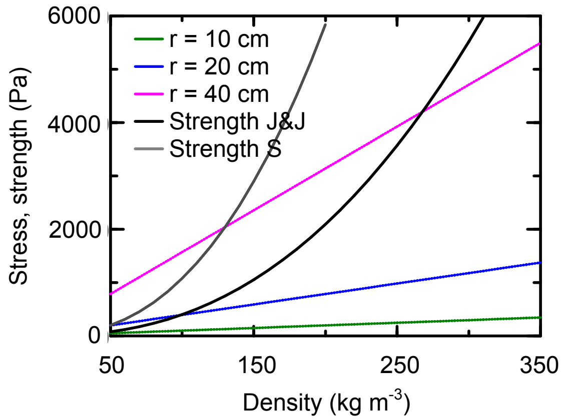

Fig. 1: Maximal tensile stress and tensile strength vs density for the simple cantilever beam approximation. The two relations for strength are based on the relations provided by Jamieson and Johnston (1990) (Eq. 4) and Sigrist (2006).

2.3 Data and data analyses

We analyzed a dataset of 1037 propagation saw tests recorded in western Canada by University of Calgary field staff during the winters 2005-2006 to 2013-2014. The PST columns were mostly 100 cm long and tests were performed according to the Canadian observation standards (CAA, 2007). For each test the critical crack length and one of the three possible fracture results were recorded: propagation to the end of the column (END), crack onset but arrest before the end of the column (ARR), and slab fracture (SF). At each test site a snow profile was observed and in most cases layer density was also measured. For analysis the average slab density and the average slab hardness index were calculated, both weighted by layer thickness. To contrast samples for the three fracture types we used the non-parametric Kruskal-Wallis test (H-test); a level of significance p = 0.05 was chosen to decide whether the observed differences were statistically significant.

3. RESULTS AND DISCUSSION

3.1 Tensile stress in cantilever beam

The maximal tensile stress using the simple cantilever beam approximation (Eq. 1) for slab thickness h = 0.3 m, and critical cut length r_c = 0.2 m is about 400 Pa for a low density slab of 100 kg/m^3. The stress increases linearly with increasing density (Fig. 1). For the tensile strength σ_s, on other hand, the relation to density can be described based on field measurements with a power law (Jamieson and Johnston, 1990):

$$\sigma_s = 79.7 \times 10^3 \left(\frac{\rho}{\rho_i}\right)^{2.39} \qquad (4)$$

where ρ_i = 917 kg/m^3 is the density of ice. For a density of 100 kg/m^3 this relation provides a tensile strength of about 400 Pa, i.e. just about as large as the maximal tensile stress.

Hence, the simple cantilever beam model suggests that for a density larger than about 100 kg/m^3 the tensile strength, assuming the parameterization provided by Jamieson and Johnston (1990), is always larger than the maximal tensile stress (for the above geometry) (Fig. 1). However, if the strength parameterization provided by Sigrist (2006, p. 58) is used, the strength is already larger than the stress for a slab density of about 60 kg/m^3. For a longer undercut (critical cut length), but also for thicker slabs this limit is slightly higher. For example, for a slab thickness of 0.7 m, and a critical length of 0.5 m, the density limit is about 210 kg/m^3.

To take into account that during dynamic crack propagation the tensile stress increases until the slab touches the substratum, the length l_t can be calculated (Eq. 3) which amounts to about 0.55 m for typical values (e.g. Δy = 1 mm) and the above used geometry, i.e. less than about twice the slab thickness. Assuming this length, the maximal tensile stress becomes very large, i.e. the slab is always too weak (for slab densities < 400 kg/m^3) and slab fracture occurs.

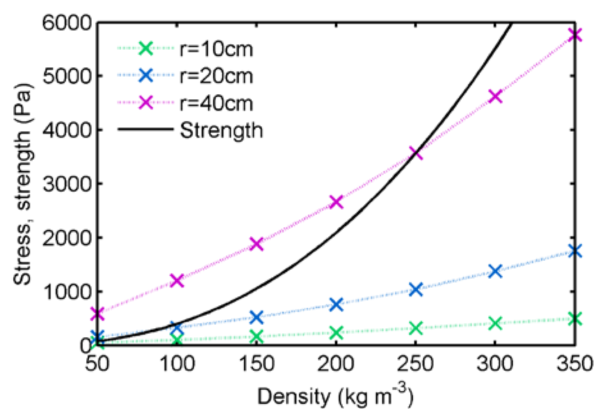

Fig. 2: FE simulation results for the maximal tensile stress at the snow surface in a PST experiment vs slab density for three different crack length (r = 10, 20, 40 cm). Also shown is the relation for tensile strength reported by Jamieson and Johnston (1990) (Eq. 4).

3.2 Finite element modeling

The above results obtained with the simple cantilever beam approximation have been confirmed by the FE simulations (Fig. 2). However, the tensile stress no longer increases linearly with slab density, but the increase is slightly stronger. This follows from the fact that we kept the weak layer and substratum density at 190 kg/m^3, while slab density increased. Furthermore, the tensile stress was obviously not maximal just above the crack tip (as in the simple cantilever beam assumption) but some distance ahead of the crack tip, at the snow surface.

The FE simulations with a layered slab, e.g. increasing density (i.e. soft over hard) or decreasing density (i.e. hard over soft) showed that the tensile stress at the snow surface was largest for a slab with increasing density (and modulus accordingly) – larger than in the case of a uniform slab with the same average density. On the other hand, in the case of decreasing slab density with depth (corresponding to the case where a hard surface layer exists) the maximal stress was slightly lower at the surface, but large stress concentrations at the crack tip occurred, indicating that this configuration favors crack propagation – in line with previous results (Schweizer et al., 2011).

So far we have only reported results approximating a PST in the flat (slope angle 0°). On the slope the maximal tensile stress increases so that slightly higher slab strength would be required to prevent slab fractures.

3.3 Field data

The vast majority (N=736) of fracture types observed in the 1037 PSTs was propagation to the end of the column (END). Fracture arrest (ARR) was recorded 201 times and slab fracture (SF) 100 times. Grain type in the weak layer was predominantly surface hoar.

Contrasting the field data with respect to the fracture type (END, ARR, SF) showed clear differences for most variables between the three groups. Propagation saw tests with slab fractures had significantly shorter critical cut length, lower slab density (Fig. 3a), smaller load on the weak layer and slabs were also softer (Fig. 3b) than in PSTs with fracture type END or ARR. All these differences were statistically significant (H-test, $p<0.001$).

This suggests that slab properties prevented full propagation and caused slab fracture. However, we do not know enough about the weak layer properties – though crack propagation depends on both weak layer and slab properties. In an attempt to compare slab and weak layer properties jointly, we assigned the weak layers, depending on grain type, a weak layer fracture energy w_f. The parameterization was based on the median values of specific fracture energy reported by van Herwijnen et al. (2014). Fig. 3c suggests that the influence of the weak layer properties was rather minor, probably mainly due the fact that similar weak layers were tested: in all three groups weak layers of buried surface hoar dominated.

Also, in those tests when fracture arrest was observed, the properties of the slab were not as different from the properties in the tests where the crack propagated to the end of the column. However, we cannot conclude whether arrest was due to changes in slab properties or weak layer properties (hypotheses 1 or 2, respectively).

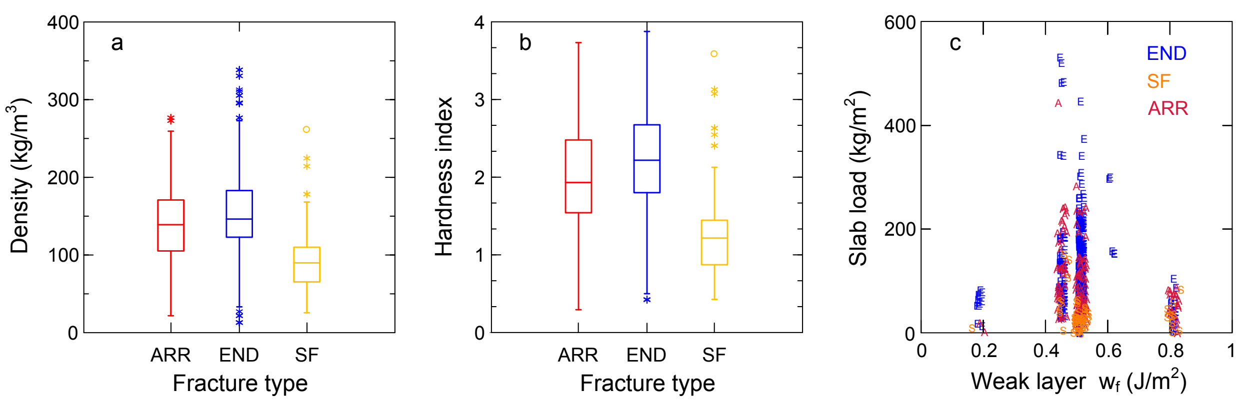

Fig. 3: Propagation saw test characteristics contrasted for the three fracture types (ARR, END and SF). (a) for average slab density (N = 863), (b) for average slab hand hardness index (N = 1021), and (c) for load on the weak layer and weak layer fracture energy (N = 879).

4. SUMMARY

We explored the effect of the tensile strength of the slab on crack propagation in propagation saw experiments. In particular, we focused on slab fractures, i.e. those cases when the running crack arrested due to a fracture trough the slab (tensile crack).

A simple cantilever beam model as well as finite element simulations suggest that in fact the tensile strength of low density, soft slabs may prevent full propagation (hypothesis 3). Whereas these slabs provide plenty of strain energy for propagation so that cracks initiate at short length, they fail due to limited tensile strength. Furthermore, the tensile stress increases during dynamic propagation as has been recently shown by Gaume et al. (2014).

The limit for density with respect to slab fractures in PSTs cannot be clearly determined but seems to be about 100 kg/m^3 based on the field measurements.

Our results suggest that the important properties of the slab with respect to propagation propensity are density, modulus and tensile strength. It seems that the lack of full propagation may well be explained by slab and possible the weak layer properties without the need for a sustainability term as suggested by Gauthier and Jamieson (2010). With respect to avalanche release clearly some of the discrepancy between PST results and avalanche triggering probability is related to the column size, in particular length (Bair et al., 2014).

Finally, we would like to point to some limits of the PST. Despite the fact that we learned a lot about crack propagation while performing propagation saw tests, not all findings can simply be transferred to the process of avalanche release. The procedure of the saw cut is peculiar and may have limited equivalence to the failure process preceding the spontaneous release of a slab avalanche. Furthermore, isolating the column from the surrounding snow cover is a strong limitation common to all snow instability tests.

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