

Validating a relationship between avalanche runout distance and frequency

Alexandra Sinickas*, Bruce Jamieson
Department of Civil Engineering, University of Calgary, Alberta

ABSTRACT: Statistical runout models can provide good estimates of extreme avalanche runout, but do not estimate more frequent runout distances such as ten or thirty-year events. These distances are critical in many hazard mapping and analysis projects. This paper presents the first validation results of a model that expresses return period as a function of runout distance. We compared modelled runout distances with observed runout distances to 129 trim lines from 34 paths in the Canadian Columbia and Rocky Mountains. We made two key findings. First, modelled runout distances were consistently downslope of observed runout distances, implying that either the model or the validation method is biased. Second, the model performed better for paths within the Rocky Mountains (median residual 3 to 11 % of path length) than for paths within the Columbia Mountains (median residual 17 to 35 % of path length).

KEYWORDS: Runout Ratio, frequency, magnitude, Poisson, avalanche runout, trim lines

1. INTRODUCTION

Snow avalanche risk assessments for development in avalanche terrain require estimates of not just extreme (100 to 300 year), but interim (<100 year) avalanche runout. Examples from the Guidelines for Snow Avalanche Risk Determination and Mapping in Canada (Canadian Avalanche Association, 2002) include:

- Ski lift tower, 10 years;
- Telephone pole, 10 years;
- Industrial road, 10 years; and
- delineating between Red, Blue & White Zones in hazard mapping, 30 years.

In addition, the Avalanche Terrain Exposure Scale uses the 30 year return period as one method of delineating between Simple and Challenging terrain (Statham et al., 2006)

Statistical models, such as the $\alpha - \beta$ and Runout Ratio, are one way to estimate extreme runout distance, but do not estimate runout distance for more frequent avalanches. McClung (2000) presented a model that extended the Runout Ratio model to include estimates of runout distance (Space) as a function of avalanche return period (Time).

The Space-Time (ST) model has many potential benefits. First, it is simple, and based on a widely used model (Runout Ratio). Second, it is probabilistically based, meaning that engineers and planners can define runout distance based on likely exceedance levels. Third, and most obvious, the model estimates runout distance for more frequent avalanches, undetermined by previous models.

McClung (2000) successfully applied the ST model to a path in Norway (Bleie), but its application to a wider set of paths has yet to be published. The goal of this study was to validate the ST model, thereby improving its value for risk analysis. The objectives were to 1) compare modelled runout distances with field-observed runout distances for interim avalanches; and 2) assess model performance across two mountain ranges.

2. HOW THE MODEL WORKS

The ST model is based on a compound distribution, meaning that two different aspects of avalanche behaviour are modelled using two different probability distributions. These aspects are 1) runout distance modelled using a Gumbel distribution; and 2) avalanche arrival rate modelled using a Poisson distribution.

This section explains how these two distributions combine. Equations are included below but are not critical to understanding how the model works.

2.1 Runout distance (Space)

Recall that the Runout Ratio model estimates the extreme (100 to 300-year) runout position (α point), using a Gumbel distribution to model the dimensionless ratio $r = \Delta x / X_\beta$ (McClung and Lied, 1986; McClung et al., 1989; McClung and Mears, 1991) (Figure 1).

The Gumbel distribution belongs to a family of distributions used to model the maximum (or minimum) values of a random variable. These distributions can estimate values higher than those recorded during the observation period. For example, the Gumbel distribution can estimate the 100-year flood height of a river, based on only 30 years of records. It is also

Corresponding author address: Alexandra Sinickas, University of Calgary, AB, Canada;
tel: +1 403 919 9737;
email: asinicka@ucalgary.ca

commonly applied to maximum storm precipitation amounts to estimate extreme snow loading in start zones.

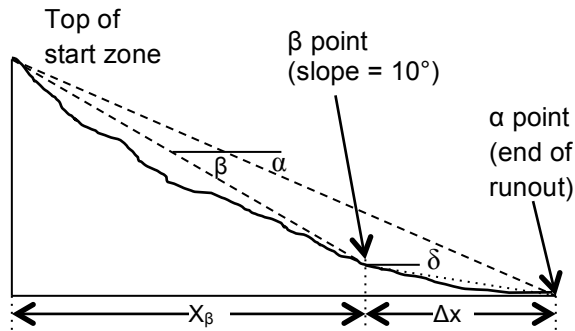


Figure 1. Typical avalanche path geometry

2.2 Avalanche arrival (Time)

Avalanche arrival rate, or frequency, is modelled with a Poisson distribution, which describes the number of discrete events within a continuous time period. It is often used to model the occurrence rate of hazardous natural events (Smith and McClung, 1997). One famous example (amongst statisticians at least) comes from the Russian, Ladislaus von Bortkewitsch, who in 1898, used the Poisson distribution to describe the number of Prussian military men killed by horse-kicks. A different kind of natural hazard.

2.3 Combined runout distance and arrival

When the two distributions are combined, the ST model can calculate either the runout distance to a given return period, or the return period to a given runout distance. In practice, inputs include:

- known return period at the β point (T_0);
- horizontal distance from the start zone to the β point (X_β);
- Gumbel parameters appropriate for the range (u and b);
- Return period(s) where the runout distance is desired or known (T_x); and / or
- Runout distance(s) where the return period is desired or known (X_T).

The extra inputs required to transition from the Runout Ratio to the ST model are shown in bold.

The underlying probability distributions allow users to place a confidence limit on the estimated runout distance. While McClung (2000) chose a Gumbel distribution to represent runout (McClung and Mears, 1991) and a Poisson distribution to represent frequency (Smith and McClung, 1997), any distribution could be used. e.g. exponential (Hamre and McCarty, 1996).

The model works most simply when the return period at the β point is known; however, if the return period at β is unknown, the return period at a reference point (RP) further upslope or downslope can be used. This requires an adjustment to the location parameter, u , where $u' = u + r_{RP}$, $r_{RP} = (X_{RP} - X_\beta) / X_\beta$ is the Runout Ratio at RP and X_{RP} is the horizontal distance from the start zone to RP.

2.4 The equations

The probability of not exceeding a given runout ratio (Space) is given by the Gumbel cumulative distribution function:

$$F(x) = \exp \left[-\exp \left(-\frac{r-u}{b} \right) \right] \quad (1)$$

where r is the given runout ratio, and u and b represent Gumbel location and scale parameters, which vary across mountain ranges (McClung and Mears, 1991; Lied et al., 1995; Johnston et al., 2012).

The probability of avalanche arrival (Time) is given by the Poisson probability density function:

$$P(\mu_0, n) = \frac{e^{-\mu_0} \mu_0^n}{n!} \quad (2)$$

where μ_0 is the expected annual arrival rate ($1/T_0$) at the β point, T_0 is the return period at the β point (years) and n is the number of random events. e.g. $n = 0, 1, 2, \dots$

The probability of an avalanche running a certain distance with a certain return period (Space-Time) is given by the compound cumulative distribution function:

$$F(x, \mu_0) = \sum_{n=0}^{\infty} P(\mu_0, n) [F(x)]^n \quad (3)$$

$$= \exp(-\mu_0(1 - F(x)))$$

where $F(x, \mu_0)$ describes position and frequency at a given point.

3. METHODS

To validate the ST model, we compared its estimated runout distances against those observed in the field for 38 paths in the Canadian Rocky, Purcell and Columbia Mountain Ranges. An additional validation, based on occurrence records collected by the BC Ministry of Transportation for 42 paths in the Coast Range, can be found in Sinickas (2013). The steps of the validation are:

1. **Survey** the centerline in the lower track and runout zone of each avalanche path to determine:
 - RP location its return period; and / or
 - β point location and its return period; and
 - Key trim lines (3 to 6 for each path).

2. **Model** runout distance to key boundaries identified in the field using ST.
3. **Compare** field-observed with ST-modelled runout distances

3.1 Step 1: Field surveys

Field surveys were conducted during the 2012 summer. Selection criteria excluded paths with runup (avalanches running up the other side of the valley), short vertical drop (< 350 m), plunging behaviour, evidence of only wet flow, defence structures and frequent avalanche control. Paths with distinct trim lines were selected in preference to those without.

For each path, the β point (slope 10°) was identified using standard methods (Sinickas and Jamieson, 2012). Return periods at key trim lines were identified through analysis of reaction wood within slices (Figure 2) and cores of selected trees.



Figure 2. Cross section (slice) of a 30 year old spruce, with reaction wood from about 20 years ago. The pencil shows the direction of avalanche flow. Photo: A. Sinickas

Additional observations included date and location of scars, broken branches, tree height, etc. Detailed vegetation analysis methods are contained in Burrows and Burrows (1976) and Canadian Avalanche Association (2011).

An example of field survey output is shown below.

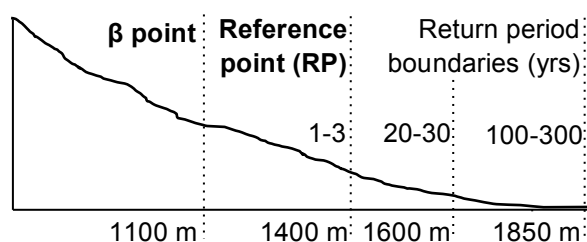


Figure 3. Sample field survey output showing changes in return period through the runout zone. Note that the return period may or may not be known at the β point.

3.1.1 Limitations

Vegetation damage analysis is subject to relatively high error (Reardon et al., 2008) for several reasons: 1) Vegetation is highly varied in itself. It can grow faster in some areas, and if subject to unseasonal temperature fluctuation can create false (intra-annual) growth rings, or skip growth rings entirely. 2) Large avalanche events can destroy trees, which contain valuable evidence of previous events. 3) Not all avalanches leave evidence in the vegetation. 4) The number of samples within a field survey can be limited. Corona et al. (2012) recommended an optimal sample of 100 to best identify avalanche events after showing that only 40 % were deciphered using vegetative analysis, but this may be impractical for many projects.

To mitigate uncertainty during this study, samples were selected based on surrounding evidence (skilled) rather than performing transects (unskilled). A minimum avalanche size (Destructive Size 2, impact pressure > 50 kPa, volume > 100 m³) was established to focus on extreme events, rather than every event. Also, observations of return period were recorded as a range, where a wider range reflected higher uncertainty. Ranges were calculated by averaging events over time, in different combinations. This method helped to identify extra-ordinary events if one range was much larger or smaller than another. For example, if a tree cross section showed reaction wood from 60, 45, 32, 27, 10 and 3 years ago, then the return period would be calculated using:

6 events since 1952 (60 years) = 1 in 10 years
 5 events since 1967 (45 years) = 1 in 9 years
 4 events since 1980 (32 years) = 1 in 8 years
 3 events since 1985 (27 years) = 1 in 9 years
 Return period range: 1 in 8 - 10 years

3.2 Step 2: Model runout distances

The ST model can calculate either the runout distance to a desired return period (X_T), or the return period at a desired runout distance (T_X). In this study, the ST model was used to calculate X_T based on the return period observed at the trim line in the field.

Inputs and typical values for the 34-path dataset are shown in Table 1. Note that T_0^* refers to the return period at either at β or RP. The Gumbel parameters (u , b) were (0.079, 0.07) for the Rocky / Purcell range and (0.105, 0.083) for the Columbia range.

Table 1. Descriptive statistics

Input	Unit	Min	Max	Average
T_0^*	years	1	20	5
X_{β}^*	m	575	2975	1620
T_X^*	years	2-3	100-300	50

*observed values

3.3 Step 3: Compare observed and modelled runout distances

For every trim line observed in the field, a minimum and maximum runout distance was modelled using the inputs listed in Table 1.

The distance between these modelled runouts and those observed in the field (a.k.a. the residual) was calculated as $X_T - X_T^*$ for all trim lines ($n = 129$). For example, if a 30-year trim line was observed in the field at 1500 m, and the model estimated the 30-year trim line somewhere between 1600 and 1800 m, then the residual = $X_T - X_T^* = 1600 - 1500 = 100$ m (Figure 4).

The smallest distance between modelled and observed runouts was always used as the residual (i.e. 100 m rather than 300 m in Figure 4). For scenarios where the observed runout (point) was within the modelled runout (range), the residual was recorded as zero. Positive values represented scenarios where modelled runout distance (X_T) was downslope of observed runout distance (X_T^*) (same configuration as in Figure 4).

Residuals were scaled by X_{β}^* to account for different path sizes using $(X_T - X_T^*) / X_{\beta}$. Residuals were then categorized into < 10 year, 10 to 20 year, 20 to 50 year, 50 to 100 year and 100 to 300 year trim lines to reflect return periods typically used in avalanche risk analysis. Ranges increase at higher return periods to reflect the scale effect of uncertainty (higher uncertainty at longer return periods).

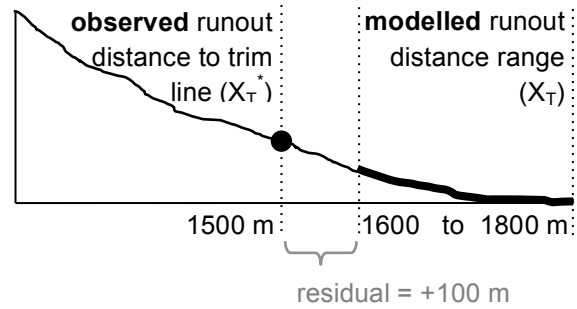


Figure 4. Sample profile showing residual calculation method for a given trim line.

4. RESULTS

Box and whisker plots of scaled residuals were created for all paths (Figure 5), and then split into trim lines within the Rocky / Purcell range and the Columbia range (Figure 6).

Plots can be interpreted by visualizing an avalanche path with the start zone above the top of the plot, and the runout zone at the bottom. Note that the vertical axis has been reversed so that positive residual values appear in the lower portion of the plot. Perfect agreement between observed and modelled runout distances lies on the zero line. As an example, for a path where $X_{\beta} = 1000$ m, a residual of + 0.2 represents a scenario where the modelled runout distance (X_T) is 200 m farther downslope of the observed runout distance (X_T^*).

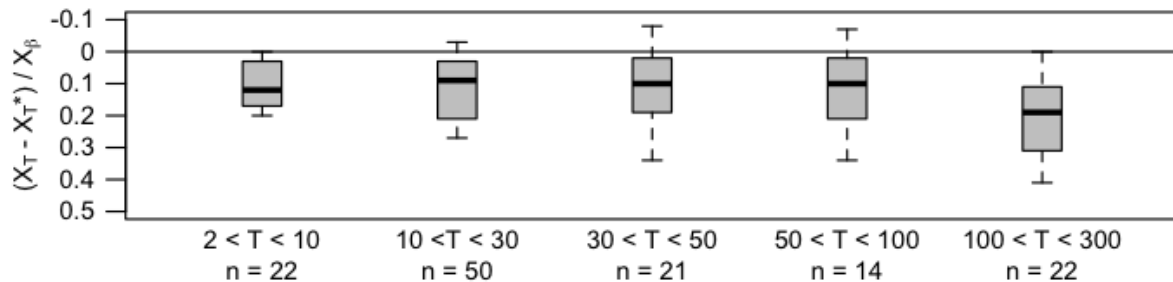


Figure 5. Scaled residuals for all paths ($n = 129$ trim lines from 34 paths). Grey boxes show interquartile range and whiskers show limits 1.5 times the interquartile range.

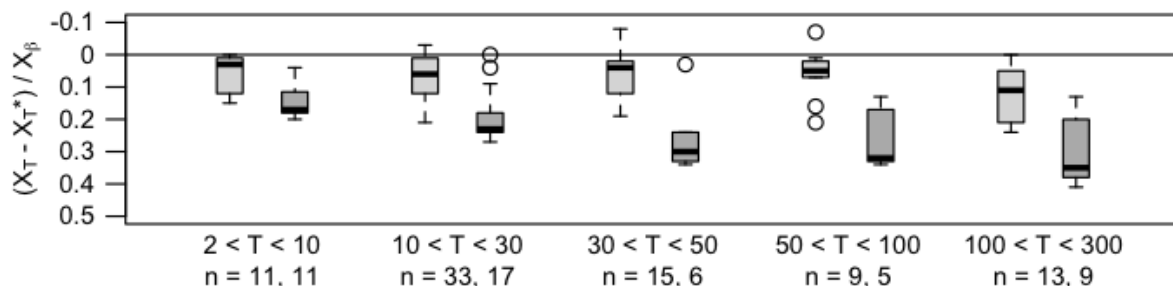


Figure 6. Same as Figure 5, but split into Rocky / Purcell paths ($n = 81$ data points from 20 paths) and Columbia paths (48 data points from 14 paths). Rocky / Purcell results are light, Columbia results are dark. Open circles are outliers.

In both plots, median residual values are positive. In Figure 5, medians remain steady around 0.1 for $T < 100$ years, and jump to around 0.2 at the 100 to 300-year trim line. In Figure 6, the Rocky / Purcell residuals are considerably smaller than the Columbia residuals; Rocky / Purcell medians hover around 0.05 while Columbia medians range from 0.17 to 0.32 for $T < 100$ years.

5. DISCUSSION

Three findings arise from the results:

5.1 Finding 1 - all medians are positive

Consistently positive medians imply that either systematic error occurred during the validation process, or the ST model is conservative.

Systematic error in the validation could arise from using vegetative damage to determine avalanche frequency (Section 3.1.1). If avalanche frequency was indeed underestimated because of these limitations, then modelled X_T would be farther downslope of the incorrectly observed X_T^* . Without reliable historical records, it is difficult to estimate the scale of this error, and to determine whether it caused the positive medians.

Arguments for systematic error in the model include:

- The ST model may produce conservative results for paths in this study because it is being used to calculate much shorter return periods ($T < 100$ years). McClung (2000) calculated return periods up to 2000 years using reference return periods of 50 and 100 years for the Bleie path in Norway.
- the selection of the reference return period at RP rather than at the β point may affect results (Sinickas, 2013)
- The Runout Ratio, upon which the ST model is based, has been shown to be conservative for high values of non-exceedance probability (P), flat runout zones and for short slopes (McClung, 2001; Jones and Jamieson, 2004). Perhaps similar properties cause conservative results in the ST model.

Also, the conservative bias observed in the results could be partly due to the adjustment of the location parameter, and partly due to validation methods.

5.2 Finding 2 - model performance varied across mountain ranges

The difference between avalanche frequency and magnitude for different mountain ranges is incorporated into the ST model

through the Gumbel location (u) and scale (b) parameters, where u describes the peak of the distribution and b describes the spread (Figure 7).

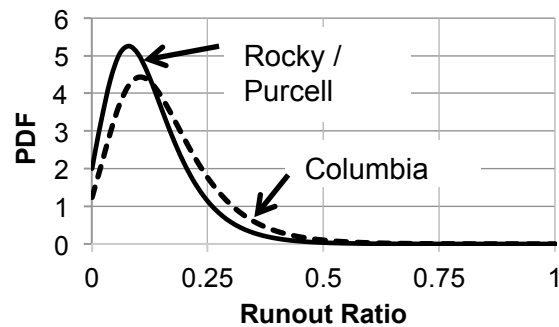


Figure 7. Runout Ratio (Gumbel) Probability Density Functions (PDF) for the Rocky / Purcell and Columbia Mountains.

McClung (2000) proposed that runout distance is mostly dependent on terrain, while frequency is mostly dependent on snow supply. In his interpretation of the Gumbel parameters, McClung showed that the b is dependent on terrain steepness and that u tends to increase with higher frequency.

The assumption that runout distance mostly depends on terrain is based on extreme avalanche behavior. We speculate that for shorter return periods, snow supply could have more influence. McClung (2003) found that predictor variables included both snow supply and path steepness in assessing average avalanche magnitude and frequency for 194 paths in BC.

Perhaps b requires modification when using the ST model to calculate interim return periods. Or perhaps the desired return periods must be closer to $T = 100$ years, for which b is calibrated.

5.3 Finding 3 – Medians increased at the extreme (100 – 300 year) trim line

Rocky / Purcell residuals increased at the 100- to 300-year trim line. Although this is an important consideration when estimating longer return periods, it is less important for this study because longer return periods (> 300 years) are not typically estimated in Canadian practice and many other methods exist for estimating 100 to 300-year runouts.

6. CONCLUSION

The ST model estimates runout distance as a function of return period down the centerline of an avalanche path. This feature is a valuable assessment tool for hazard mapping.

The goal of this study was to validate the ST model based on field-based observations of runout distances and return periods. The

primary objective was to compare modelled runout distances with those observed in the field. The secondary objective was to compare model performance across mountain ranges.

Two key findings were drawn from the results. First, modelled runout distances were consistently downslope of observed runout distances. This implies that either the ST model is conservatively biased, and/or the validation is biased. Both options are plausible. Second, median residuals (modelled distance - observed distance) were smaller for the Rocky / Purcell dataset (3 to 11 % of X_β), than the Columbia (17 to 35 % of X_β) datasets. This implies that although the model adjusts for different terrain and climate characteristics, it does not perform equally across mountain ranges.

Limitations included uncertainty surrounding return period estimation using vegetation damage (Section 3.1.1 and 5.1) and a small sample size (129 trim lines from 34 paths).

This paper presents part of the first systematic validation of the ST model. See Sinickas (2013) for complete results. The findings should increase confidence and use by hazard mappers. Users should expect estimates to be conservative, and accuracy to vary across mountain ranges.

7. FURTHER RESEARCH

To implement the ST model more widely, research could focus on applying the model to more paths in more regions. If return periods are determined through analysis of vegetation damage, then more samples could be collected to improve confidence in the results.

Additional research could focus on the effect of using the return period at RP rather than at β on estimated runout distances. This would help to identify the source of bias, or lack thereof, in the ST model.

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