A PHASE-TRACKING SNOW MICRO-STRUCTURE MODEL

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ABSTRACT: Depending on the thermal conditions, snow metamorphoses via two processes: kinetic or equilibrium metamorphism. Kinetic growth occurs due to temperature gradients within the snow and results in faceted snow crystals. Equilibrium growth occurs in the absence of a gradient and results in rounded crystals. Generally, snow models are limited in their ability to model these micro-structural changes, especially in three dimensions, and rely on effective material properties. To enhance the tools available to avalanche researchers a new 3D finite element (FE) model capable of modeling snow metamorphism is being developed using techniques common for modeling phase transitions and crack propagation in alloy solidification. The FE model operates at the micro-structural level and is capable of tracking vapor deposition and sublimation within the snow. This is accomplished using a level set approach that combines features of front-tracking and fixed domain methods. It incorporates adaptive meshing techniques to efficiently track heat and mass transport and includes micro-convection on the phase-change boundary. This work is the part of an ongoing research project that aims to demonstrate the ability to model snow at the micro-structural level and move away from the more common coarse, effective property modeling techniques. The model will also serve as the deterministic basis for a multi-scale, stochastic model of snow that will account for uncertainties such as poorly understood growth properties and measurement variability. Future applications may also include external forces, yielding a thermo-mechanical model that could evolve.

1. INTRODUCTION

Over the past ten years the United States has averaged 25 fatalities annually from avalanches (CAIC, 2012), which is of similar magnitude as those resulting from lightning strikes (37), tornadoes (108), floods (78), and hurricanes (114) (NWS, 2012). In particular, faceted snow is of importance; it has been shown to account for up to 73% of human-triggered avalanches (Schweizer and Jamieson, 2001, Tremper, 2001). Despite this, modeling has generally been limited to one dimensional models that rely on effective material properties. To enhance the current abilities for modeling snow metamorphism, a 3D finite element (FE) model capable of tracking the phase transitions at the micro-structure level is under development.

2. BACKGROUND

Analytical work surrounding the thermal behavior of snow has evolved from research as early as 1892 that examined temperature profiles of snow (see LaChapelle, 1960) to research using a continuum mechanics theory of mixtures (e.g., Brown et al., 1999) to a thermal non-equilibrium approach (Bartelt et al., 2004). To date, perhaps the most comprehensive model developed is the SNOWPACK model (Lehning et al., 2002): a one-dimensional, three-phase model that accounts for heat transfer, water transport, vapor diffusion, and mechanical deformation with special conditions for wind drifting and snow ablation (Bartelt and Lehning, 2002). Miller and Adams (2009) provided a unique approach for modeling kinetic crystal growth that assumed spherical geometry. Recent approaches are based on 3D images of the snow micro-structure. Kaempfer and Plapp (2009) demonstrated that methods similar to those proposed herein may be applied to snow metamorphism.

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3. METHODS

The basic methodology model under development is derived from methods used for predicting phase transitions in alloy solidification (Samanta and Zabaras, 2005). This work employs a finite element solution for the mass, momentum, energy, and species transport equations; the governing equations of which are provided in equations (1)–(4), respectively.

$$\frac{\partial \rho_k}{\partial t} + \nabla \cdot (\rho_k \vec{v}_k) = 0, \tag{1}$$

$$\frac{\partial \rho_k \vec{v}_k}{\partial t} + \nabla \cdot (\rho_k \vec{v}_k \vec{v}_k) = \nabla \cdot \vec{\sigma}_k + \vec{b}_k, \tag{2}$$

$$\frac{\partial \rho_k h_k}{\partial t} + \nabla \cdot (\rho_k h_k \vec{v}_k) = -\nabla \cdot \vec{q}_k, \tag{3}$$

$$\frac{\partial \rho_k C_k}{\partial t} + \nabla \cdot (\rho_k C_k \vec{v}_k) = -\nabla \cdot \vec{j}_k, \tag{4}$$

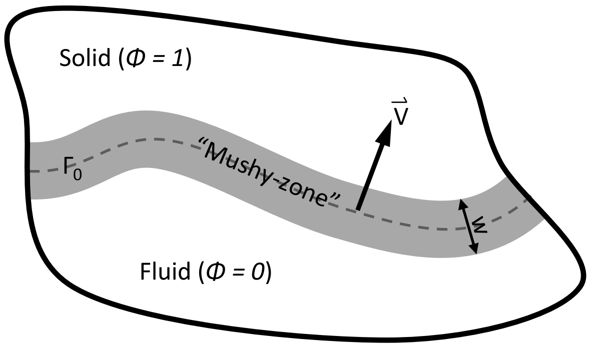

Figure 1: Schematic of phase-transition region of the snow micro-structure model.

where the subscript k specifies the phase (solid or fluid), ρ is the density, $\vec{v}$ is velocity, h is enthalpy, t is time, σ is the stress tensor, $\vec{b}$ is the body force, $\vec{q}$ is the heat flux, and $\vec{j}$ is the species diffusion flux. The fluid is assumed to be Newtonian, thus $\vec{\sigma}$ is a function of pressure and the enthalpy is assumed to be an explicit function of temperature.

The solution strategy for the governing equations includes applying a volume averaging scheme, which allows the above equations to apply to the entire domain regardless of phase. This introduces the function $\Phi(\vec{x}, t)$ of equation (5), where $\vec{x}$ is position. The volume-averaged versions of the equations are then solved using a stabilized finite element method, the details of which are beyond the scope of this paper (see Zabaras and Samanta, 2004, Samanta and Zabaras, 2005).

$$\Phi(\vec{x}, t) = \begin{cases} 1 & \phi > w, \\ 0 & \phi < -w, \\ \frac{\phi}{2w} + 0.5, & \phi \in [-w, w], \end{cases} \tag{5}$$

where ϕ is the distance to the phase interface (Γ_0) that has a width w, as shown in figure 1.

Along the phase-change boundary a "mushy-zone" is established. This zone is modeled as a porous media comprised of solid and fluid components and includes micro-scale convection due to the phase change and shrinkage (Zabaras and Samanta, 2004). The interface position is defined implicitly in the aforementioned function ϕ and its motion is governed by the level set equation:

$$\phi + \vec{V}|\nabla\phi| = 0, \tag{6}$$

where $\vec{V}$ is the interface velocity (Tan and Zabaras, 2006). This velocity is computed according to the work conducted by Miller and Adams (2009), which computes velocity as a function of crystal habit and excess vapor pressure and allows for sublimation and deposition to occur. Equation (6) is solved using a discontinuous Galerkian formulation (Di Pietro et al., 2006).

The model is being developed as an open-source C++ framework that relies on the libMesh finite element library (Kirk et al., 2006). As such, it includes adaptive mesh coarsening and refinement, relies on domain decomposition for optimum parallel performance, and is capable of running on massively parallel machines.

4. RESULTS AND VALIDATION

The research program established to develop this model is only half-way through the two years allocated; thus, the model is currently under development. The model, once complete and tested, in the next six months, will serve as the deterministic basis for a stochastic analysis that will account for uncertainties in the model such as poorly understood growth properties and measurement variability in the model inputs.

Validation will use μ-CT data from two institutions: Montana State University and the Swiss Federal Institute for Snow and Avalanche Research, see Schneebeli and Sokratov (2004) and Stanton et al. (2012) for μ-CT image examples. The aforementioned stochastic framework that will be built around the deterministic model will produce a range of possible outcomes for any given simulation scenario, making the comparison to measured data, in the form of μ-CT images, straightforward: If the model is accurate the measured data should fall within the range of possible outcomes provided by the stochastic analysis.

5. CLOSING REMARKS

The ability to model snow is vital to improving the ability to predict and prevent avalanches, the ongoing research presented in this paper aims to provide the tools necessary to accurately predict the microstructural evolution of snow with numerical simulations and shift away from the more usual coarse, effective material property modeling techniques. The model will serve as the deterministic basis for a multiscale, stochastic model of snow that will account for uncertainties in the system providing a distribution of possible micro-structures, as is observed in nature. Future applications may also include external forces, the introduction of liquid phase, and incorporation into regional scale models, yielding a true thermomechanical model for avalanche prediction.

6. ACKNOWLEDGMENTS

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7. REFERENCES

Bartelt, P., O. Buser, and S. Sokratov, 2004: A nonequilibrium treatment of heat and mass transfers in alpine snowcovers. *Cold Regions Science and Technology*, **39**, 219–242.

Bartelt, P. and M. Lehning, 2002: A physical SNOWPACK model for the Swiss avalanche warning. Part 1: numerical model. *Cold Regions Science and Technology*, **35**, 123–145.

Brown, R., M. Edens, and M. Barber, 1999: Mixture theory of mass transfer based upon microstructure. *Defense Science Journal*, **49 (5)**, 393–409.

CAIC, 2012: Colorado Avalanche Information Center: Avalanche Accident Statistics (CAIC). `http://avalanche.state.co.us/acc/acc_stats.php`.

Di Pietro, D., S. Lo Forte, and N. Parolini, 2006: Mass preserving finite element implementations of the level set method. *Applied Numerical Mathematics*, **56 (9)**, 1179–1195.

Kaempfer, T. and M. Plapp, 2009: Phase-field modeling of dry snow metamorphism. *Physical Review E*, **79 (3)**, 31 502.

Kirk, B. S., J. W. Peterson, R. H. Stogner, and G. F. Carey, 2006: `libMesh`: A C++ Library for Parallel Adaptive Mesh Refinement/Coarsening Simulations. *Engineering with Computers*, **22 (3–4)**, 237–254, `http://dx.doi.org/10.1007/s00366-006-0049-3`.

LaChapelle, E., 1960: Critique on Heat and Vapor Transfer in Snow. *U.S. Department of Agriculture, Alta Avalanche Study Center, Progress Report No. 1*.

Lehning, M., P. Bartelt, R. Brown, and C. Fierz, 2002: A physical SNOWPACK model for the Swiss avalanche warning. Part III: Meteorological forcing, thin layer formation and evaluation. *Cold Regions Science and Technology*, **35**, 169–184.

Miller, D. and E. Adams, 2009: A microstructural dry-snow metamorphism model for kinetic crystal growth. *Journal of Glaciology*, **55 (194)**, 1003–1011.

National Weather Service (NWS), 2012: Office of Climate, Water, and Weather Services: Natural Hazard Statistics. URL `http://www.weather.gov/os/hazstats.shtml`, URL `http://www.weather.gov/os/hazstats.shtml`.

Samanta, D. and N. Zabaras, 2005: Modelling convection in solidification processes using stabilized finite element techniques. *International Journal for Numerical Methods in Engineering*, **64 (13)**, 1769–1799, doi:10.1002/nme.1423, URL `http://dx.doi.org/10.1002/nme.1423`.

Schneebeli, M. and S. Sokratov, 2004: Tomography of temperature gradient metamorphism of snow and associated changes in heat conductivity. *Hydrological Processes*, **18 (18)**, 3655–3665.

Schweizer, J. and J. B. Jamieson, 2001: Snow cover properties for skier triggering of avalanches. *Cold Regions Science and Technology*, **33 (2-3)**, 201–221.

Stanton, B., D. Miller, and E. Adams, 2012: Analysis of surface hoar growth under simulated meteorological conditions. *International Snow Science Workshop*, Anchorage, AK.

Tan, L. and N. Zabaras, 2006: A level set simulation of dendritic solidification with combined features of front-tracking and fixed-domain methods. *Journal of Computational Physics*, **211 (1)**, 36–63.

Tremper, B., 2001: *Staying Alive in Avalanche Terrain*. The Mountaineers, Seattle, WA.

Zabaras, N. and D. Samanta, 2004: A stabilized volume-averaging finite element method for flow in porous media and binary alloy solidification processes. *International journal for numerical methods in engineering*, **60 (6)**, 1103–1138.