FINITE ELEMENT MODELS AND SENSITIVITY ANALYSIS OF THE VULNERABILITY OF AN AVALANCHE PROTECTION GALLERY

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ABSTRACT: Our research work has focused on the Montaulever avalanche protection gallery. Horizontal and vertical structural elements are modelled herein using the finite element (FE) code Abaqus, under both static and dynamic avalanche loads. A comparison of certain vulnerability indicators, such as maximum displacements or stresses in concrete and rods, and the damage zone, demonstrates the need for dynamic analysis when designing this kind of structure. A probabilistic sensitivity analysis of these indicators with respect to uncertain FE model parameters for a protection gallery column is also proposed. The input of uncertain parameters, like Young's modulus of concrete and avalanche amplitude (vertical and tangential dynamic), are modelled by random variables that follow a log-normal law. Next, vulnerability indicator means and square deviations are approximated using a probabilistic approach, making it possible to draw conclusions on column behaviour.

KEYWORDS: avalanche protection gallery - finite element model - probabilistic sensitivity study.

1. INTRODUCTION

In the field of natural risk management, the vulnerability of protective engineering structures subjected to various kinds of phenomena, such as fire, flooding and earthquakes, may be defined as a predictor of structural damage for a range of intensities relative to the considered action. Damage behaviour is extensively used in studies of structures and geomaterials; it assumes a complex and generally nonlinear material behaviour, with respect to irreversible states under extreme loadings. The quantification of vulnerability thus requires complex constitutive laws and models, in addition to knowledge about the potential real load. Back-analyses, like that conducted on Taconnaz deviatory teeth (Berthet-Rambault, 2007), reveal the importance of taking the dynamic behaviour of both load and structure into account. This importance has been confirmed by a study of the horizontal and vertical elements in a protective gallery. Moreover, the authors demonstrate the critical nature of a structure’s free period as well as the dynamic characteristic of avalanche loading in structural damage (Daudon, 2009; Ying, 2008).

It is also helpful to quantify the sensitivity of damage response of these structural elements to the variability of certain design parameters related to either the concrete or the actual avalanche load amplitude. Such parameters could in fact differ slightly between the structure in reality and results from the design step. This paper will thus present a complex finite element model of a column in a recently-built protective gallery. Its damage response consists of a number of vulnerability indicators, including maximum displacements or stresses in both concrete and rods. After studying the behaviour upon undergoing a real dynamic load, the discussion will focus on studying the sensitivity of these indicators to both the Young's modulus of concrete and avalanche amplitude.

2. FINITE ELEMENT MODEL

The present study concerns the Montaulever avalanche protection gallery (Cemagref, 2000 - see Fig. 1) and examines the behaviour of a current column subjected to: its own weight, earth pressure, and an avalanche load (Szczurowska, 2008).

![Figure 1: a) Schematic view of the gallery section, b) Characteristic sequence of the tunnel and components](image)

2.1 Geometry and boundary conditions

The structure's geometry is depicted in Figure 1. The model features two main components: the 4.45-m column height with a cross-section equal to 1.8 m², and the 6.0-m long transverse beam (cross-section: 1.1 m²). The boundary conditions applied to this model correspond to the column's position within the structure.

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The bottom of the column is assumed to be embedded. The transverse beam part has only been fixed in the horizontal direction. The numerical model consists of 4-node tetrahedral elements for the concrete column and linear elements for the reinforcement. The approximate overall size of elements equals 0.2 m. The reinforcement is defined as wire elements bonded to the existing concrete. Just the primary reinforcement element has been represented.

2.2 Material models

Let's now consider a nonlinear constitutive law for both concrete and steel rods in order to describe structural behaviour in the ultimate state. The Concrete Damage Plasticity (CDP) constitutive model (Lodygowski, 2005) has been chosen for these purposes. Both compression hardening and tension stiffening of the concrete are taken into account. The constitutive parameters correspond to those of concrete class C50 and are listed in Table 1.

Table 1: CDP model parameters for Class C50 concrete (Lodygowski, 2005)

<table>
<thead>
<tr>
<th>Material parameters</th>
<th>CDP model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete C50</td>
<td>β 38°</td>
</tr>
<tr>
<td>E [GPa]</td>
<td>f=f_{cu}/f_{c} 1.12</td>
</tr>
<tr>
<td>N</td>
<td>γ 0.666</td>
</tr>
</tbody>
</table>

The classical Mises elastoplasticity model for metals is applied here for the reinforcement rods (Table 2).

Table 2: Parameters of the steel plasticity model

<table>
<thead>
<tr>
<th>Type</th>
<th>Orientation</th>
<th>Pressure (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exceptional avalanche</td>
<td>Normal</td>
<td>14</td>
</tr>
<tr>
<td>Tangential</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Snow cover</td>
<td>Vertical</td>
<td>Initial 9, Final 17</td>
</tr>
</tbody>
</table>

2.3 Loads applied to the structure

In our studies, we have taken into account the structure's own weight, the earth pressure generated from the soil located behind the tunnel, and the avalanche impact, in considering the dynamic feature of this load.

The structure's own weight (roof, main beams and crossbeams) is applied to the column as a vertical reaction. Eurocodes recommend a moderation factor of 1.35, in applying densities of 2,500 kg/m³ for concrete and 7,800 kg/m³ for steel. The resultant vertical loads of 432 and 237.5 kN respectively are then applied to each column and between two columns.

The earth pressure introduced stems from indirect loads acting on the column, such as vertical and horizontal reactions. These reactions equal 62.3 and 57.3 kN, respectively, according to soil characteristics typical for gravels (γ=18 kN/m³, Φ = 34.5°, ID = 0.7).

Two avalanche impact loads are applied, i.e.:

1. A static normative design load, estimated by a dynamic pressure E = 360 kPa for an exceptional reference avalanche;

2. An actual transient load, derived from the magnification of an avalanche signal measured at the Col du Lautaret peak (winter 2007).

The dynamic pressure E = 360 kPa corresponds to an exceptional reference avalanche with the following characteristics: velocity = 30 m/s, flow thickness = 3 m, snow density = 400 kg/m³. The vertical and tangential pressures on the roof gallery, denoted \( P_n \) and \( P_t \), can then be calculated (OFROU, 2007).

\[
P_n = \frac{Eh_0 \sin \beta}{L} \quad \text{and} \quad P_t = cP_n
\]  

where \( \beta \) is the deviation angle between the gallery and the ground slope (equal to 20°), \( h_0 \) an avalanche thickness before the break in slope, \( L \) a distance between the avalanche edge and the break in slope (around 20-25 m), and \( c \) a coefficient with a value between 0.3 and 0.4. This steady dynamic pressure leads to loading the column (see Table 3).
The actual transient avalanche load profiles shown in Figure 3 describe both the normal and horizontal loads applied to the gallery column. They are derived from a finite element calculation conducted to estimate reaction load on the column due to an avalanche pressure measured at the Col du Lautaret peak (Ying, 2008). It is considerably magnified, by a factor of 1.6, in order to obtain plasticity within the horizontal structural concrete elements. The two static loads consist of: a reference one. These results are part of a more comprehensive study (Daudon, 2009), which reveals that a static analysis should be complemented by a dynamic analysis, as a result not only of these greater values, but also of the need to adapt reinforcement rod geometry to dynamic action.

The concrete can also be damaged before reaching the static reference load.

Moreover, it seems worthwhile to study the sensitivity of such a dynamic load to the variability in material parameters.

3. SENSITIVITY ANALYSIS OF THE AVALANCHE PROTECTION COLUMN MODEL

This section will present a sensitivity study of the avalanche protection column FE model introduced above. The aim here is to quantify the effect of variability in uncertain model parameters on the variability of vulnerability indicators. The uncertain input parameters in this instance are the Young’s modulus of concrete and the dynamic load, while the vulnerability indicators consist of: the Von Mises stress in steel rods, stresses in concrete, and maximum column head displacement.

3.1 Presentation of the probabilistic approach

The variability of uncertain parameters can be taken into account by modelling input parameters using random variables \( Y = \{Y_1, \ldots, Y_5\} \) with known probability laws. The output parameters \( Z = \{Z_1, \ldots, Z_5\} \) also need to be characterized. The FE model is represented as a function \( f \), such that \( Z = f(Y) \). Let \( g \) be the composite function \( (f \circ T) \) of the mechanical response function \( f \), linking \( Z \) and \( Y \), and the function \( T \) linking \( Y \) with a standard random variable \( X \) (Gaussian) (Baroth, 2007). For the sake of simplicity, let’s consider just the scalar output \( Z \). The mean \( \mu_Z \) and square deviation \( \sigma_Z \) of \( Z \) are approximated such that:

\[
\mu_Z = \sum_{i=1}^{4} \omega_i \cdot g(x_i) \quad \text{and} \quad \sigma_Z = \sum_{i=1}^{4} (g(x_i))^2 \cdot \omega_i - (\mu_Z)^2
\]

with four integration points and weights \((x_i, \omega_i)_{i=1}^{4}\) (Millard et al., 2000). The coefficient of variation of \( Z \), denoted \( \text{Cov}(Z) = \frac{\sigma_Z}{\mu_Z} \), can therefore also be approximated.

In addition, the mechanical response \( Z \) may be approximated using these points and weights, in writing the approximation \( \tilde{Z} \) of \( Z \) as a development of Lagrange polynomials (Baroth et al., 2007), denoted \( L_i \) where for this study \( i \) varies from 1 to 4:

\[
\tilde{Z}(x) = \sum_{i=1}^{4} g(x_i) \prod_{\substack{k=1 \atop k \neq i}}^{N} \frac{x-x_k}{x_i-x_k} = \sum_{i=1}^{4} g(x_i) \cdot L_i(x)
\]

Monte Carlo simulations might eventually be applied to this response surface in order to...
obtain an approximation $p_2$ of the probability density function (PDF) $p_Z$. From this PDF, the probability $P(Z < z^*)$ of remaining below the limit value $z^*$ can be evaluated, provided this probability is not less than a few percent (Humbert et al., 2009).

### 3.2 Effect of variability with the Young’s modulus of concrete

Variability in the Young’s modulus of concrete $E$ will be considered first; this variability is modelled by a lognormal variable, with a mean of 30 GPa and Cov($E$) lying between 2.5% and 15%. Figure 4 displays the evolution in the Cov of vulnerability indicators for various Cov($E$) values. The relation between Cov($E$) and the Cov of maximum concrete stresses is mainly linear, yet such relations become nonlinear and sensitivity becomes more significant regarding both displacement and Von Mises stress (a Cov of 15% and 8% respectively for Cov($E$) = 20%). Figure 5 indicates the evolution in probability when observing a tensile stress of up to 3 MPa in the column for the range of Cov($E$) values. The nonlinear relationship is then perceptible, along with a significant probability that the Young’s modulus coefficient of variation exceeds 5%.

### 3.3 Effect of dynamic load variability

The uncertain parameter considered is the intensity of the dynamic real snow avalanche. This load is modelled as a random Gaussian variable $G$, with a mean of 1 and coefficient of variation between 5% and 20%, making it possible to consider characterizing an average load $y(t)$ by a confidence interval (Fig. 6).

### 3.4 Effect of normal load variability

The normal avalanche load component $y_n(t)$ is defined as the product of the normal top load, equal to 457.5 kN, multiplied by Profile no. 2 of the actual load (Fig. 3). The vulnerability indicator means are nearly constant, even though the coefficients of variation increase from 5% to 20%. The mean Von Mises stress in steel rods equals 40.7 MPa, while the mean maximum tensile stress in concrete is 2.93 MPa and the mean maximum compressive stress in concrete is 9.8 MPa. The mean of the maximum pile head displacements is equal to 3.66 mm. Figure 7 depicts the evolution in coefficients of variation for displacement, Von Mises stress and concrete compressive stress for different coefficients of variation specific to load intensity. Linear relations and the strong dependence of compressive stress can be noticed, along with a non-significant dependence of displacement and Von Mises stress.

### 3.5 Effect of horizontal load variability

The horizontal avalanche load component $y_h(t)$ has been defined in Figure 3. Table 6 presents the means, square deviations, coefficients of variations and characteristic values of $X_k$ for certain parameters. This value of $X_k$ is derived such that 95% of outcomes are less than $X_k$.

The vulnerability indicator means are no longer constant for the different coefficients of variation on uncertain parameters. Displacement and maximum plastic strain are particularly sensitive to variations in the horizontal avalanche component. The coefficients of variation for these parameters are thus respectively 54% and 20% for a 20% variation in horizontal load. Since the effect of these parameters on the normal load is quite insignificant, it can be deduced that the maximum displacement and plastic strain of steel rods are also highly sensitive to the ratio of...
normal to horizontal loads \( y/y_h(t) \).

![Figure 7: Evolution in the Cov of vulnerability indicators vs. coefficient of variation for the vertical dynamic load](image)

**Table 6:** Means, square deviations, Cov and characteristic values of vulnerability indicators, for horizontal load Cov values of 10% and 20%

<table>
<thead>
<tr>
<th>Cov of upper avalanche (%)</th>
<th>Max. concrete tensile stress [MPa]</th>
<th>Max. displacement [mm]</th>
<th>Max. plastic strain ([10^{-5}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>2.90</td>
<td>3.63</td>
<td>3.7</td>
</tr>
<tr>
<td>Mean</td>
<td>11.7</td>
<td>13.2</td>
<td></td>
</tr>
<tr>
<td>Dispersion</td>
<td>0.14</td>
<td>0.56</td>
<td>0.89</td>
</tr>
<tr>
<td>Cov (%)</td>
<td>4.8</td>
<td>15</td>
<td>24</td>
</tr>
<tr>
<td>Characteristic value</td>
<td>2.96</td>
<td>4.36</td>
<td>5.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>15.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>26.5</td>
</tr>
</tbody>
</table>

Horizontal load variations however do not appear to significantly influence the variability in concrete tensile stress \( f_t \). For example, probabilities can be quantified in order to observe a tensile stress of up to 3 MPa, as denoted \( Pr(f_t > 3 \text{ MPa}) \); these values equal 0.58% and 3% for horizontal load variations of 10% and 20%, respectively.

**CONCLUSION**

This paper has presented a sensitivity study of the finite element model for a column at the Montaulever avalanche protection gallery. A probabilistic analysis of sensitivity to uncertain FE model parameters, including vulnerability indicators such as maximum displacements and stresses in concrete and reinforcement rods, has been proposed. Uncertain input parameters like Young’s modulus of concrete and dynamic avalanche load intensity (vertical and tangential) are modelled by random variables, in following a log-vertical law. The means and square deviations of vulnerability indicators have also been approximated. A number of results were presented; statically designed, the gallery is secure and in particular features a high sensitivity of maximum steel rod displacement and plastic strain to the ratio of normal-to-horizontal dynamic loads, as well as to the choice of C coefficient.

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1 OPALE: Acronym for Protective Structures and Residential buildings exposed to the action of avalanches: Loading, response and design