Dynamic Avalanche Modeling in Natural Terrain

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Abstract

The powerful avalanche simulation toolbox RAMMS (Rapid Mass Movements) is based on a depth-averaged hydrodynamic system of equations with a Voellmy-Salm friction relation. The two empirical friction parameters μ and ξ correspond to a dry Coulomb friction and a viscous resistance, respectively. Although μ and ξ lack a proper physical explanation, 60 years of acquired avalanche data in the Swiss Alps made a systematic calibration possible. RAMMS can therefore successfully model avalanche flow depth, velocities, impact pressure and run out distances. Pudasaini and Hutter (2003) have proposed extended, rigorously derived model equations that account for local curvature and twist. A coordinate transformation into a reference system, applied to the actual mountain topography of the natural avalanche path, is performed. The local curvature and the twist of the avalanche path induce an additional term in the overburden pressure. This leads to a modification of the Coulomb friction, the free-surface pressure gradient, the pressure induced by the channel, and the gravity components along and normal to the curved and twisted reference surface. This eventually guides the flow dynamics and deposits of avalanches. In the present study, we investigate the influence of curvature on avalanche flow in real mountain terrain. Simulations of real avalanche paths are performed and compared for the different models approaches. An algorithm to calculate curvature in real terrain is introduced in RAMMS. This leads to a curvature dependent friction relation in an extended version of the Voellmy-Salm model equations. Our analysis provides yet another step in interpreting the physical meaning and significance of the friction parameters used in the RAMMS computational environment.

Key words: snow, avalanche, dynamics, modeling, natural terrain, curvature

1. Introduction

Avalanche models are based on hydrodynamical partial differential Eq. (1). They describe the time evolution of the flow height h and the flow velocity $\mathbf{u}$, summarized in the state vector $\mathbf{V}(\mathbf{x},t) = (h, h\mathbf{u})^T$. $\mathbf{F}(\mathbf{V}(\mathbf{x},t))$ represents the transport flux and $\mathbf{S}(\mathbf{V}(\mathbf{x},t)$ are the source terms.

$$\partial_t \underbrace{\mathbf{V}(\mathbf{x},t)}_{\text{State vector}} + \nabla \cdot \underbrace{\mathbf{F}(\mathbf{V}(\mathbf{x},t))}_{\text{Flux}} = \underbrace{\mathbf{S}(\mathbf{V}(\mathbf{x},t))}_{\text{Source Terms}} \quad (1)$$

The dynamics of any avalanche or free surface flow are governed by driving and resisting forces which are represented by the source terms in Eq. (1). The net driving forces arise from gravitational and an opposing frictional forces. Because of its influence on the resulting net driving force the surface curvature of the avalanche path is of particular interest. Advanced mathematical models derived from first principles and confirmed by laboratory experiments provide a fundamental basis for existing flow models (Gray et al. (1999), Pudasaini and Hutter (2007)). In order to apply the theory to real world problems both expert knowledge and large representative data sets are necessary to calibrate the models and eventually interpret the simulation results. An tendency in the past years has been the development of two separate branches of avalanche research, one focussing on a further analysis and development of a sophisticated theoretical foundation, the other on a practical application of relatively simple models applied to avalanche events. This work is a step forward to combine both approaches to a unified theory.

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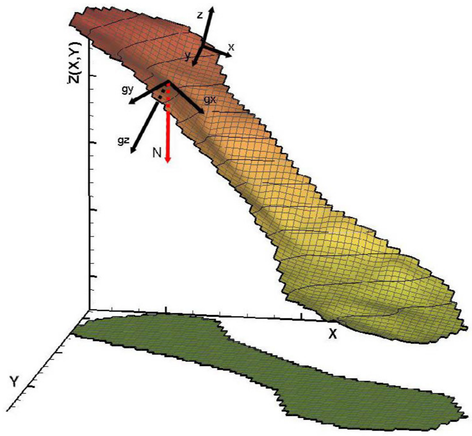

Figure 1: The topography $Z(X, Y)$ of the Salezertobel, Davos given in a Cartesian coordinate System (DEM). The surface induces a local coordinate system x, y, z and therefore the components of the gravitational acceleration $\mathbf{g} = (g_x, g_y, g_z)$, N points in the direction of the gravitational acceleration.

Because the mountain topography of the avalanche path is taken into account, we are able to identify small curvature effects in the empirical calibrated friction parameters. Although this approach is not capable of reproducing the calibration of the empirical models. It gives an understanding of the underlying physical process.

The mountain topography is constructed through a *Digital Elevation Model* (DEM) with spatial resolutions between $2 - 25$ m, producing a locally orthogonal coordinate system x, y, z (Fig.1). Furthermore, the physical model assumptions and typical avalanche scales induce restrictions for which the models are valid.

2. Voellmy-Salm approach

The Voellmy-Salm (VS) model (Voellmy (1955)) has been successfully applied to avalanche simulations since the mid 1960s. The dynamics of the flow are predominately influenced by the net forces in X and Y directions denoted by $S_i, i \in \{X, Y\}$ and given by a superposition of gravitational acceleration and frictional resistance.

$$S_i = \underbrace{hg_i}_{\text{gravitational force}} - \underbrace{\frac{u_i}{||\mathbf{u}||}\left(h\mu g_Z + \frac{||\mathbf{g}||}{\xi}\mathbf{u}^2\right)}_{\text{frictional forces}} \quad (2)$$

flow height: h, *grav. acceleration:* $\mathbf{g} = (g_X, g_Y, g_Z)$ (Fig. 1), *velocity:* $\mathbf{u} = (u_X, u_Y)$, *friction parameters:* μ, ξ.

The two empirical friction parameters μ and ξ represent dry Coulomb friction and turbulent friction, respectively. In the original Voellmy-Salm approach μ is independent of material properties but varies with the mountain profile. However experiments on the laboratory scale (Pudasaini and Hutter (2007); Platzer et al. (2007)) and on the large field scale (Bartelt et al. (2006); Sovilla et al. (2008)) indicate, that this assumption is not true in general. Bartelt and Buser extended the VS model (Bartelt et al. (2006),Buser and Bartelt (2009)) that provides an additional evolution equation for the kinetic energy of the fluctuating motion inside an avalanche. Furthermore they couple the Coulomb friction coefficient μ to this additional field variable. The velocity dependent frictional force is characterized by the parameter ξ and was first stated by Voellmy (1955) as 'turbulent' friction. Modeling results have been promising, however the real origin of this contribution to the basal topography and thus the friction law was subject to discussion. Later, Salm (1993) stated that ξ should mainly depend on the terrain geometry and interpreted it as 'viscous' friction.

Although μ and ξ still lack a proper physical explanation, 60 years of acquired avalanche data in the Swiss Alps made a systematic calibration possible. Today, practical avalanche simulations rely on a rigorous project to calibrate the friction parameters μ and ξ (Gruber and Bartelt (2007)). According to the Swiss Guidelines (Salm et al. (1990); Gruber (1998)), values for μ and ξ can be prescribed manually, or alternatively for large scale simulations, automated procedures based on GIS that classifies the terrain and determines the friction coefficients (Fig.2). This process is based on a DEM analysis to distinguish between different terrain features such as open slope, channeled, gully, forested or non-forested.

3. Models including Curvature

Savage and Hutter (1989) developed a continuum mechanical model for rapid granular flows. Various extensions to this theory appropriately account for a complex bottom topography. Gray et al. (1999) (GWH) proposed a two-dimensional depth-integrated theory for gravity driven free surface flow over a moderately curved surface in the downslope direction. A further generalization is attributed to Pudasaini and Hutter (2003) (PH), which introduces the effects of curvature as well as twist of channels into the avalanche models. In their approach a predefined

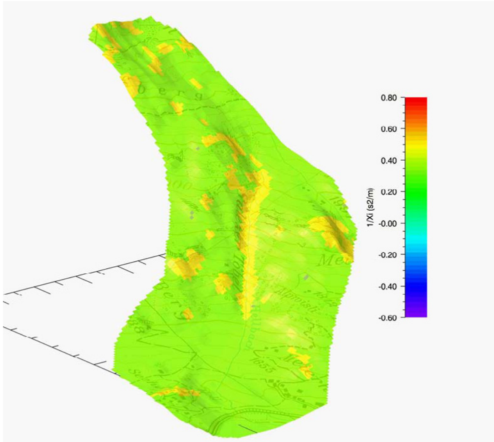

Figure 2: Map of the Salezertobel, Davos (10 m resoution) showing the calibrated $||\mathbf{g}||/\xi$. Bright colors indicate higher friction values.

master curve is adjusted to a flow channel. In a next step a curvilinear coordinate system is constructed originating from the master curve. The transformation into the space-curve based coordinate system gives insight into the effects of non-uniform curvature and torsion of the avalanche path.

We discuss the effect of the geometric terms on the dynamics by again having a closer look on the net driving forces $S_i, i \in \{x, y\}$ (where $\{x, y\}$ are curvilinear coordinates along a master curve).

$$S_i = \underbrace{hg_i}_{\text{grav. force}} - \underbrace{\frac{u_i}{||\mathbf{u}||} h \tan\delta\,(g_z + \kappa\,\mathbf{u}^2)}_{\text{frictional force}} \qquad (3)$$

Where δ is the bed friction angle, accounting to the dry coulomb friction ($\mu = \tan\delta$). κ accounts for the surface curvature of the avalanche path.

Although the basic derivation of the avalanche flow model (GWH,PH) is quite different to the VS approach, the model Eq. (3) are of the same general form as Eq. (2). However, in Eq. (3) the arising parameters have a different, physical interpretation. By transforming the model equations into a curvilinear coordinate system, additional terms in the frictional force arise. The normal force (overburden pressure) gains an extra component, which accounts for a 'centrifugal force' associated with the curvature κ (Fig. (3)). Again this contribution to the basal friction depends on the underlying topography. Due to the proper physical explanation the contribution can be directly calculated from the DEM and a calibration step is not necessary.

4. Curvature in natural terrain

From empirical calibration, the VS model is capable of reproducing very good simulation results for flow run-outs, flow velocities and impact pressures. The extended theories due to Gray et al. (1999) and Pudasaini and Hutter (2003) on the other hand provide a way to simulate flow in complex topography, and in principle they get along without labor-intensive calibration. Their results are tested and confirmed on laboratory experiments (Pudasaini and Hutter (2007)). However it remains a challenge to implement them for generic mountain topographies.

One of the main difficulties of the curvature definition in natural terrain is the downslope and the flow direction respectively. The terrain is described by a general DEM, therefore the downslope direction is arbitrary and changes throughout the avalanche flow. The curvature dependent models are based on coordinate systems aligned to the predefined downslope direction and chosen to be parallel to the 'mean' downslope topography (GWH) or the predefined thalweg (for channelized flow (PH)). This preprocessing, twist induced channel overflow and splitting of the flow into multiple avalanche branches requires special attention during implementation for such complicated flows. From a technical point of view it is possible to use a channel based coordinate system. However, these approaches have their own limitations in practical application. The terrain induced calibration of friction parameters in the VS is direction independent, therefore it can be applied to any mountain topography but is not applicable to curvature along the avalanche path.

The curvature definition in natural terrain applied in the context of the extended VS model (Fischer (2009)) is directly coupled to the avalanche flow direction and path. Thus a non orthogonal local coordinate system is adopted which can be applied directly on the given DEM data. The extra component in the frictional resistance arises from the projections of acceleration and curvature in the normal and flow direction (see Fischer (2009)). We therefore propose a combined approach that still makes use of the good performance of the Voellmy-Salm avalanche modeling approach, while at the same time uses the additional insight from an intense study of the extended models to overcome problems concerning the right DEM resolution and the application to region, where only limited data is available. We discuss two aspects of a unified interpretation.

4.1. A curvature dependent friction relation

Instead of interpreting both friction parameters in the VS model as empirical values, that are accessible only through calibration work, we split the velocity dependent friction contribution into two parts. The components $(S_i, i \in \{X, Y\})$ of the net driving force are:

$$S_i = h\, g_i - \frac{u_i}{||\mathbf{u}||} \left(h \mu\, g_Z + \xi_t\, \mathbf{u}^2 + \xi_v\, \mathbf{u}^2 \right) \quad (4)$$

$$\xi_t = \mu\, h\, K, \quad \xi_v = \frac{||\mathbf{g}||}{\xi}.$$

A static part, denoted ξ_v (v stands for *viscous*) accounts for the empirical viscous friction and is assumed to be independent of the mountain profile. It represents the influence of the snow properties. In analogy to the PH model, a second terrain dependent contribution ξ_t (t stands for *terrain*) originates from the surface curvature K, and is an additional contribution to the normal force as an extra 'centrifugal' term. Hence ξ_t contains all the spatial variation of the classical Voellmy-Salm friction parameter ξ whereas ξ_v is static for the whole simulation. This approach ('extended VS model' in the following) is well justified by comparing Fig. 2 and Fig. 3. An obvious advantage of our combined avalanche theory is that it avoids the necessity of defining different terrain categories as well as the independent calibration for each of them. Only the calibration of the static ξ_v remaining, whereas the other term has a clear physical explanation and can be determined directly from the DEM.

An approach like this might seem irrelevant for avalanche simulations in the Swiss Alps as all the calibrating work has already been done and can easily be extracted from the Swiss Guidelines. However, it becomes important, when trying to calibrate the numerical software for other mountain regions in the world. Due to different general weather, snow and topography conditions a region specific calibration might be possible. With our approach to interpret the friction relation, a much smaller data set is required to calibrate the simulation tool.

4.2. Resolution threshold

Elaborate techniques to acquire topography data, such as LIDAR laser, have been developed. Hence the available DEM resolution for a particular avalanche path continuous to be improved but it is not at all obvious what the optimal terrain resolution for the corresponding avalanche simulation should be. Calculating curvature with the DEM data suggests a way to determine a reasonable terrain resolution based on an estimate of the profile scale.

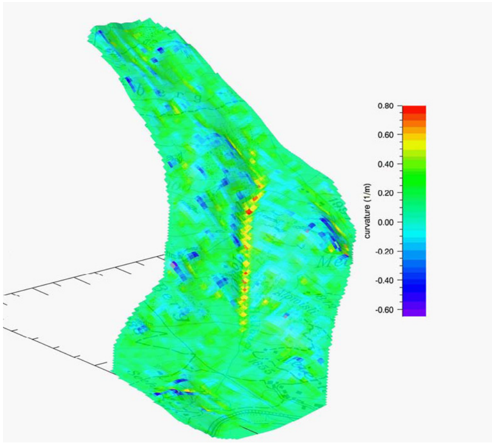

Figure 3: Map of the Salezertobel, Davos. Showing the computed absolute curvature κ. Bright colors indicate high values of curvature.

A first result of this analysis is that intuitively: *Higher resolution of the topography information implies better results* is not true in general. In fact a reasonable simulation on the basis of high resolution DEMs is only possible with a preprocessing topography resampling onto an optimal resolution. The calculated curvature of the DEM data is dependent on the chosen spatial resolution. Dealing with this surface curvature and taking characteristic length, height and profile scale (L, H, λ Pudasaini and Hutter (2003)) into account we find a threshold relation for the curvature ($\lambda/L \geq K$) and therefore a maximal limit of the DEM resolution. This means, if the DEM resolution is to high with respect to the typical length scales, the model assumptions do not hold anymore. This implies that the DEM resolution has to be chosen depending on the surface geometry and the typical avalanche scales.

5. Simulation Example

5.1. RAMMS

RA**pid **M**ass **M**ovement**S (RAMMS) is a computer simulation model designed by the SLF (Institute for Snow and Avalanche Research) as a practical tool for avalanche engineers (Christen et al. (2009)). The theoretical model is solved with first and second-order accurate numerical schemes.

RAMMS has been calibrated and tested on a series of well-documented avalanches of the Swiss avalanche database. Therefore it is able to predict run-out distances, flow velocities, flow heights and flow momenta. A graphical visualization of the numerical calculations is provided to evaluate the simulation results. Geo-referenced maps or aerial photographs can also be imported into RAMMS and then superimposed on the computational domain which makes the interpretation of model results a lot easier.

To perform numerical calculation RAMMS needs three basic input quantities.

1. *Release zone area and fracture height plus snow cover entrainment heights*. This information provides the knowledge about the avalanche volume and therefore has a strong influence on the avalanche dynamics
2. *The DEM.* All information about the complexity and geometry of the natural terrain are provided by the DEM. The spatial DEM resolution has an enormous impact on the flow dynamics and avalanche path.
3. *Model friction parameters*. The model incorporates two friction parameters, μ and ξ which represent dry coulomb and viscous friction, respectively.

For the following simulation example the friction relation implemented in RAMMS has been adjusted according to section 4.1.

5.2. Examples in natural terrain

As a first step in interpreting the combined friction relation, a numerical calculation on three avalanche paths is performed. The curvature effects are highly dependent on the underlying topography as well as the actual avalanche path, which is defined by the release area, the release volume and the avalanche dynamics. To get an idea of the extended model performance three different avalanche pathes are simulated: Salezertobel (Davos), Vallee de la Sionne, Val Prada. The given DEM resolution is 10 m and fullfills the limiting model assumptions for all three examples, see Section 4.2.

According to the Swiss guidelines large avalanches of a 300-year return period are simulated. The corresponding initial data for an avalanche of this kind are an approximate volume of > 60000 m^3, this corresponds to a snow mass of > 18000 t $(300$ kg/m^3

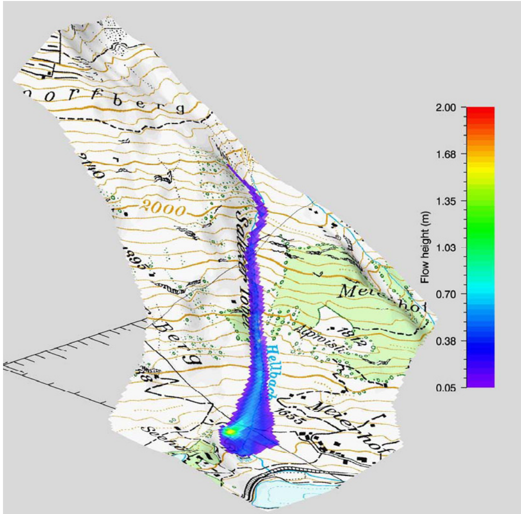

Figure 4: Map and DEM of the Salezertobel, Davos. Avalanche simulation, showing flow height.

snow density). The general suggestion of the Swiss Guidelines for a first avalanche analysis is $\bar{\mu} = 0.2$, $\bar{\xi} = 2000$. For comparison we perform the simulations with three slightly different friction approaches:

1. Standard VS approach with varying friction parameters (μ and ξ according to the Swiss Guidelines, Eq. 2)
2. Standard VS approach with constant friction parameters ($\mu = \bar{\mu}$, $\xi_v = \bar{\xi}$, Eq. 2)
3. Extended curvature VS approach with constant friction parameters ($\mu = \bar{\mu}$, $\xi_v = \|\mathbf{g}\|/\bar{\xi}$) and terrain dependent friction parameter ξ_t (Eq. 4)

In our numerical study we compare the influence of ξ, ξ_v and $\xi_v + \xi_t$ of the different approaches respectively. In Figure 5 we plot the maximal ξ values over time against the corresponding ξ according to the Swiss Guidelines.

Naturally the ξ values of the Swiss Guideline (red line) appear as the bisectrix. On the other extreme the constant friction values are represented by a constant (black line) and show no correlation at all. However, the interesting result is that the correlation between the extended VS model and the classical VS model using the Swiss Guidelines highly depends on the chosen avalanche path. In particular, we see that it is not the shape of the flow channel itself, but rather the degree of alignment of the main flow direction and the channel. In the Salezertobel (cyan line) the transit is mainly in a longitudinal channel direction that

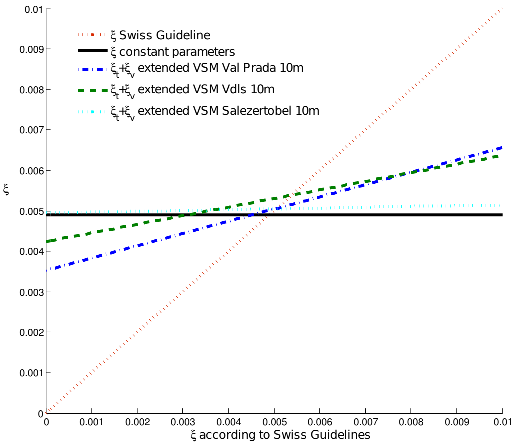

Figure 5: linear interpolated friction coefficients along the avalanche path for the extended VS model, VS model with varying- and VS model with constant friction coefficients.

shows a poor correlation. The typical release zone in the Valee de la Sionne (green line) induces a flow direction slightly oblique to the relevant topography. An additional curvature effect is due to the run up on the opposite valley side. Finally the Val Prada (blue line) avalanche path is highly twisted and thus the flow experiences significant curvature effects. The same tendency is observed when comparing the maximum height and velocity values directly.

In general, we see a significant correlation between the extended VS model and the classical VS model using the Swiss Guidelines as long as curvature terms are considered that are aligned with the flow direction. For further discussion and complete analysis (also regarding the influence of the spatial resolution), see Fischer (2009).

6. Conclusion and Outlook

Avalanche dynamics are influenced by the effects resulting from the avalanche paths curvature in natural terrain. Using the modified friction relation is a step forward of a systhematic combination of practical and theoretical concepts in avalanche modeling. The extended model provides a deeper understanding of the physical processes and thus a more realistic simulation of avalanches in natural terrain. This combination of theory and application is capable of describing the dynamics of velocity dependent friction throughout an avalanche flow due to curvature. However, for a complete physical explanation of the friction parameters further effects must be considered.

A more sophisticated understanding of frictional variation requires further examination of the properties of flowing snow.

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