

Fundamental studies on applications of MPS method for computing snow avalanches

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This paper presents some considerations on applicability of the MPS method for snow avalanches. The MPS method is a powerful tool to reproduce flow phenomena with large scale surface deformation. In order to apply this method to snow avalanches, we modified the original model to introduce a constitutive equation of Bingham fluid and effects of drag force due to air. The modified model was applied to some simple cases and evaluated the model performances through comparisons with experimental results. Then, the modified model was applied to the energy dissipating phenomena by a snow absorbing frame. The computational results showed that the present numerical model can capture the fundamental aspects of snow avalanche phenomena satisfactorily and can be a powerful tool for prediction and prevention of disasters due to snow avalanches.

KEYWORDS: snow avalanche, numerical simulation, Bingham fluid, MPS, Lagrange simulation

1 INTRODUCTION

It is important to establish reliable prediction method for occurrence and magnitude of snow avalanche from the viewpoint of disaster prevention. However, development of a predictive tool is very difficult because the snow avalanche contains a lot of unknown factors. In order to reduce the damage from snow avalanche, many kinds of snow absorbing frames have been proposed. The planning and design of snow absorbing frames have been performed using experimental and empirical approaches. However, so far, a well-established designing method does not exist. Unfortunately, field observation data of snow avalanches are very scarce. Therefore, in order to construct effective snow absorbing structures economically, a development of a reliable numerical method, which can predict occurrence conditions, flow behavior and stopping process of snow avalanches, is of keen importance.

Many kinds of numerical methods for simulating snow avalanches have been proposed and can be classified into the following 4 models; (1) fluid model, (2) density current model, (3) mass center model and (4) particle flow model. The MPS method used in the present work is categorized into the fluid model. The MPS method computes the fluid flow using a Lagrange approach and can efficiently simulate

the large deformation of fluid surface, such as wave breaking phenomena etc. Therefore, the MPS seems to be an adequate numerical tool for snow avalanches. However, since the snow is basically not consisted from continuous materials but discreet powders, there are some challenging matters. The first simple problem is how to simulate the stopping process of a snow avalanche using MPS method, which is a fluid model. Takahashi et al (1998) used a particle flow model and simulated successfully the stopping process of snow avalanches introducing a deposition rate. A three dimensional analysis of snow avalanche using MPS has been reported by Saito et al (2006) although they have not succeeded in reproducing the stopping process. Another problem is how to introduce the drag force from air. This problem is important for application to a powder snow avalanche.

In this study, we modified the MPS method from the following viewpoints.

a) In order to simulate the stopping process of snow avalanches, the constitutive equation is modified by assuming the snow as Bingham fluid.

b) The introduction of the drag force from air is considered for simulating powder snow avalanche with a large velocity.

The validation of the modified model is performed through the comparison with experimental results. We use the experimental data of the slump tests for the concrete for the validation of the constitutive equation. The experimental data of the flow of a lot of ping-pong balls are used for considering the model of air drag force. The present modified model is applied to the snow avalanche

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impinging on a snow protecting frame and fundamental behavior of the numerical model is examined.

2 NUMERICAL TECHNIQUE

2.1 Constitutive model of Bingham fluid

For simulation of stopping process of snow avalanche, a incompressible Bingham fluid is assumed for the constitutive equation. The continuity and momentum equations are described as follows.

$$\frac{D\rho}{Dt} = 0 \quad (1)$$

$$\rho \frac{\partial u_i}{\partial t} = -\frac{\partial p}{\partial x_i} - \rho \delta_{i2} g + \frac{\partial \tau_{ij}}{\partial x_j} \quad (2)$$

where ρ is the density, u is the velocity, p is the pressure, τ_{ij} is the shear stress, g is the gravity. Shear stress of Bingham fluid is expressed as

$$\begin{cases} \tau_{ij} = \eta \dot{\gamma}_{ij} + \tau_0 & \tau_{ij} \geq \tau_0 \\ \dot{\gamma} = 0 & \tau_{ij} < \tau_0 \end{cases} \quad (3)$$

where η is the plastic viscosity, $\dot{\gamma}_{ij}$ is the shear strain rate, τ_0 is the yield value of Bingham fluid.

In his study, τ_0 is not a constant value, but given by the following coulomb equation.

$$s = c + \sigma \tan \phi \quad (4)$$

where s is shear strength, c is cohesion, σ is normal stress, ϕ is internal friction angle. We used τ_0 in Eq. (3) in place of s in Eq. (4), and used a model of Moriguchi et al (2005), in which σ is replaced with p as:

$$\tau_{ij} = \eta \dot{\gamma} + c + p \tan \phi \quad (5)$$

This model contains only two parameters of c and ϕ . We can easily get values of c and ϕ of snow in previous studies (e.g., Matsuzawa et al., 2007) and can apply those values for the present model. If eq. (5) is divided by the shear strain rate, $|\dot{\gamma}|$, we can get the equivalent viscosity to Newtonian viscosity.

$$\eta' = \eta + \frac{c + p \tan \phi}{|\dot{\gamma}|} \quad (6)$$

The shear strain rate can be calculated by the following equation.

$$|\dot{\gamma}| = \sqrt{\frac{1}{2} \dot{\gamma}_{ij} \dot{\gamma}_{ij}} \quad (7)$$

Because the function of equation (3) is discontinuous at $\tau_{ij} = \tau_0$, we used the following approximated formula proposed by Papanastasiou (1986).

$$\eta' = \eta + \frac{c + p \tan \phi}{|\dot{\gamma}|} (1 - e^{-m\dot{\gamma}}) \quad (8)$$

where m is the stress growth exponent, which has a dimension of time. Fig. 1 shows the relation between τ/τ_0 and $\dot{\gamma}$ at $\eta/\tau_0 = 1$. We

can see that the accuracy of the approximated equation increases as m becomes larger. $m = 1000$ (s) was used for the present work.

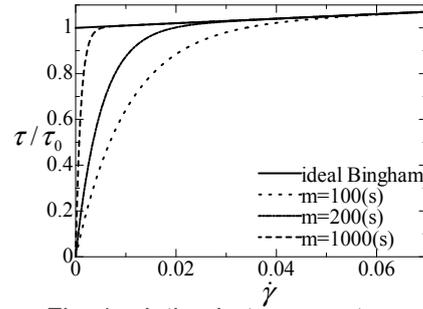


Fig. 1 relation between τ/τ_0 and $\dot{\gamma}$

2.2 Introduction of air drag

If snow avalanches are assumed to be composed of a Newtonian fluid, the momentum equation becomes:

$$\rho \frac{\partial u_i}{\partial t} = -\nabla p + \rho \nu \nabla^2 u_i - \rho \delta_{i2} g - F_a \quad (9)$$

where ν is kinematic viscosity and F_d is an air drag.

Air drag F_d is given as:

$$F_a = \frac{1}{2} \rho_a C_D S u |u| \quad (10)$$

where ρ_a is air density, C_D is drag coefficient and S is the project area. The drag coefficient C_D is evaluated using Schiller and Naumann's formula, which is a function of Reynolds number Re as:

$$C_D = \begin{cases} \frac{24}{Re} (1 + 0.15 Re^{0.687}) & Re \leq 1000 \\ 0.4 & Re > 1000 \end{cases} \quad (11)$$

2.3 Gradient and Laplacian models for MPS

MPS method is Lagrange type technique for solving the incompressible flow. It is categorized as a particle method. Each term in Eq. (9) is expressed as interactions of particles.

MPS method replaces the continuum with limited number of particles. The interaction area of particle is expressed by the weight function as:

$$w(r) = \begin{cases} \frac{r_e - r}{r_e} & r \leq r_e \\ 0 & r > r_e \end{cases} \quad (12)$$

where r is a distance of particles and r_d is the influence radius of the interaction.

Differential operators in eq.(9) are Gradient and Laplacian. In MPS, those are described as:

$$\langle \nabla \phi \rangle_k = \frac{d}{n^0} \sum_{l \neq k} \left[\frac{\phi_l - \phi_k}{|r_l - r_k|^2} (r_l - r_k) w(|r_l - r_k|) \right] \quad (13)$$

$$\langle \nabla^2 \phi \rangle_k = \frac{2d}{\lambda n^0} \sum_{l \neq k} [(\phi_l - \phi_k) w(|r_l - r_k|)] \quad (14)$$

where r is a location vector of particle, l is number of neighbourhood particles, d is a spatial

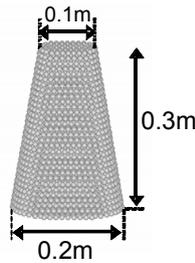
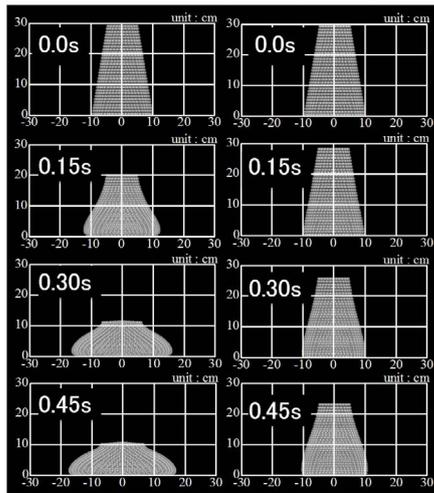


Fig.2 Initial shape of slump test

Table 2 Computational conditions

	Case1	Case2	Case3
yield point (Pa)	300	900	1500
plastic viscosity(Pa·s)	100	100	100


 Fig. 3 Computational results
Case 1(left) and Case 2(right)

dimension, n^0 is a particle number density obtained from the initial condition, λ is a coefficient introduced to fit an increase of the statistical variance of distribution to an analytical solution and is given as:

$$\lambda = \frac{\sum_{l \neq k} [w(|r_l - r_k|) |r_l - r_k|^2]}{\sum_{l \neq k} [w(|r_l - r_k|)]} \quad (15)$$

Particle number density n is defined using the weight function as:

$$\langle n \rangle_k = \sum_{l \neq k} w(|r_l - r_k|) \quad (16)$$

Eq.(16) expresses the sum of the weight between particle k and neighbourhood particles.

The algorithm of computation is same as Koshizuka (2005).

3 MODEL REFINEMENT

3.1 Gradient and Laplacian models for MPS

(a) Outline of refinement

A motion of a Bingham fluid occurs when a shear stress exceeds a threshold. Concrete is a typical Bingham fluid. TOYAMA et al (2007) applied MPS method for concrete slump test. We try to modify MPS method to compute stably at

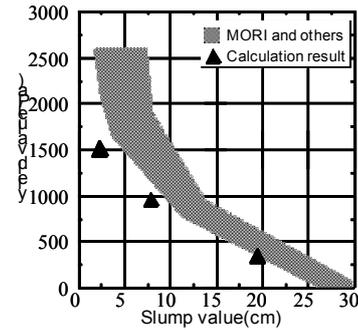


Fig.4 Comparison of slump value

the region of very small shear strain using the formula by Papanastasiou (1986).

Shear strain rate in (7) can be written as:

$$|\dot{\gamma}| = \sqrt{\frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j} \right)} \quad (17)$$

Evaluation of derivative of velocity is done using gradient model (eq.(13)). Eq.(6) becomes infinity if the strain rate becomes 0. To avoid this, η' is set to be 0 when the strain rate becomes 0. In order express the stop of snow avalanche, eq.(8) is used instead of eq.(3) when $d\dot{\gamma}/dt > 0$ and $\dot{\gamma} < 0.005$. If $d\dot{\gamma}/dt < 0$, the velocity of particles with $\dot{\gamma} < 0.005$ is set 0.

(b) Test computation

We adopted the slump test of fresh concrete as the test condition. Initial shape of the concrete is shown in Fig.2. The mean particle interval and number of particles are 0.01m and 5640, respectively. The main parameters are listed in table 1. The plastic viscosity η and density ρ are given as 100 Pa·s and 2300 kg/m³, respectively.

Fig.3 shows the comparison of behavior in Case 1 and 2. The change of deformation process due to different yield points can be reproduced clearly. The comparison with the experimental results by Mori and Tanigawa (1987) is shown in Fig.4. Since the accuracy in case of smaller yield points is better, this method seems to be applicable for snow avalanche, which yield point is much smaller than concrete.

3.2 Gradient and Laplacian models for MPS

(a) Outline of refinement

In order to introduce the effect of air drag, we tried to simulate the avalanche experiment of ping-pong balls. The avalanche of ping-pong balls can be assumed as a simplified snow avalanche because the density of ping-pong balls are almost same as snow avalanche and the size of particle is uniform.

We can assume the cube with an edge-length of mean particle distance as a group of ping-pong balls with same volume. The project area S in eq.(10) is evaluated considering sum of project area of ping-pong balls located at a

Table 2 Computational conditions

	? in viscosity term (m^2/s)	? in bed friction term (m^2/s)	Air drag
Case1	1.0×10^{-2}	1.0×10^{-2}	No
Case2	1.0×10^{-2}	1.0×10^{-2}	Yes
Case3	1.0×10^{-4}	1.0×10^{-7}	Yes
Case4	1.0×10^{-5}	1.0×10^{-7}	Yes
Case5	1.0×10^{-4}	1.0×10^{-8}	Yes
Case6	1.0×10^{-5}	1.0×10^{-8}	Yes

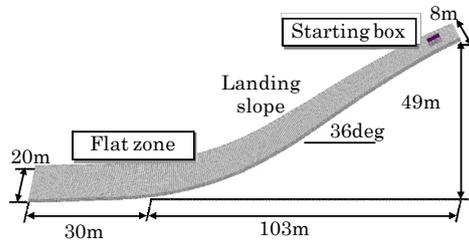


Fig. 5 Schematic diagram of experimental site (Miyanomori ski jump In Sapporo, Japan)

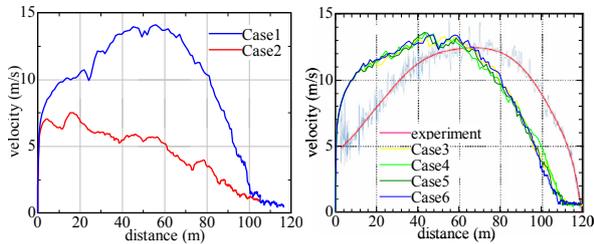


Fig. 6 Comparison of front velocities in Case 1-6

surface of the cube. The air drag term in eq.(9) is only introduced if the particle is located at the flow surface. The particle at flow surface is judged with $n^* < 0.97 n_0$ (n^* : particle number density and n_0 : mean particle number density).

(b) Test computation.

Computational conditions are listed in Table 2. In Case 1, air drag is not considered and is considered in Case 2-6. Case 2-6 have different kinetic viscosity. The density of ping-pong balls is set as 87.7 kg/m^3 . The mean particle distance of MPS is set as 0.2m. The experiment was performed at a ski jump site (Miyanomori Shanze, Sapporo Japan). The schematic diagram of the experimental site is shown in Fig.5.

Fig.6 shows the comparison of the velocity of avalanche front and distance in the horizontal direction. The viscosity of Case 1, in which air drag is not considered, is tuned to make the front velocity close to the experiment. The maximum velocity in Case 2 is much smaller than Case 1. The results in Case 3-6 show no clear differences. In those result, if the air drag is introduced and kinetic viscosity is tuned well, the model can reproduce the maximum velocity of avalanche. However, in all case of 1-6, the location of the maximum velocity emerges earlier than the experimental result. One of the reasons is supposed to be in the difference of initial conditions between numerical and experimental cases. The comparison of the shape of flowing ping-pong balls is shown in Fig.7. The left figure shows the experimental result and the right is the

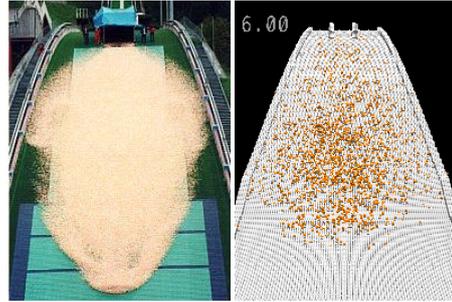


Fig. 7 Comparison of front shape (left: experiment, right: computation)

numerical result with Case 3. The experimental result shows that the ping pong balls flow as one group and a clear front is formed. However, in the numerical result, the particles seem to flow independently. One of the reasons seems to be that the number of flow particles in present computation is too small to reproduce real phenomena. We should consider further on the effects of the particle number.

4 APPLICATION TO SNOW PROTECT FRAME

4.1 Computational conditions

The modified MPS model is applied to realistic snow phenomena, i.e., a snow avalanche impinging on a snow protecting frame. Fig. 8 shows the schematic diagram of the flow domain. A slope is gradually connected with a horizontal plate at the downstream part. At the downstream horizontal plate, multiple piers, which are mimicking the snow protecting frame, are attached. The height of the pier is 1.5m. We performed 3 cases of computations. In Case A, piers are not installed. In Case B and C, the intervals of piers are 1.5m and 0.8m, respectively. The average particle diameter is set as 0.2m and total number of fluid particles is 3744.

4.2 Results and discussions

Fig.9 shows the comparison of average snow velocities computed in Case A-C at 0.4 m downstream of the snow protecting frame. The figure indicated that the velocity of the snow becomes smaller if the interval of the piers decreases. Fig.10 shows the comparison of flow patterns in Case A (left) and C (right). It is observed that the velocity of particles at the flow surface is larger than the inner particle. At the downstream of the piers, the range of spreading of particles in Case C is much smaller than that in Case A. In Case C, the accumulation of particles at the upstream region of the piers can be seen. Fig.11 shows the number of deposited particles at upstream and downstream of the piers in Case C. This result also confirms the effect of a snow protecting frame. The simulated

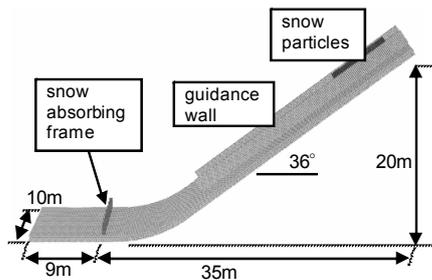


Fig.8 Flow domain with snow protecting frame

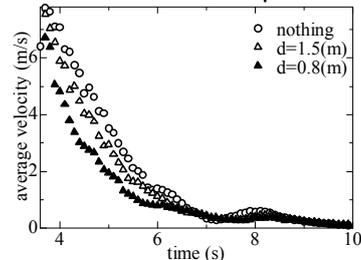


Fig.9 Comparison of snow velocities

results seem to be realistic and reasonable though the results were not compared with the experimental results.

5 CONCLUDING REMARKS

We proposed the refined MPS method for simulating snow avalanches considering air drag and Bingham fluid. The model with effect of air drag could simulate satisfactorily the experimental results of ping-pong balls avalanche. The stopping process of the snow could be expressed by introducing Bingham fluid type constitutive equation. The present model could compute the snow avalanche impinging on a snow protecting frame reasonably. Those results indicated that the present refined MPS method is a powerful tool for predicting the occurrence and behavior of snow avalanches.

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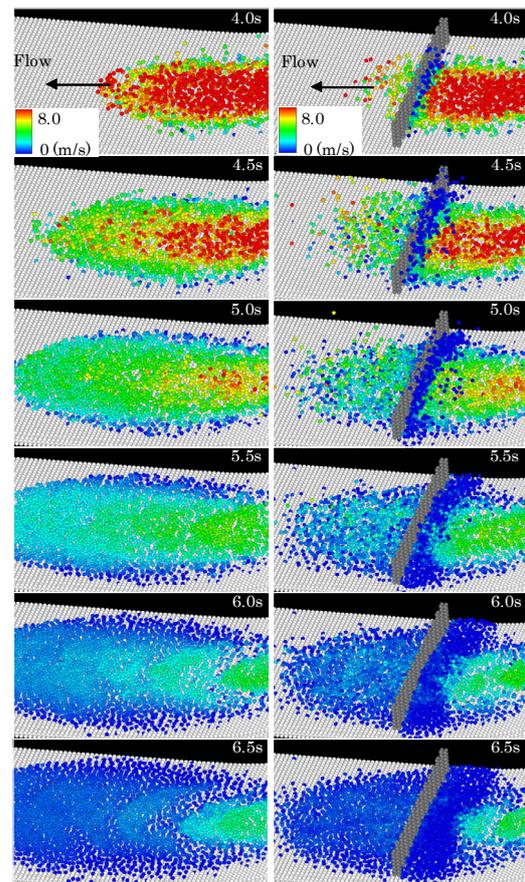


Fig. 10 Comparison flow patterns (left: no piers, right: interval 0.8m)

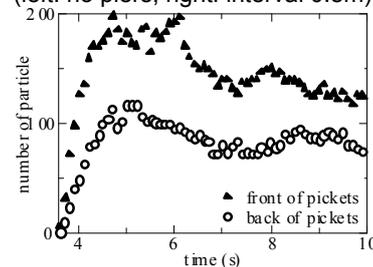


Fig.11 Comparison of deposited particle numbers

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