\( \beta \) - \( \beta \) model: Can we learn more from the statistical avalanche model with respect to the dynamical behavior of avalanches

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ABSTRACT. A meaningful risk assessment in avalanche prone areas involves the estimation of the runout and impact pressures of potential avalanches. Two categorize methods for determination of the runout exists. The first method is based on statistical methods such as the well known \( \beta \) - \( \beta \) model. The second is based on numerical avalanche models such as the PCM-model or Voellmy-Salm type models. More sophisticated models are based on similar rheological models. Dynamic models have the advantage that they, in addition to the estimated runout distance, also provide velocity and impact pressure distributions along the avalanche track. However, the success of the dynamical models depends on the knowledge of appropriate rheological models describing the behavior of flowing snow and their parameters.

We re-evaluate avalanche data from Norway, Austria, and other parts of the world, on which the well-known statistical \( \beta \) - \( \beta \) model is based, with respect to dynamical measures. As all those avalanche data belong more or less to extreme events (i.e. avalanches with return periods of 30 - 300 years or more) the dynamical measures can give hints for an appropriate rheology for dynamical models suitable for extreme avalanche events. The analysis raises reasonable doubt whether the classical ansatz for the retarding acceleration with additive terms involving Coulomb-friction and a velocity squared dependency, which is used as basis in many avalanche models, is adequate.

Keywords: avalanche dynamics, statistical model, observations

1 INTRODUCTION

Hazard assessment for landuse planning in snow avalanche prone areas requires, besides knowledge of return periods, the specification of expected runout distances. For a complete risk assessment, additionally, the intensity of the event, often expressed in terms of impact pressures, and the corresponding vulnerability of endangered objects are needed. The success of the well known \( \beta \) - \( \beta \) model (Lied and Bakkehøi, 1980) suggests that the runout of extreme avalanches is mainly a function of the path topography. The \( \beta \) - \( \beta \) model, however, does not provide necessary estimates on velocities and impact pressures along the avalanche track. To this end, numerical models are used. However, their success strongly depends on the choice of appropriate rheological models.

In the following, we briefly outline the re-evaluation of avalanche data from Norway, Austria, and other parts of the world, on which the statistical \( \beta \) - \( \beta \) model is based, with respect to dynamical measures. As all those avalanche data belong more or less to extreme events (i.e. avalanches with return periods of 30 - 300 years or more) the dynamical measures can give hints for a proper rheological model for dynamical models suitable for extreme avalanche events.

2 DATA

For the re-evaluation, we focus mainly on data, which served as basis for the derivation of the \( \beta \) - \( \beta \) model.
(Bakkehøi et al., 1983; Klenkhar and Weiler, 1994). In addition, data from Austria, France, Iceland, and Norway published within the CADZIE project (Domaas et al., 2002), data published in (Meunier and Ancey, 2004; Ancey et al., 2004; Ancey, 2005), data from Switzerland (Gubler et al., 1986), Russia (Kotlyakov et al., 1977), and some data from Canada (Delparte et al., 2008) are also included. For comparison, avalanche data originating from the full-scale test sites Col de Lauterat (COL) (Meunier et al., 2004), Ryggfonna (RGF) (Gauer et al., 2009), and Vallée de la Sionne (VDLS) (SLF, 2006) are analyzed too.

The data mainly consist of information about the avalanche path profile, estimated position of the crown in the avalanche release zone, and the (estimated) position of the maximum runout (tip of the deposition). Here, we focus on runout distance of the dense or fluidized part, respectively, of the regarded avalanches. In some cases, the data included information on deposition masses/volumes. For few events velocity distributions along the track are available.

3 ANALYSIS

For the analysis, we stayed with parameters used for the statistical $\alpha - \beta$ model, i.e. we use the $10^\circ$-point, here defined as the furthest downhill point where the terrain gradient is $10^\circ$ and/or the corresponding $\beta$-angle is greater or equal $15^\circ$, and the $\beta$-angle as reference. Although the choice of the $10^\circ$-point is rather arbitrary, $\beta$ may be regarded as a good approximation for the mean slope angle of the avalanche track and therefore, $g \sin \beta$ as the mean driving force per unit volume. Figure 1 shows a profile of an avalanche path and the relevant points used in the following analysis.

$$M \frac{dU^2}{2 ds} = M g \sin \phi + M \alpha_{ret}, \quad (1)$$

where $U$ is the velocity of center of the mass block, $M$ is its mass, $s$ the distance along the track, $g$ the acceleration due to gravity, and $\phi$ is the local slope angle. Here, we use the relation $dU/dt = dU^2/2 ds$. The retarding acceleration, $\alpha_{ret}$, is commonly given by (e.g. Mellor, 1968)

$$\alpha_{ret} = -a_0 \max(0, g \cos \phi + \kappa U^2) - a_1 U - a_2 U^2. \quad (2)$$

where $a_i$ are parameters and $\kappa$ is the curvature of the track. The first term on the right can be interpreted as Coulomb-friction and the third as a velocity squared dependent friction. The second term is more often than not neglected. However, in our approach of a “fictitious mass block”, $\alpha_{ret}$ not only includes all external retarding forces, but might also include potential coupling forces between the regarded avalanche head and the avalanche body, like pressure gradient forces, which could also act as driving force. Starting from (1), it is possible to derive a relation for the mean retarding acceleration,

$$|\alpha_{ret}| = \frac{g H}{S}, \quad (3)$$

which is a measure for the energy dissipation. Here, $H$ is the total fall height and $S$ the total travel distance along the track. In the following, we focus on this mean retarding acceleration. The symbol $\alpha_{ret}$ will also be used for the amount of the retarding acceleration, $|\alpha_{ret}|$, as confusion can be excluded.

![Figure 1: Main parameters in the $\alpha - \beta$ runout model.](image1)

![Figure 2: De-trended mean retarding acceleration, $\alpha_{ret} - \alpha_{fit}$, vs. $g \sin \beta$.](image2)
Figure 2 shows the de-trended mean retarding acceleration, \( a_{\text{ret}} - a_{\text{fit}} \), versus \( g \sin \beta \). The linear trend is given by

\[ a_{\text{fit}} = c_1 g \sin \beta + c_0 \]  

(4)

derived from 320 extreme events, where, \( c_1 = 0.82 \) and \( c_0 = 0.51 \text{ m s}^{-2} \), and the standard deviation \( \sigma = 0.45 \text{ m s}^{-2} \). This plot implies that the mean retarding acceleration increases (linearly) as the mean driving force represented by \( g \sin \beta \) increases.

Figure 3 plots \( a_{\text{ret}} - a_{\text{fit}} \) versus the total fall height. No correlation between \( a_{\text{ret}} - a_{\text{fit}} \) and \( H \) is identifiable, which implies that \( a_{\text{ret}} \) is independent of \( H \).

Based on velocity observations from Rogers Pass, British Columbia (McClung and Schaerer, 2006, Fig. 5.32), McClung and Schaerer perceived that the maximum speed along the avalanche track scales with the square root of the total fall height, \( H \). They prosed an upper limit of \( U_{\text{max}} \approx 1.5—1.8 \sqrt{H} \). Figure 4 shows a collection of avalanche data from VDLS, RGF, Khibin, Austria, and Switzerland, where velocity measurements, either based on photo or video analysis, or based on radar measurements, are available. Not all of those avalanches were “extreme events”, but all can be regarded as considerable events. The data agree well with the proposed trend.

4 CONCLUSION

In the preceding section, we briefly outline some results from a re-evaluation of avalanche data from Norway, Austria, and other parts of the world with respect to dynamic measures. The analysis raises reasonable doubt whether the classical ansatz (c.f. equation (2)) for the retarding acceleration with additive terms involving Coulomb-friction and a velocity squared dependency, which is used as basis in many avalanche models, is adequate. For example, observations originating from Rogers Pass (McClung and Schaerer, 2006; McClung, 1990) and data shown in Fig. 4, suggest that the maximum (front) speed is \( \propto \sqrt{H} \). At the same time, observations do not reflect a fall height dependency of \( a_{\text{ret}} \) (Fig. 3). Both observations together with the clear tendency of \( a_{\text{ret}} \) to increase with increasing mean slope angle (Fig. 2) are in contradiction with an ansatz like (2). This failure of the commonly used avalanche models requires a high degree of experience from the practitioner when he chooses suitable model parameters. None of the parameter recommendations reflect the observed slope dependency of \( a_{\text{ret}} \). Instead a volume or mass dependency is presumed, which actually seems to be not as obvious as commonly believed (corresponding data are not present here).

We hope that measurements from the full-scale test sites Col de Lauterat, Ryggfonn, and Vallée de la Sionne as well as from small-scale experiments will provide more in-depth insight in the dynamics of avalanches. Although, it should be kept in mind that these measurements need to be related to the wide range of potential avalanche paths. The test sites having \( \beta \)-angles of approximately 27°, 29°, and 21°, actually range in the lower half of the regarded avalanche paths with respect to the \( \beta \)-angle. Therefore, it is desirable to have more high quality avalanche observations that cover a wide range of varying topographies,
masses, and, where available, velocities.

ACKNOWLEDGMENTS

This work was partly funded through NGI’s SIP-program “Avalanche research”. We like to thank all those who were involved in any way in the data collection and to encourage the continuation of avalanche observations in the future.

REFERENCES


