Satellite observed SCA and gamma distributed snow in the HBV model

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ABSTRACT: Areas having a seasonal snowpack experience a risk of melt-floods. The magnitude of such a flood is highly dependent on the available water stored in the snow reservoir. Therefore it is of vital importance to estimate the reservoir as accurate as possible. The proposed method models the snow reservoir dynamically throughout the winter season, only using precipitation and temperature as input. Every snowfall and melting event, as well as the total accumulated snow reservoir, is spatially distributed according to a gamma distribution. The model has the ability to mimic the observed distributional change from highly right-skewed early in the accumulation season, a more normal distribution near the snow maximum, to an increasingly right-skewed as the melt season progress. This behavior is important for the evolution and producing of snow free areas and hence for the discharge dynamics. Also; a model that can predict a realistic development of bare ground is a prerequisite for utilizing satellite derived snow covered area (SCA), thus making it possible to update the snow reservoir for instance due to faulty model input. In incorporating remotely sensed SCA we have to make sure not to introduce faulty estimations that would propagate to a reduction in the accuracy of the discharge predictions. The work will be presented by means of the HBV-model for two test catchments situated in the mountainous parts of South-Norway. SCA-values for both catchments are derived from AVHRR scenes utilizing the Norwegian-Linear-Reflectance-to-snow-cover (NLR) algorithm.

KEYWORDS: Snow modelling, satellite observed SCA, SWE, snow distribution.

1 INTRODUCTION

Snowmelt is a major contributor to flooding in Norway. In order to accurately forecast melt floods and combined melt and rain floods, we need good estimates of both the extent and volume of the snow reservoir, together with precipitation and temperature. Operationally the HBV model (Bergström, 1992; Sælthun, 1996) is being used for flood forecasting in Norway. This model includes a snow accounting routine (Killingtveit and Sælthun, 1995), which simulates the snow water equivalent (SWE) and the snow covered area (SCA) at different altitude levels. To capture the runoff dynamics in the melt season it is imperative to model the SCA correctly, as the interaction of bare ground on snow covered areas is important for the process of runoff generation.

Skaugen et al. (2004) put forward an alternative formulation for the spatial distribution of snow, in stating that the distribution of accumulated events of individual gamma distributed snowfalls are also gamma distributed with parameters derived from the individual events and the number of accumulations. This model accounts for the changed statistical features of the spatial distribution during the winter season, in a way that the snow reservoir will start off as a right-skewed distribution, approach a more normal distribution when the maximum snow amounts occur and finally be more and more right-skewed during the melt season.

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2 METHODOLOGY

A major advantage of the gamma-model initially described in Skaugen et al. (2004) and further refined in Skaugen (2007) is that the spatial distribution of snow can be quantified at all times. Together with a more realistic development of the snow reservoir, we now have the means for linking SWE with SCA and thus be able to adjust faulty modelled values.

With reference to figure 1 & 2 and the equations below, we estimate the change in SCA due to melting:

$$A1 = \int_0^X x f_a \qquad A2 = \int_X^\infty x f_a \tag{1}$$

$$S1 = \int_0^X x f_s \qquad S2 = \int_X^\infty x f_s \tag{2}$$

$$a = \int_0^X f_a \qquad a' = 1 - a \tag{3}$$

$$s = \int_0^X f_s \qquad s' = 1 - s \tag{4}$$

f_a and f_s are the spatial probability density functions of accumulation and melt, respectively. The parameters of f_a and f_s are modelled as temporally correlated identically distributed variables. The parameters of a melt event are chosen to be identical to that of a snowfall. This is motivated by the fact that the ultimate distribution of melt has to be identical to that of the accumulation and that we have limited information on the true nature of the melt distribution on the catchment scale. The basic assumption is that all areas with SWE values less than the point where the frequencies of f_s match the frequencies of f_a will be left snow free after a melting event. In addition there will be generated partially snow free areas where the frequencies of melt are lower than those of accumulation, so that the total relative area left snow free appears as $p = a + s' = \int_0^X f_a + \int_X^\infty f_s$.

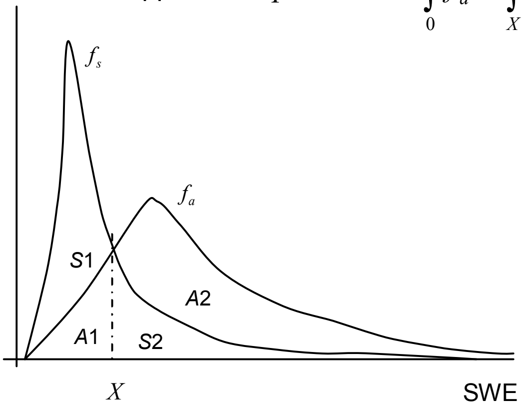

Figure 1. The crossing point X where the frequencies of the distributions of accumulation and melt match.

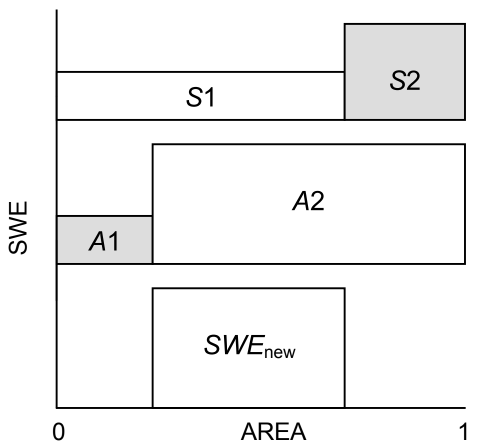

Figure 2. Illustration of the snow covered area that disappear after a melting event. The gray blocks disappear from the accumulation distribution and generates the new mean value SWE_{new} with the reduced spatial extent.

In the HBV model the mean value of the distribution of melt S, is estimated as a function of air temperature through an index on a column of snow. So that the amount given as melt can be considered as a potential melting event, that is, conditioned on complete coverage of snow. S is thus the difference in mean between the conditional mean prior to the melting event and the unconditional mean posterior to the melting event.

The new relative coverage of the SCA is the complement of p, $p' = 1 - p$ and the new conditional mean SWE for this area is denoted SWE_{new}, which assessed for the area prior to the melting event is $SWE_{new}(1 - p)$. We can now formulate the balance equation:

$$\begin{aligned} A - S &= SWE_{new}(1 - p) \\ \frac{1}{1-p}(A - S) &= SWE_{new} \end{aligned} \tag{5}$$

A and S are conditional means prior to the melting event, whereas $SWE_{new}(1 - p)$ is an unconditional mean posterior to the melting event. In a rainfall-runoff model like the HBV both A and S is known at any given time, so the main task is to determine the crossing point X. Once X is known we can also determine p, which in turn gives us the new mean areal SWE.

2.1 Updating from remotely sensed SCA

Let us say that we obtain an estimate of SCA from a satellite observation which differs from the simulated value from the HBV model. Given that the observed SCA is correct, there are two possible scenarios:

- The distribution of SWE in the model is wrong, but the mean and hence the meteorological input is correct. In this case we can simply adjust the SCA and carry on.
- Both the distribution and the mean are wrong. This case demands adjustment on both the SCA and the parameters of the distribution.

The latter case will arise if the meteorological input is faulty, which is known to be a general rule of such data. We therefore place confidence in this scenario and establish the tools for correcting the snow reservoir based solely on

SCA information. The link between SWE and SCA is presented in Skaugen et al. (2004) and is briefly reviewed above. We apply a similar reasoning for updating the spatial distribution of SWE based on information of SCA, the procedure differs however in that we now go *from* known SCA *to* an updated distribution of SWE. In Skaugen et al. (2004) a known change in SWE (accumulation or melt) was used to update the SCA.

2.2 Case of satellite derived SCA<modelled

A satellite observation of SCA tells us that the modelled SCA is too high. This implies that there is less snow in the catchment than the model predicts. In order to correct this we have to identify the unknown parameters of the balance eq. (5):

$$\frac{1}{1-p}(A-S)=SWE_{new}$$

Here we search for a SWE_{new} and do this by increasing S until the desired SCA is matched. When the modelled SCA is matched with the observed one, SWE_{new} is the conditional mean of the updated distribution.

2.3 Case of satellite derived SCA>modelled

A satellite observation of SCA indicates that there is more snow in the catchment than the model predicts. As above we identify the unknown parameters of eq. (5):

$$\frac{1}{1-p}(A-S)=SWE_{new}$$

In this case we let the current accumulation distribution serve as SWE_{new} and search for a distribution of A that is greater than SWE_{new}. When A provided with a melting event S give us the desired change in SCA, we have the new conditional mean of the updated distribution. Here we assume a new accumulation distribution that provided with a melting event give us the desired reduction in coverage *from* the new observed SCA *to* the old modelled SCA. This way of reasoning is quite the opposite of the case when the observed SCA is less than the modelled SCA.

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