Aerodynamic and surface characteristic length scales
of snow covered flat planes

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ABSTRACT: Velocity profiles over fresh fallen snow surfaces have been measured in the SLF cold atmospheric boundary layer wind tunnel. Aerodynamic roughness lengths z_0 have been estimated from log-law velocity profile fitting. In parallel, photographs of the snow surfaces have been taken and evaluated using digital image analysis giving snow surface contour line coordinates. Applying structure functions to the snow surface coordinates and statistical fitting analyses resulted in sets of surface characteristic length scales (random field approach). The synthesis of both approaches suggests a linear relationship between z_0 and one of the surface characteristic length scales.

KEYWORDS: aerodynamic roughness length, digital image analysis, snow surface roughness, structure functions, surface characteristic length scales, turbulent boundary layer flow.

1 INTRODUCTION

The surface roughness is of great importance for all kind of turbulent exchange processes within the near-ground part of the atmospheric boundary layer. For atmospheric flows, the aerodynamic roughness lengths z_0 is commonly used to express the surface roughness implications on wind flow and turbulent momentum transport. It is a measure of the capability of the surface to extract momentum from the flow (e.g. Raupach et al., 1991). Despite its significance, still a considerable uncertainty on the appropriate choice of the parameter z_0 for snow covered surfaces exists. This is reflected by the variety of values for z_0 which, even for orographic less structured areas, spans several orders of magnitude (e.g. Andreas et al., 2004).

In the SLF cold atmospheric boundary layer wind tunnel (situated on 1650 m a.s.l. in Davos, Switzerland) flows over snow covered surfaces and atmosphere-cryosphere interaction processes are studied. So far, the wind tunnel has been used to investigate threshold wind speeds for snow transport (Clifton et al., 2006), to investigate snow ventilation (Clifton et al., 2008), saltation of fresh snow (Guala et al., 2008) and to improve saltation models (Clifton and Lehning, 2008).

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This paper is devoted to snow surface characteristics, in particular to surface roughness features. Two different approaches have been followed to characterize snow surface roughness, (i) aerodynamic roughness lengths z_0 have been estimated from log-law fitting of measured vertical profiles of horizontal mean velocities from turbulent boundary layer flows over snow covered surfaces and (ii) surface characteristic length scales have been determined from second order structure functions of snow-air interface lines obtained from surface photographs and digital image analysis (random field approach), see Manes et al. (2008).

The final aim of these investigations is to bring both approaches together, i.e. to analyze correlations and to establish possible relationships between the two sets of roughness length scales.

2 APPROACH

2.1 Velocity measurements

Velocity profiles of turbulent boundary layer flows over snow surfaces were measured. The experiments were performed with naturally fallen snow which was collected in custom-made trays positioned outside the wind tunnel facility in a wind sheltered area. The snow laden trays were placed in the wind tunnel providing an 8 m long natural fetch of snow on which a boundary layer developed.

Horizontal and vertical velocity fluctuations were measured using a 2D Hot-Wire Anemome-

ter (HWA) which was traversed at many positions z above the surface along a single vertical located at 1.5 m from the downstream end of the snow fetch. The aerodynamic roughness lengths z_0 were determined by fitting the data of the mean horizontal wind velocities u(z) to the logarithmic velocity profile

$$\frac{u(z)}{u_*} = \kappa^{-1} \ln\left(\frac{z-d}{z_0}\right) \qquad (1)$$

with u_* the friction velocity of the constant flux layer, d the displacement height and κ the von Kármán constant (κ = 0.4).

During each experiment, the wind tunnel flow velocities were kept low to prevent snow drift and to ensure that the snow surface properties remained unaltered.

2.2 Random field approach

Before and after each wind tunnel experiment several photographs of the snow surface were taken. The photographs were processed by digital image analysis in order to work out the contour lines of the snow-air interface (Figure 1).
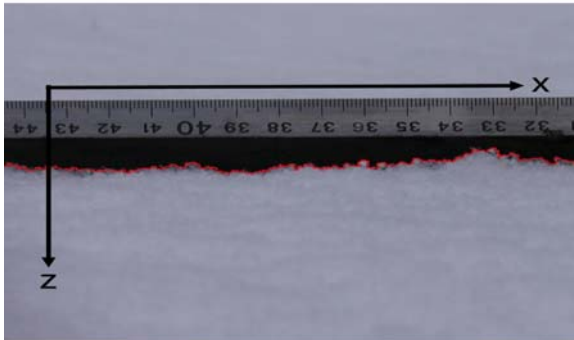

Figure 1. Photograph of snow surface with snow-air interface contour line.

Each point in the contour line was then associated to a horizontal and a vertical coordinate (x and z, respectively) which were used to compute the second order structure functions $D_2(r)$

$$D_2(r) = \left\langle [z(x+r) - z(x)]^2 \right\rangle \qquad (2)$$

for many values of the spatial lag r.

Applying a three-linear regression analysis to such second order structure functions lead to define four characteristic length scales; two horizontal length scales, the crossover length l and the saturation length L, and their corresponding vertical length scales $[D_2(l)]^{1/2}$ and $[D_2(L)]^{1/2}$. The characteristic length scales mark three distinct scaling regions with scaling exponents $\alpha_{2,i}$ being the slopes of the curves in the double log presentation (Manes et al., 2008).
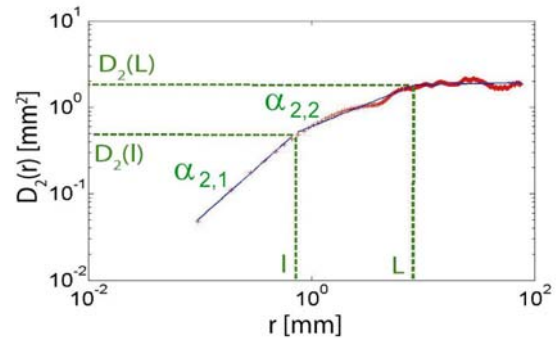

Figure 2. Diagram of second order structure function $D_2(r)$ with characteristic length scales and scaling exponents.

3 RESULTS

3.1 Velocity measurements

A series of 4 wind tunnel experiments with turbulent boundary layer flow over fresh fallen snow was performed in the winter season 2008/2009. Figure 3 shows the inner scaled log-law fitted vertical velocity profiles indicated by their dates (ddmmyy).
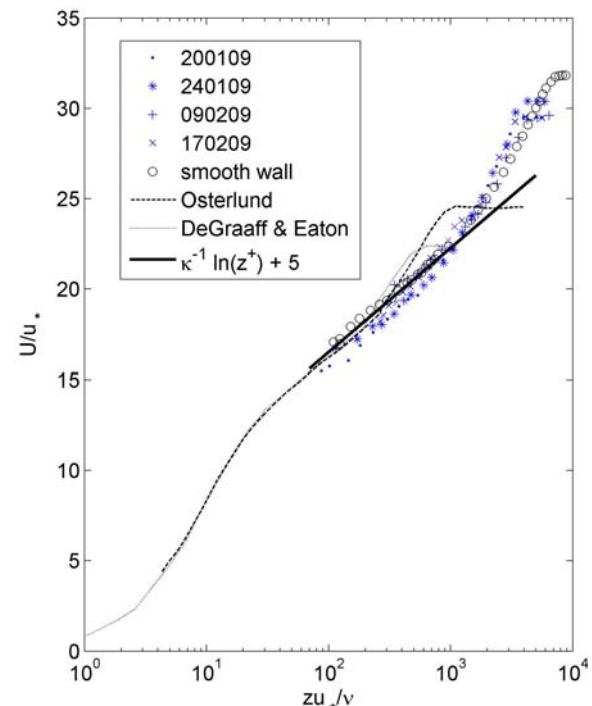

Figure 3. Vertical velocity profiles normalized according to inner scaling and offset by the roughness function $\Delta U/u_*$.

For the purpose of comparison and general wind tunnel setup validation, one smooth wall velocity profile, measured in the SLF cold atmospheric boundary layer wind tunnel, and turbulent boundary layer flow data of Osterlund,

(1999) and DeGraaff and Eaton, (2000) are shown together with the curve $\kappa^{-1} \ln(z^+) + 5$ (Raupach et al., 1991) in Figure 3. The 5 is an integration constant for smooth surfaces arising from the asymptotic matching analysis for u(z). Within the core of the log-law region ($100 < z^+ < 2000$), a collapse of all data sets into a narrow band is visible. This indicates the general capability of the SLF cold wind tunnel for boundary layer studies and the proper choice of the log-law parameters u_*, $\Delta U/u_*$ respectively z_0 and d, which have been determined by a combination of least square based linear regression analysis and manual adjustment, see Table 1.

Figure 4 shows the outer scaled velocity profiles with the velocity defect law depicted along the axis of ordinate. Again, a satisfactory collapse of the measurement data is evident, confirming the capability of the SLF cold wind tunnel to generate boundary layer profiles.
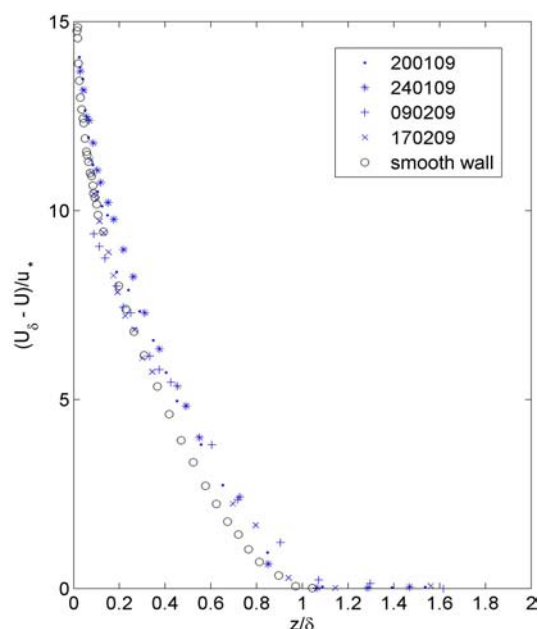

Figure 4. Vertical velocity profiles normalized according to outer scaling.

| | U_δ | u_* | δ | z_0 | d |
	[m/s]	[m/s]	[m]	[mm]	[mm]
200109	5.53	0.24	0.25	0.130	0.0
240109	5.50	0.24	0.28	0.190	0.0
090209	5.51	0.27	0.24	0.320	0.0
170209	5.46	0.25	0.23	0.190	0.0
smooth	14.0	0.44	0.27	0.005	0.0

Table 1. Velocity profile parameter.

Here z_0 was calculated from the roughness function $\Delta U/u_*$ by

$$z_0 = \frac{v}{u_*} \exp\left\{\kappa\left(\frac{\Delta U}{u_*} - 5\right)\right\} \qquad (3)$$

The aerodynamic roughness lengths z_0 found (Table 1), are in general agreement with literature data (Andreas et al., 2004, Clifton et al., 2006). Displacement heights d were estimated to zero for all experiments. This is attributed to the relatively flat snow surface and to the quite small roughness element elevations. The rather small scatter of estimated z_0 values (within a factor of 2.5) is ascribed to the relative constant boundary conditions. All experiments were performed with fresh fallen snow within a few hours after snowfall under ambient temperatures ranging from -5°C to -2°C, so that significant snow metamorphosis processes with implications on changes in the snow surface roughness can be neglected. Moreover, since the snow deposited naturally in a wind sheltered area, effects of mean wind and turbulence as well as snow drift on the deposition process can be excluded.

Based on the values given in Table 1, roughness Reynolds numbers Re_R of the snow experiments were calculated. Calculations for the roughness Reynolds number resulted in $Re_R = z_0^+ = z_0/(v/u_*) = 1.9, 2.7, 5.4, 2.9$ in accordance to the order given in the table. Thus, with the exception of the first experiment (200109), there is evidence of aerodynamically rough regimes ($z_0^+ > 2.5$) with flow independent aerodynamic roughness lengths z_0 (e.g. Schlichting and Gersten, 2000). However, this is no final proof for the presence of aerodynamically rough regimes. A strict proof would require several experiments with different flow Reynolds numbers, i.e. different velocities, giving all the same z_0.

3.2 Random field approach

The characteristic length scales (l, $[D_2(l)]^{1/2}$, L, $[D_2(L)]^{1/2}$) and the scaling exponents $\alpha_{2,i}$ resulting from the three-linear regression analyses of second order structure functions of snow-air interface lines (Figure 2) are summarized in Table 2. The length scales given are mean values of 5 surface photographs taken at different positions along the snow fetch immediately after each wind tunnel experiment.

	l	$D_2(l)^{1/2}$	$\alpha_{2,1}$	L	$D_2(L)^{1/2}$	$\alpha_{2,2}$
200109	0.49	0.40	1.16	12.6	1.29	0.69
240109	0.53	0.43	1.21	13.6	1.63	0.78
090209	0.71	0.43	1.26	4.1	0.80	0.75
170209	0.55	0.34	1.23	9.2	0.91	0.68

Table 2. Surface characteristic length scales of snow roughness (units: mm).

While the horizontal crossover lengths l are of snow crystal size scale, the horizontal saturation lengths L are one to two orders of magnitude larger, reflecting typical aggregation size scales (Lowe et al., 2007). Consequently, $D_2(l)$ and $D_2(L)$ are associated with typical vertical lengths on the crystal and aggregation size

scale, respectively. Regarding the scaling exponents $\alpha_{2,i}$ it can be seen that they do not vary strongly but rather show constant values for the two scaling regions with means of $\alpha_{2,1} = 1.22$ and $\alpha_{2,2} = 0.73$. The mean value found for $\alpha_{2,2}$ is close to the estimate for the scaling exponent inherent a deposition process called Ballistic Deposition, which retains universal scaling behaviour. Following theoretical considerations, the Ballistic Deposition scaling exponent was determined to $\alpha_2 = 0.76$ (Barabási and Stanley, 1995). Moreover, Manes et al. (2008) revealed further evidence for the self affinity of naturally deposited snow surfaces. Calculating higher order structure functions $D_p(r)$, they found $\alpha_{p,i}$ to increase linearly with p to a good approximation, i.e. $\alpha_{p,i} = p \cdot \alpha_{1,i}$.

The applicability of the random field approach shall be fortified by the diagrams given in Figure 5. Here, the second order structure functions were normalized by the crossover length scales l and $D_2(l)$ (top diagram) and by the saturation length scales L and $D_2(L)$ (bottom diagram). Both normalizations result in a narrow band collapse within the corresponding scaling regions.
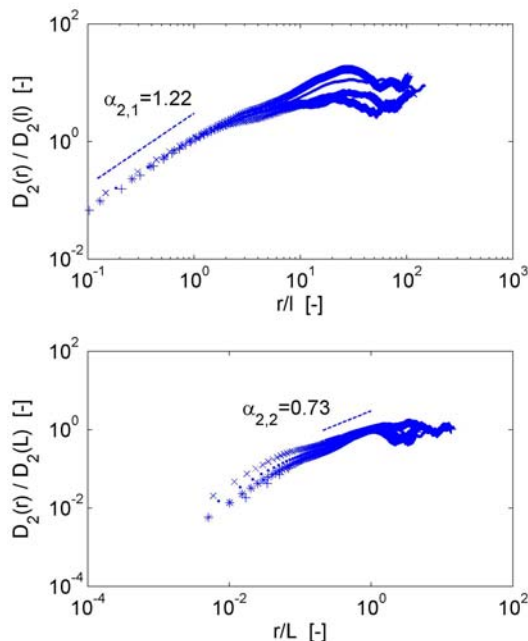

Figure 5. Second order structure functions $D_2(r)$ scaled with crossover (top) and saturation (bottom) length scales.

4 DISCUSSION

The aim of this chapter is to give a synthesis of the surface length scales obtained from the velocity measurements, i.e. the aerodynamic roughness length z_0 and characteristic surface length scales obtained from the random field approach. In Figure 6, diagrams of z_0 values plotted against the surface characteristic length scales l, $D_2(l)^{1/2}$, L and $D_2(L)^{1/2}$ are provided.

Though the data basis of aerodynamic roughness lengths z_0 and characteristic length scales is rather small, we have calculated the correlation coefficients R for each possible combination. This coefficient represents a measure for the strength of a linear relationship between the response variable z_0 and the predictor variables l, $[D_2(l)]^{1/2}$, L and $[D_2(L)]^{1/2}$. Additionally, the hypothesis of "no correlation" was tested. This test gives the probability P for obtaining an equal correlation coefficient R by random chance, when there is no correlation at all. The results of the statistical analysis are summarized in Table 3.
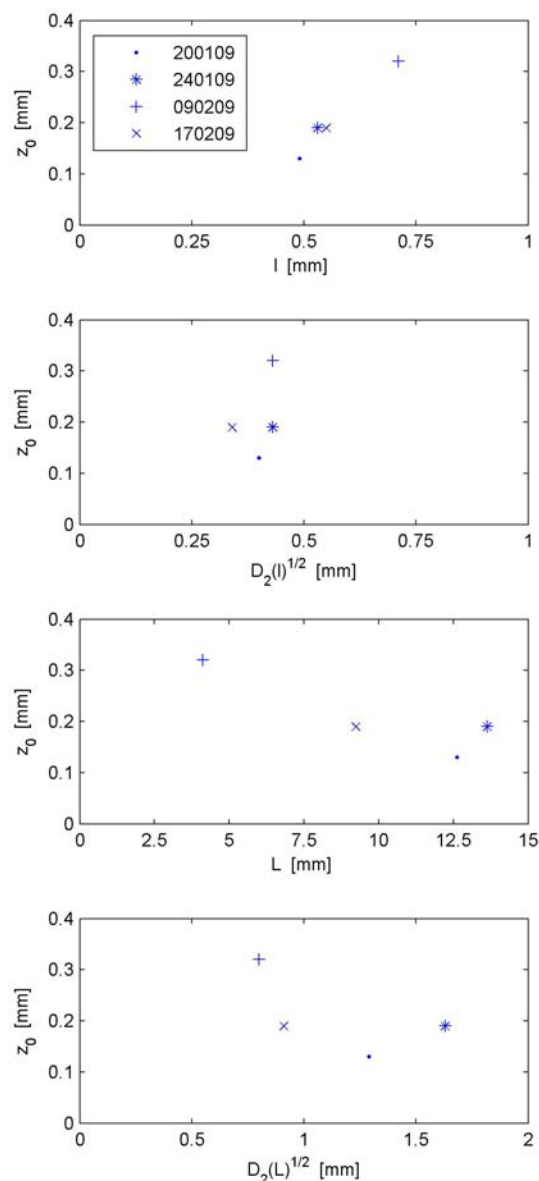

Figure 6. Aerodynamic roughness length z_0 versus surface characteristic length scales l, $D_2(l)^{1/2}$, L and $D_2(L)^{1/2}$.

	l	$[D_2(l)]^{1/2}$	L	$[D_2(L)]^{1/2}$
R	0.99	0.38	-0.89	-0.60
P	0.01	0.62	0.11	0.40

Table 3. Correlation coefficients R and "no correlation" hypothesis probabilities P for characteristic length scales and z_0.

The correlation is said to be significant when the corresponding P value is less than 0.05. Based on this criterion, the statistical analysis suggests a linear relationship between the crossover length l and the aerodynamic roughness length z_0, whereas for the other combinations no linear relationship is evident. The statistical analysis is confirmed by the diagrams shown in Figure 6, where the pair of values l and z_0 appears to lie to a good approximation on a straight line, but no such behaviour is apparent for the other combinations.

The correlation between the aerodynamic roughness length z_0 and the crystal size length scale l contains some controversy. In general, z_0 is mostly influenced by the largest length scales characterizing a rough surface and not the smallest. However, this is true only if there is a sufficient scale separation between the two. Unfortunately, in our experiments, grain and aggregation length scales are often comparable (Table 2). We must consider, for the set of experiment presented here, that the crystal size length scales are more relevant than the aggregation length scales for the definition of a representative aerodynamic roughness length z_0.

However, since our data basis is still very small, we did not further pursue to establish a functional relationship between the aerodynamic roughness length z_0 and the crossover length l.

5 CONCLUSIONS

The SLF cold wind tunnel has proved to be capable of boundary layer flow studies over naturally fallen snow surfaces, as was shown by the inner and outer scaling diagrams shown in Figures 3 and 4. Aerodynamic roughness lengths z_0 obtained from boundary layer flow wind tunnel experiments are in overall agreement with available literature data.

As a result of the random field approach, we were able to give further evidence of the deposition nature of snow fall. Scaling exponents correspond to that of Ballistic Deposition which was also noted by Lowe et al. (2007) and Manes et al. (2008). Moreover, the resulting characteristic length scales could be assigned to typical particle size and aggregation size length scales.

The synthesis of both sets of surface length scales is limited as the data basis is still too small. However, the experimental results sug-

gest a linear relationship between the aerodynamic roughness length z_0 and the crossover length l. This finding contains some controversy, as z_0 seems not to depend on aggregation size length scales but rather on the particle size. We argue that the less pronounced scale separation between particle and aggregation size scales might be a possible explanation for the apparent relevance of l on z_0.

The limited amount of experiments does not allow us to provide a final robust relationship between roughness length scales and z_0. Further experiments are necessary to enlarge the data base.

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