Modeling snow slab release using a visco-elastic, temperature dependent constitutive model and weak layers

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Abstract

A widely accepted theory of slab avalanche formation assumes the existence of "super-weak" layers. Because a weak layer cannot effectively transmit the shear forces from the overburden snow, stress and strain-rate concentrations at the ends of the weak layer are generated. Fracture initiates at the higher end when the strain-rate concentration exceeds a critical value.

This theory, which predicts the necessary length of the weak layer for avalanche release, is based on simple temperature-independent, linear-viscous material behaviour for both the homogeneous and weak snow layers. It is inadequate to describe the influence of loading rate and temperature increase because the highly non-linear visco-elastic, temperature dependent behaviour of snow is not included in the formulation.

This paper presents a two-dimensional, temperature dependent, visco-elastic finite element model for avalanche formation. Multi-layered snowpacks on uniform slopes and on slopes with ground perturbations are modeled using periodic boundary conditions and special finite elements to account for weak layers and snowpack glides. Example simulations are shown and the results are compared to established theories.

Temperature dependent Material behaviour

The relationship between yield stress, strain-rate and temperature can be expressed as a power law (Glen, 1955) based on the Arrhenius law:

\[ \dot{\epsilon}_i = A \sigma^{n_i} \exp \left( -\frac{Q}{RT} \right) \]

where \( \dot{\epsilon}_i \) is the applied strain-rate in \( \text{s}^{-1} \), \( A \) is a density dependent material parameter in \( \text{kPa}^{-n_i} \), \( Q \) is the activation energy in \( \text{kJ mol}^{-1} \), \( R \) is the gas constant \( \text{kJ mol}^{-1} \text{K}^{-1} \), \( T \) is the temperature in \( \text{K} \), \( \sigma_y \) is the resulting yield stress in \( \text{kPa} \) and \( n_i \) is a dimensionless exponent.

Stress - strain-rate relationship

Fig. 1 shows typical stress-strain curves for snow at different temperatures. Fig. 2 shows the relationship between applied strain-rate and yield stress in a log-log plot.

The stress - strain-rate relationship obeys the power law (Eq. 1). Note that the exponent \( n \) of Eq. 1 represents the slope of the plotted lines (Fig. 2).

\[ \epsilon_y = A \sigma^n \]

where \( \epsilon_y \) is the applied strain-rate and \( \sigma_y \) is the measured yield stress. The measured yield stresses and \( n \) resp. \( n_1 \) and \( n_2 \) are the calculated exponents. The activation energy for the temperature range \( T_1 < T < T_2 \) is obtained from Eq. 3.

\[ Q = R \left( n_1 \ln \sigma_y - n_2 \ln \sigma_y \right) \]

Fig. 4 shows the calculated activation energy as a function of the density for three different temperature ranges.

Material Law for Snow

In order to apply the experimental results in a finite element model, the \( \eta-p \) and \( Q-p \) isochrons (Figs. 3 and 4) were parameterised. The relationships presented above were employed in a plane-strain constitutive law for steady-state creep, used by Bader and Salm (1990)

\[ \sigma_{ij} = [N] \left( \dot{\epsilon}_{ij} + \frac{\sigma_y}{m-2} \right) \]

where,

\[ [N] = \begin{bmatrix} 0 & \eta \sigma \eta \sigma_y \eta \sigma_y \eta \sigma_y \\ \eta \sigma \sigma_y \sigma_y \sigma_y & 0 \end{bmatrix} \]

\( \eta \) is a stress, density and temperature dependent viscosity

\[ \frac{1}{\eta} = A \exp \left( \frac{Q}{RT} \right) \]

and \( m \) is the inverse of the viscous analogue of Poisson's ratio.

The constitutive law given by Bader and Salm was introduced into a finite element model. In a first step, the stress state is determined from an elastic calculation. Then, the steady-state strain-rates are determined according to Eq. 1.
Finite Element formulation

The force balance equation is solved for the displacement u.

\[-\nabla \cdot (\sigma) = f - f_e + f_t\]

Forces on the body are the gravity force \(f_e\) (weight/body force) and an external force \(f_t\) applied on the boundary of the model domain. This differential equation is solved by the standard finite element method with linear shape functions.

The boundary conditions are zero velocity on the ground and no outward stress at the top.

\[v_i|_{\text{ground}} = 0\]

\[\sigma_{ij} n_j \mid_{\text{surface}} = 0\]

"Weak layer" model

\[k = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} v_n \\ v_f \end{bmatrix}\]

Should the weak layer have a residual shear strength \(k_1 > 0\).

Model Calculations

Fig. 6 shows the first model domain. The length \(a\) of the weak layer, the density \(\rho_1\) and temperature \(T_1\) were varied. This model is taken from the work of Bader and Salm.

\[d_0 = d_1 = 1m, \rho_0 = 350kg/m^3 \text{ and } T_1 = -2^\circ C\]

The second model shown in Fig. 7 has a ground perturbation at the middle of the domain. It is modeled as a Gaussian bell shape with a length of 10m and a varying height of 0.4 to 1m.

For both models the top layer properties were varied in the relation below.

\[
\begin{cases}
\sigma_1 & = k_0 \\
\sigma_2 & = 0 \\
v_n & = 0 \\
v_f & = 0
\end{cases}
\]

Results

Weak-Layer

\[\text{Shear strain rate for different temperatures with a bump height of 1.0m and } \rho = 2000kg/m^2\]

In Fig. 9 the density dependence is shown. As can be seen, for higher densities the strain rate peaks are much smaller. Compared to Bader and Salm who used a density of more than 400kg/m^2 this seems to be a much more realistic result.

\[\text{Shear strain rate for different bump heights at } \rho = 2000kg/m^2\]

The influence of the weak layer length is shown in Fig. 10. For a weak layer length of about 18m the strain rate reaches a critical value of about 10^-4s^-1. This is according to the literature sufficient for a slab avalanche to be triggered.

\[\text{Shear strain rate for different weak layer lengths at } T_1 = -2^\circ C\]

Ground perturbation

The temperature dependence shown in Fig. 11 indicates that a sudden change of temperature form either warm (higher than -2°C) or cold (less than -10°C) to around -6°C might be an avalanche trigger event. Here one sees, that it is important to take the snow temperature into account for stability simulations.

\[\text{Shear in x [l]}\]

Conclusions

The main results of Bader and Salm could be corroborated; however, with realistic model parameters. A weak layer needs a certain length, about 20m to be the cause of an avalanche. However, the results are very temperature and density sensitive. It seems that if large weak layers under high density snow exist the event of an avalanche is unlikely.

Weak layers are not the only triggering element - even if there is no explicit weak layer.

References

