Fracture toughness for dry slab avalanches

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Abstract: Dry slab avalanches release by propagating shear fractures. Since alpine snow is a quasi-brittle strain-softening material, it is implied that fracture toughness not shear strength is the fundamental quantity related to instability. In this paper, the first estimates of fracture toughness for dry snow slabs are given based on field data in relation to the fundamental fracture mechanics size effect law and mechanical properties of snow slabs.

Keywords: fracture, dry slab avalanche

Fracture mechanical size effect law for triggering of dry slab avalanches

Alpine snow is a pressure sensitive, dilatant strain-softening material with significant temperature and rate dependence when failed in shear. Since it is a strain-softening material it is classed as quasi-brittle and as a result there will be a fracture mechanical size effect which governs fracture initiation. Thus, the failure cannot be described in terms of plasticity (Bažant and Planas, 1998) but fracture mechanics must be used. If plasticity governed, the mechanical failure criterion could be developed in terms of stress and strain and their invariants, i.e., geometrically small and large snow slabs would fail at the same maximum stress, or at the same nominal stress, defined as the average shear stress applied to a cross section of the slab, with no size effect dependence.

The necessity of size effects on natural failures involving strain-softening materials was recognized by Palmer and Rice (1973) and they illustrated the concept for slides in over-consolidated clay. Using non-linear fracture mechanics they showed that there is a finite size associated with the fracture process zone for which strain-softening is taking place (shear stress drops from peak to residual) and they derived relationships for the Mode II fracture energy required to form a crack of unit area. The formalism of Palmer and Rice was applied to the snow slab by McClung (1979, 1981, 1987) following the discovery (McClung, 1977) that alpine snow fails as a dilatant, strain-softening material in simple shear.

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Fracture mechanical size effect law for the dry snow slab from non-linear fracture mechanics with finite residual stress

Bažant, Zi and McClung (2002) developed the theory for the dry snow slab assuming quasi-brittle (strain-softening) behaviour in the weak layer with a finite sized fracture process zone and finite residual shear stress downslope from the fracture process zone. The theory takes the assumption of elastic slab behaviour when it is failing with strain-softening behaviour taken as the failure process in the weak layer. See Bažant, Zi and McClung (2002) for additional assumptions and qualifiers on the theory, which include: ignoring visco-elastic effects and the assumption that the snow-pack material below the weak layer has a much higher modulus than the slab.

From Bažant, Zi and McClung (2002) the expression for the nominal shear stress at failure, τ_N for large sizes is given by:

$$\tau_N = \frac{1}{\alpha_0}\sqrt{\frac{2EG_{II}}{D}} + \tau_r \qquad (1)$$

where α_0 is a material constant, τ_r is residual stress and the expression is written for $D >> c_f$ (where $2c_f$ is length of the fracture process zone). McClung (1979) and McClung and Schweizer (1999) have given estimates of c_f derived from strain-softening shear failure experiments on alpine snow. Figure 1 is a schematic showing slab geometry and the lengths: D and $2c_f$. In (1), E is the effective Young's modulus at the base of the snow slab. The nominal shear stress in this case for a planar snow slab on a slope of constant angle (ψ) is estimated as: $\tau_N = \rho g D \sin\psi$ where g is the magnitude of acceleration due to gravity, ρ is mean slab density and ψ is slope angle.

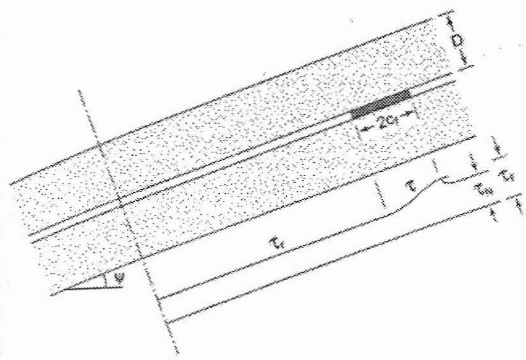

Figure 1: Geometry and length scales for snow slab fracture. Slab thickness is D, the fracture process zone is $2c_f$ and τ_f is peak shear stress.

Following Bažant, Zi and McClung (2002) with the assumption that τ_r is negligible, the Mode II fracture toughness may be written:

$$K_{IIc} = \sqrt{EG_{II}} = \tau_N \alpha_0 \sqrt{\frac{D}{2}} \qquad (2)$$

for $D >> c_f$

Thus, for large sizes, fracture toughness scales as: $\tau_N D^{1/2}$. In (2), there are two important effects: the dependence of τ_N on D (shown below to depend on creep effects under the slab overburden) and the square root dependence derived from non-linear fracture mechanics. Both of these effects must be considered in order to explain the Mode II fracture toughness of the snow slab, which is central for discussing snow slab release.

Dependence of nominal shear strength scaled from field measurements at fracture lines

The fracture mechanical size effect contained in Bažant et al., 2002 is derived from the assumption that the material is the same underneath the snow slab at all sizes. The field data presented below show clearly that this condition will not be satisfied on mountain slopes. The reason is that the weight of the snow above the weak layer and the age of snow at the weak layer have major effects on the properties of snow at the base of the snow slab. An increase in thickness, which is often accompanied by increased age of snow at the base, causes densification and bonding which increases the Young's modulus and enhanced bonding increases the nominal shear strength. The combined effects of creep under load and bonding must be taken into account in order to adequately describe fracture toughness. These effects combine to increase the fracture toughness in addition to the fracture law size effect. For the present analysis, it would not be correct to attribute these effects to shear strength since the concept of shear strength does not apply to snow slab release governed by fracture and strain-softening (Bažant, Zi and McClung, 2002). Instead, fracture toughness is the fundamental quantity.

Field data from fallen snow slabs

I have collected data taken from the 187 fracture lines of dry fallen snow slabs. The data contain the mean slab density, the average slope angle that the slabs have initiated on and the average depth D of the slabs at the fracture line. The sources of the data include: Perla, 1976; Stethem and Perla, 1980 and data from the collection of the Dr. Jürg Schweizer, Swiss Federal Institute for Snow and Avalanche Research. For some of the data (123 cases), I also have weak layer failure temperatures. A least squares regression line for τ_N as a function of D was determined. The equation of the least squares line is:

$$\tau_N = CD^{1.22} \qquad (3)$$

In (3), 82% variance of τ_N with D is explained ($R^2 = 0.82$, where R is Pearson correlation coefficient, standard error is 0.33 for 187 values). The value of $C = 1.36$ kPa/m$^{1.22}$. The 95% confidence limits on the power of D are between 1.13 - 1.30.

Fracture toughness scaling law

Combination of equations (3) and (2) gives a fractal scaling law for the fracture toughness from the fracture scaling size effect law and the material effects of creep and bonding on τ_N:

$$K_{IIc} = \left(\alpha_0 C / \sqrt{2}\right) D^{1.72} \qquad (4)$$

Equation (4) implies that $K_{IIc}^2 \propto D^{3.44} \propto G_{II}$. Thus, the fracture toughness squared has a fractal power of approximately 3.44 and it is proportional to the energy for the Mode II fracture energy of snow, i.e. the energy required to form a unit area of sliding crack.

For slabs with $D \geq 0.50$ m, expression (4) is replaced by:

$$K_{IIc} = \left(\alpha_0 C / \sqrt{2}\right) D^{1.78} \qquad (5)$$

which implies $K_{IIc}^2 \propto D^{3.56}$. This last relation results from a least squares analysis yielding: $\tau_N = 1.37\ D^{1.28}$ with $R^2 = 0.70$, standard error 0.31 for 118 dry slabs.

For this analysis, 95% confidence limits on the power of D are 1.13-1.44.

Summary and Conclusions

Dry slab avalanches initiate by Mode II shear fracture in a weak layer preceded by strain-softening. This implies a size effect law for fracture and that the fracture toughness, K_{IIc} governs slab release not the shear strength as classical analyses using plasticity predict. The classical stability index (e.g. Roch, 1966): the ratio of shear strength to shear stress at a weak layer does not apply to snow slab release in general. Any yield criterion based solely on stress or strain is analogous to plasticity for which fracture size effects do not exist. Snow slab release is a problem involving strain-softening, size effects and fracture initiation.

In addition to creep and bonding processes and the fracture mechanics size effect law, there are other aspects of fracture toughness which may enter including temperature effects and anisotropic mechanical properties of the weak layer. These will be presented in a future paper.

The answer to the question: Why are big (thicker) snow slabs rarer than thin ones? is contained in fracture toughness. Thicker snow slabs have increased toughness due to creep and bonding processes under weight applied at the weak layer and they are subject to the fracture size effect law.

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