Estimating the strength of faceted snow layers for an avalanche forecasting model

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Abstract: Current multivariate forecasting models make limited use of stability indices partly because the required manual measurements of snowpack properties are not available daily. Using 10 years of strength measurements of weak layers of faceted grains from two ranges in western Canada, a regression model is developed for estimating the strength of weak layers of faceted grains within eight days of manual snowpack measurements. Model testing shows promising results for the Columbia Mountains.

Keywords: Snow stratigraphy, snow properties, snow strength, avalanche forecasting, slab avalanches, snow crystals

1. Introduction

Meteorological and avalanche occurrence data are commonly used for forecasting avalanches. Up to now computer-assisted forecasting models have made little use of snowpack variables including stability indices, even though such indices have been shown to correlate with avalanche activity (e.g. Föhn, 1987; Jamieson and Johnston, 1993). Such indices use the strength of buried weak layers, which are potential failure layers for slab avalanches.

Chalmers (2001) successfully estimated shear strength changes of surface hoar layers up to eight days past the day in which a manual snow profile was observed. In the Columbia Mountains, a stability index for skier-triggering, based on the strength estimates, correlated with skier-triggered avalanches within 100 km of the study plot in which the snow profile was observed.

Since layers of surface hoar and faceted crystals form the failure layers for many skier-triggered avalanches in the Columbia Mountains, our objective was to develop a similar model for layers of faceted crystals. Specifically we wanted to:

1. develop a model for estimating the shear strength of faceted layers (without shear frame tests) on a day in which a snow profile was observed,
2. extrapolate shear strength over time by developing a model for the rate of change of shear strength of faceted layers,
3. assess the combined model with strength measurements,
4. discuss the physical role of the predictor variables.

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2. Literature review

Compared to layers of precipitation particles, decomposing and fragmented particles and rounded grains, layers of faceted grains exhibited increased variability of tensile and shear strength (Sommerfeld, 1973; Jamieson and Johnston, 1990, 2001).

For data from the Columbia Mountains of western Canada, Chalmers (2001) and Chalmers and Jamieson (2002) regressed current strength and the rate of strength change for surface hoar layers on measurements from manual snow profiles. When these two regression models were combined, estimates of strength and stability within eight days of a snow profile were promising.

Johnson (2000) related the shear strength of weak layers of faceted crystals in the Columbia Mountains (mostly Intermountain snow climate) and Rocky Mountains (Continental snow climate) to easily measured snowpack properties. He found positive correlations with load, slab density, slab thickness, hardness of the facet layer, snowpack thickness and age of the facet layer. In the first four to six weeks after the layers formed, the increase in shear strength was roughly proportional to the loading rate in each of the two snow climates. Attempts to estimate strength of the facet layers from Kojima's (1967) densification model were unsuccessful.

3. Methods

3.1 Field methods

Most observations were made according to the Canadian observation guidelines for snow profiles (Canadian Avalanche Association, 2002). There are three exceptions. The thickness of weak layers was measured to the nearest millimetre at three or more places on the pit wall and averaged. The shear strength of weak layers was measured with the shear frame as described in Sommerfeld (1984) and Jamieson and Johnston (2001). The load (overburden) was the average weight of cylindrical samples from the snow surface to the weak layer divided by the cross-sectional area of the cylinder.

3.2. Dataset

Data from the Columbia and the Rocky Mountains of western Canada were selected to develop a model to forecast the shear strength of faceted layers. The grains in 53% of the layers consisted of ICSSG Type 4 (faceted crystals), and the remainder were classified as Type 4c (rounding facets) (Colbeck and others, 1990). Included were 17 time series (7 Continental, 10 Intermountain) with a total of 86 shear strength measurements (32 Continental, 54 Intermountain) from the years 1993 to 2002. This dataset included 69 strength changes (25 Continental, 44 Intermountain) for the analysis. From the model development, we excluded:

- measurements with weak layer temperatures > -1°C, to avoid the late season effect where the strength of a weak layer (as measured with a shear frame) decreases, but does not result in increased avalanche activity,
- time series in which the measurement intervals exceed 25 days, since these were statistical outliers,
- data points for which strength change rates were less than -120 Pa d^{-1} over a measurement interval, since we assume these unlikely strength changes were due to spatial variability at the study site,
- sets of measurements for which shear frame operators noted that the measurements were inconsistent or the fractures non-planar, and
- two complete time series for model testing purposes (one from a Continental snow climate and one from an Intermountain snow climate).

3.3. Model building

The goal was to find a model for estimating shear strength on a forecasting day on which no snowpack observations are performed. Following Chalmers' (2001) Interval model for estimating surface hoar shear strength, we use the same form of equation to forecast the strength of faceted layers:

$$\Sigma_j^* = \Sigma_i^* + \Delta t_{ij} \, (\Delta\Sigma/\Delta t)_{ij}^* \qquad (1)$$

where Σ_i^* and $(\Delta\Sigma/\Delta t)_{ij}^*$ are functions of snowpack observations on day i; Σ_i^* is the estimated shear strength on day i (kPa), $\Delta t_{ij} = t_j - t_i$ is the time interval between day i and day j, $(\Delta\Sigma/\Delta t)_{ij}^*$ is the estimated rate of change in shear strength (kPa d^{-1}) between day i and day j, and Σ_j^* is the forecast shear strength on day j (kPa). Since Equation 1 is intended to estimate the shear strength of a faceted layer a number of days after the manual snowpack measurements, we refer to it as the Forecasting model.

This approach required that two formulas be developed:

1. shear strength on the day of manual snowpack observations, and
2. rate of change of shear strength between the day of the last snowpack observation and the day to be forecasted.

The response variables were Σ_i and $(\Delta\Sigma/\Delta t)_{ij}$. In the dataset, Σ_i ranged from 0.6 to 6.7 kPa with a mean of 2.3 kPa. Values of $(\Delta\Sigma/\Delta t)_{ij}$ varied from -117 Pa d^{-1} to 336 Pa d^{-1} with a mean of 46 Pa d^{-1}.

Neither response variables were normally distributed. Shear strength had an extended tail of higher values, and was, of course, truncated at zero. The distribution of strength change rate was skewed towards higher positive values with a gradual decrease in the frequency of negative strength change rates.

The set of predictor variables included snowpack and weather observations as well as elaborated variables (Table 1).

Table 1: Possible predictor variables

Abbreviation	Explanation
Age	Age of the faceted layer (days)
HS	Height of snowpack (cm)
Slope	Inclination of slope (°)
Load	Weight per unit area of overlying snow (kPa)
H	Thickness of overlying slab (cm)
h	Hand hardness exponent (Geldsetzer and Jamieson, 2001)
HH	Hand hardness index = 4^{h-1}
TG	Temperature gradient measured over 10 cm across failure plane in facet layer (°C/m)
Ta/HS	Average snowpack temperature gradient (°C/m)
Twl	Temperature of the weak facet layer (°C)
Emin	Minimum grain size of facets (mm)
Emax	Maximum grain size of facets (mm)
Ta	Air temperature (°C)
Thick	Thickness of the weak layer (cm)
SlabDens	Density of overlying slab (kg m^{-3})
Crust	Existence of a crust below the weak layer

The Age of the layer represents the number of days since the layer was buried. This parameter has to be viewed with caution since some of the observed layers in the Rocky Mountains initially consisted of surface hoar crystals that metamorphosed to facets.

In the analysis of shear strength change rate, we also included the potential predictors:

- initial strength of the facet layer,
- average values of HS, Load, *H*, *h*, TG, TG/Twl, Twl, Ta, SlabDens over the preceding measurement interval,
- average daily change in HS, Load, *H*, TG, Twl, Ta over the preceding measurement interval.

3.4. Analytical methods

Spearman rank correlations were applied to assess the relative importance of each individual predictor variable and provide a basis for discussing the physical effect of the predictors on the response variables.

Simple linear regression analysis was used to examine the relationship between two variables (Allen, 1997, p. 16). We used the most significant variables from the Spearman rank correlation as predictor variables. Shear strength and strength change rate were the response variables.

Multivariate analysis was used to model the relationship between the predictor variables and the response variables: shear strength and shear strength change rate. A comparison of the performances of the simple and the multivariate regression models was made.

4. Analysis

4.1. Spearman rank correlations

The following predictor variables were significantly correlated ($p < 0.05$) with shear strength: Age, HS, Load, *H*, *h*, TG, Twl, Emax, SlabDens. Johnson (2000, p. 57-63) used the same analysis in his study. We used two additional years of data but due to the exclusions mentioned in Section 3.2, our dataset included less shear strength measurements. Our results were similar to Johnson's except that Emax correlated positively to strength in our dataset, probably because we continued to test layers of large crystals after they become relatively strong.

The Spearman rank correlations between each predictor and the rate of strength change showed only a weak correlation with HS ($p = 0.05$). Consequently, the regression analysis was omitted and average rates of loading for the two snow climates were used in Section 4.3.

4.2 Physical interpretation of predictors

Four of the predictors that yielded significant Spearman rank correlations with shear strength were of particular interest for the regression analysis in the next section:

- Load: The positive rank correlation showed that greater loads typically overlie stronger facet layers. Load causes densification (Kojima, 1967; Conway and Wilbour, 1999) and pressure sintering between the load-bearing crystals and increased bonding.
- *H*: The positive rank correlation indicates that thicker slabs typically overlie stronger facet layers. This is likely because slab thickness is strongly correlated to Load.
- SlabDens: The positive rank correlation indicates that denser slabs typically overlie stronger facet layers. Since denser slabs are usually older and apply more load than less dense slabs, the underlying facet layer is likely stronger due to densification and pressure sintering (in a snow climate that favors equilibrium metamorphism).

- *h:* The positive rank correlation indicates that harder facet layers usually had greater shear strength. This is expected since hand hardness and shear strength are both measures of bonding. While hand hardness of thick layers is usually easier to measure than shear strength, hand hardness is partly subjective and is difficult to estimate for thin layers.

For a physical interpretation of other predictors, see Johnson (2000, p. 57-63).

4.3 Rate of shear strength change

Since the correlations with the rate of strength change were weak or non-significant, we chose to use the average loading rate for the snow climate as the strength change rate (Table 2) because:

1. Densification and pressure sintering provide physical explanations why increased load contributes to increased strength.
2. Using a large dataset mostly from the Intermountain snow climate, Jamieson and others (2001) found that strength-load ratios averaged 0.98.
3. Load and strength increase from approximately 0 over the same time (age of the layer).
4. Based on daily average snowfall of 7.8 cm per day from December through March during the winters of 1966 to 1986 at Mt. Fidelity (Schleiss, 1989), and a typical new snow density of 80 kg m^{-3}, the daily average loading rate was 62 Pa d^{-1}. This illustrates the strong effect of load on the shear strength of facet layers in the Intermountain snow climate since facet layers at our study sites at Mt. Fidelity and Mt. St. Anne—which has a snow climate similar to Mt. Fidelity—exhibit an average daily strength increase rate of 58 Pa d^{-1}.
5. Strength change rates in our Continental data averaged 23 Pa d^{-1} whereas loading rates at Bow Summit averaged 18 Pa d^{-1} (Johnson, 2000, p. 60), which we chose to represent our continental snowpack sites.

We recognize that while this simple estimate of the strength change rate may include the influence of lagged load (Chalmers, 2001, p. 79-84), it ignores effects such as temperature, temperature gradient and microstructure.

Table 2 Loading rates

Snow climate	Average loading rate (Pa d^{-1})
Continental	18
Intermountain	62
Combined	40

4.4 Shear strength regression

A simple linear regression of measured strength on Load for the combined dataset resulted in an r^2 value of 0.74. However, a plot of the residuals vs the predicted values (Figure 1) shows that the variance increased with strength.

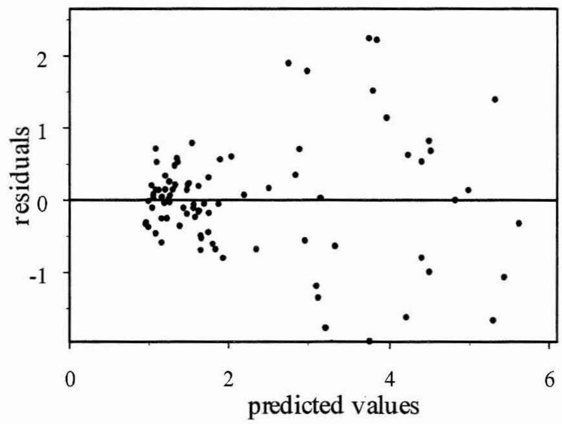

Figure 1: Scatter of residuals of load regression model to predict shear strength

Consequently, we regressed ln Σ on ln Load to get

$$\text{Ln}\,\Sigma = \ln A + B \ln \text{Load} \qquad (2)$$

which can be re-written as a power law

$$\Sigma = A\,\text{Load}^B$$

Results for the Continental, Intermountain and combined datasets are shown in Table 3. The r^2 values are similar to the regression model before the transformation, but the effect of higher variance for higher strength is stabilized as shown in Figure 2 for the combined model. The intercept shows almost no influence of snow climate.

The exponents of Load range from 0.32 to 0.57 indicating that, as observed by Johnson and Jamieson (2001), shear strength does not increase as fast as Load for high values of Load.

Table 3 Regression results for Equation 2

Snow climate	No. of obs N	Coefficients A	B	Coef. of deter. r^2	Signif. level p
Continental	32	1.35	0.32	0.21	0.01
Intermountain	54	1.34	0.57	0.74	10^{-16}
Combined	86	1.36	0.55	0.74	$< 10^{-20}$

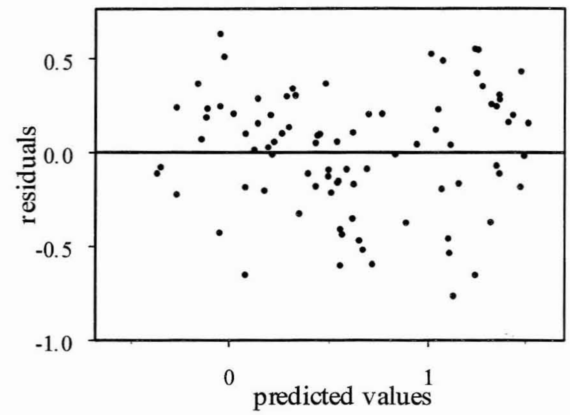

Figure 2: Scatter of residuals of logarithmic load model for predicting shear strength

When applied to the residuals in Figure 2, the hypothesis of normality is not rejected at the 5% level by the Shapiro-Wilk test ($W = 0.98, p = 0.18$).

The fit of the Continental data to Equation 2 was not good ($r^2 = 0.22$), presumably because of strong effects other than Load. The fit of the Intermountain and combined data to Equation 2 was substantially better ($r^2 = 0.74$) indicating the predictive potential of load for shear strength of facets, at least for our data from an Intermountain snow climate.

The dataset for the multivariate regression for estimating shear strength included 86 shear strength measurements. Stepwise regression (forwards and backwards) selected the predictors *H*, HH, Thick and SlabDens. The coefficient of multiple determination equaled 0.78 for the combined dataset. Again the distribution showed a higher variance for higher strength values. Applying the log transformation on the four predictor variables as well as on strength resulted in a better fit including four variables ($r^2 =$ 0.82). Because we were cautious about fitting four variables to only 86 data, stepwise deletion of the least significant variable resulted in the following equation:

$$\text{Ln}\,\Sigma = \ln D + F \ln H + G \ln \text{SlabDens} \qquad (3)$$

which can be written as

$$\Sigma = D\, H^F\, \text{SlabDens}^G$$

When applied to the Continental dataset, the coefficient of *H* was not significant ($p = 0.36$). The results of regressing the Intermountain and combined strength data with Equation 3 are shown in Table 4.

Table 4 Regression results for Equation 3

No. of obs N	Coefficients D	F	G	Coef. of det. r^2	Signif. level p
Intermountain					
54	-5.84	0.47	0.82	0.75	10^{-16}
Combined					
86	-6.22	0.43	0.92	0.75	$<10^{-20}$

Considering the inherent variability of faceted layers and the range of strength of these layers, the fit for the Intermountain and the combined model is adequate with $r^2 = 0.75$. As for the Load model (Equation 2), the multivariate regression model is not capable of predicting shear strength for our Continental data, perhaps because the dataset was too small and contained too much variability for model building.

5. Model selection

The coefficients of determination for fitting the shear strength data to Equations 2 and 3 were similar. In the following analysis, Equation 2 was used because:

1. The coefficient of *H* was not significant in the multivariate regression for the Continental dataset in the multivariate regression.
2. Load was the most significant predictor of strength for both Continental and Intermountain snow climates in Johnson's (2000, p. 57-58) analysis.
3. The effect of load on strength can be explained based on densification (Kojima, 1967; Conway and Wilbour, 1999) and increased pressure sintering.
4. The predictors *H* and SlabDens in Equation 3 were strongly correlated with Load (Johnson, 2000, p. 59), and SlabDens is not easier to measure than Load.

Using the strength change rate for the snow climate (Table 2), the Forecasting model is rewritten:

Continental: $\Sigma_j^* = 1.35\ \mathrm{Load}_i^{0.32} + 0.018\ \Delta t_{ij}$ (1a)

Intermountain: $\Sigma_j^* = 1.34\ \mathrm{Load}_i^{0.57} + 0.062\ \Delta t_{ij}$ (1b)

Combined: $\Sigma_j^* = 1.36\ \mathrm{Load}_i^{0.55} + 0.040\ \Delta t_{ij}$ (1c)

Based on these equations, Table 5 summarizes the coefficient of determination between estimated and measured shear strength at the start and end of the intervals in the model building dataset:

Table 5 Fit of Forecasting model to data used to build the model

	Coefficient of determination r^2					
	Continental		Inter-mountain		Combined	
	Interval		Interval		Interval	
	Start	End	Start	End	Start	End
1a	0.21	0.03	-	-	-	0.69
1b	-	-	0.74	0.63	-	0.70
1c	-	0.01	-	0.64	0.74	0.71

Since the fit for the Continental data to Equation 1a was poor, data for this snow climate were excluded from further analysis and model testing.

Equation 1b is for estimating shear strength in areas with a snow climate similar to our Intermountain data. The r^2 for this Intermountain model at the end of the intervals was 0.63 ($N = 44$, $p = 10^{-10}$) and the average error of estimation was 32%. Some of the variability is likely due to using the average loading rate from Mt. Fidelity for estimating the strength change rate for study plots in the Columbia Mountains with varying elevation, as well as for periods with above or below average loading rates. However, the lagged response of shear strength to load (Chalmers, 2001, p. 79-84) and the length of the measurement intervals (mostly 4-8 days) reduce the sensitivity of the strength model (but not stability!) to one or two day periods with above or below average loading. This equation is tested with independent data in the next section.

6. Model testing

Two time series, one from Continental snow climate and one from Intermountain snow climate, were excluded from the analysis so that they could be used to test the model. Since the model did not work for our Continental data, only the layer that formed at Mt. St. Anne on 7 January 2002, (12 points over 78 days) was tested. In Figure 3, the estimated strength at the start of an interval (Equation 2) and forecast strength at the end of each interval (Equation 1 b) are plotted along with measured values. At the start and end of each interval, the Forecasting model for the Intermountain snow climate explained 87% and 88% of the variability in the test series, respectively (Figure 3).

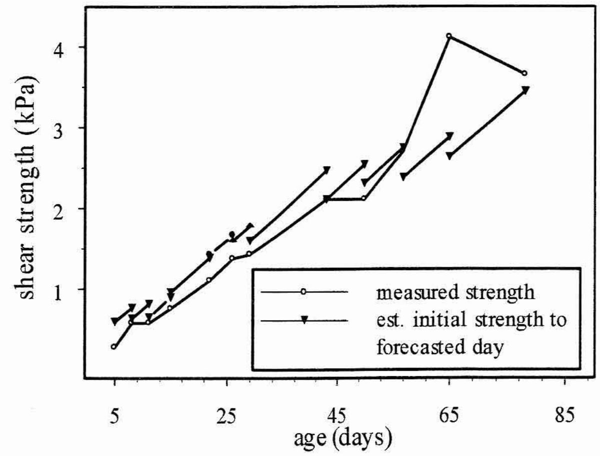

Figure 3: Model testing to forecast shear strength at Mt. St. Anne (faceted layer formed 07 January 2002)

Up to Day 50, the model overestimated the strength by an average of 14% and 21% at the start and end of the intervals, respectively. On Day 65 (13 March 2002), the model underestimated the measured strength significantly (30% of 3.5 kPa), but in view of the good estimate (5%) for the next measurement (Day 82), the measured strength on Day 65 may have been higher than average values in the study site due to spatial variability. The average error of the estimations at the start and end of all forecast intervals was 21% and 18%, respectively.

In areas with a snow climate similar to Mt. Fidelity and Mt. St Anne, the model shows potential for estimating shear strength over periods up to eight days based on a snow profile (to identify the weak layer) and load measurement.

7. Summary

Load and variables related to load were the most influential factors in the present study for estimating shear strength of facet layers.

The difficulties predicting shear strength for areas with a Continental snow climate may be due to the limited number of measurements in this snow climate. Specifically, the strength of faceted layers in areas with a Continental snow climate likely evolve differently due to higher temperature gradients and resulting effects on strength that compete with densification and pressure sintering.

Estimating shear strength on the day of a snow profile and several days ahead is promising using the developed model for layers of facets and rounded facets in areas with a snow climate similar to Mt. Fidelity and Mt. St. Anne. For the test dataset, the average error of the estimates at the start and end of the measurement intervals were 21% and 18%.

Use of the average loading rate as the strength change rate is simplistic and produced a model that did not satisfactorily fit our data from a Continental snow climate. Improvements to the model may be possible by considering the load lagged by a few days (Chalmers, 2001, p. 79-84) and microstructural parameters such as grain size. Such analysis will require a larger dataset and, ideally, a physically based model including pressure-sintering as well as temperature and temperature gradient.

Future research will involve using daily estimates of the strength and stability of buried facet layers, along with weather and recent avalanche activity in a nearest neighbour forecasting model.

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