theoretical and experimental approach of snow-slab stability using cohesive materials

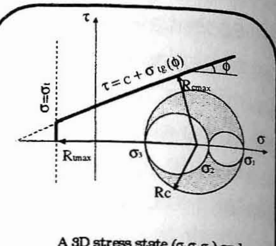
Dominique DAUDON[1], Francois LOUCHET[2]
[1] Laboratoire 3S, [2] Laboratoire LTPCM, Joseph Fourier University, BP 53, 38041 Grenoble Cedex, France
Contact : dominique.daudon@hmg.inpg.fr, francois.louchet@inpg.fr

STATIC EQUILIBRIUM APPROACH : 3D STABILITY INDEXES F_{SHEAR}, $F_{TENSION}$

based on general (3D) stress state and static equilibrium at each point of the snow cover
It assumes a granular material with an elastoplastic behaviour (in our case : a Mohr Coulomb plasticity criterion with tension cut off)

$$F_S = \frac{R_{smax}}{R_c} = \frac{\sin\Phi(2C\cot g\Phi + \sigma_1 + \sigma_3)}{\sigma_1 - \sigma_3},$$

$$F_t = \frac{R_{tmax}}{R_c} = \frac{\sigma_1 + \sigma_3 - 2\sigma_t}{\sigma_1 - \sigma_3},$$



A 3D stress state ($\sigma_1\sigma_2\sigma_3$) and a snow cover elastoplastic criterion (C, Φ) and a σ_t tension cut-off, E young modulus

•Theoretical instability in shear [tension] is assumed when F_s [F_t] is less than 1,

• it could be judicious using security factor a of 1.5 or 2

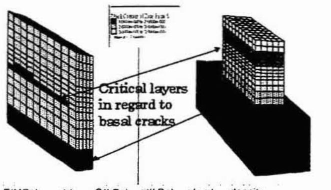
•The indexes are implemented in a finite differences code which estimates the stress inside the snow cover.

•Maps of indexes give a localisation of potential instable layers and areas

SOME APPLICATIONS

The "numerical Rutchblock" : knowing which layer leads to instability

Calculation of the F_s indexes to point out the shear critical layers by implementation in a stress strain code



E(MPa)	φ	C(kPa)	σt(kPa)	e(cm)	density	
0.8	28	1.5	0.9	20	140	cche8
4	36	4	2.9	20	230	cche7
4	28	1.5	1.3	6	220	cche6
4	28	1.5	1.3	12	220	cche5
8	28	1.5	1.9	12	250	cche4
80	29	3	5.3	20	370	cche3
200	41	6	6	25	390	cche2
600	53	12	8.3	25	440	cche1

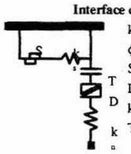
Mesh Model
half Rutchblock on a 35° rigid bedrock slope
no other loading than gravity

F_s is the smallest for layer 4 and 1.
Slab mechanism should be due to shear failure in layer 4

A 3D snow cover slope : effect of local defect of mechanical properties in the critical layer

Snow cover meshed geometry:
•homogeneous top slab (50 cm) under gravity (E=4 Mpa,C= 5 kPa,f=38.7,σ_t=3.6 kPa,d=250 kg/m³)
•The thin critical layer is assumed to be an interface without weight and depth

Interface critical layer modelling



k_s=1.8 MPa
ϕ = 27.5°
S = 1.5 kPa (shear rupture)
D=0 (dilatancy)
k_n= 4MPa
T= 1.9 kPa (tension rupture)

default size (m²)	Ft
0	11
1*2	2.02
1*3	1.26
1*4	1.19
2*2	1.13
2*3	1.03

F_t as a function of the size of the defect (Interface Properties divided by 2)

value of Ft inside the upper slab layer(α=35°)

•crest or comb (40 m wide, 100 m long)
•slope angle of 35° (majority of slabs triggering)
•boundary conditions :
4rigid bed rock
4only vertical displacements due to elastic behaviour allowed at the lateral boundary

The static equilibrium approach shows only the release model of the Griffith approach.

CRACK STABILITY WITH GRIFFITH APPROACH

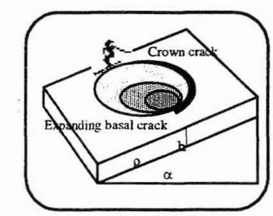
Hypothesis : based on the evolution of an initial crack due to creep or skier triggering
Two different release modes : scenarios may arrive according whether or not the basal crack fulfills Griffith's instability criterion before crown crack opening.

<ins>**Shear stress at the slab substrate interface :**</ins>

$$\tau = \tfrac{1}{2}\rho.\ g.h.\ \sin(2\alpha)$$

<ins>**tensile stress in the slab at the upper tip of the basal crack:**</ins>

$$\sigma = \rho.\ g.\ a_s.\ \sin(\alpha)$$



Mode 1 **splitting slab fragments**

Mode 2 **breaking wave slab**

<ins>Release Mode 1</ins> : ***basal crack expands, and σ increases with the skier progression across the slab.***
A tensile crack starts opening at the crown of the basal crack when the tensile stress σ reaches the tensile failure stress σf the snow. Crack propagation is controlled by the energy balance between stresses relaxation and free surface creation (Jamieson and Johnson 91)

so $a_t = \sigma_f / \rho.\ g.\ \sin(\alpha)$,

where a_t is the size of the basal crack in mode 1.

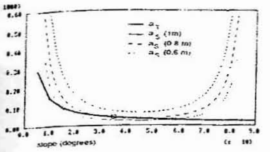
<ins>Release Mode 2</ins> : ***basal crack meets alone Griffith's condition for unstable growth*** before the tensile stress σ at the crown reaches the critical value σf for tensile failure. the basal crack becomes unstable when the shear stress concentration factor $\tau(\pi a_s)^{1/2}$ exceeds the interface shear toughness K_{II} The basal crack is already overcritical as the crown crack expand and the whole slope release. This suddenly open may be responsible of the "bang" heard in some avalanche.

So $a_s = 1/\pi.\ (\ 2\ K_{II}/\rho.\ g\ .h.\sin(2\alpha)\ )^2$,

where a_s is the size of the basal crack in mode 2.

TRANSITION BETWEEN THE TWO TRIGGERING MODES

The competition between slab splitting and breaking wave modes is illustrated on this figure



For a given slope , the type of avalanche is determined by the smallest of the 2 crack size a_s or a_t

The transition between the two modes is obtained for :**a_t=a_s and $da_s/d\alpha$ $da_t/d\alpha$** that leads to **α = 35.6° independent of the snow properties** (thougness, density ...)

EXPERIMENTAL STUDIES ON REDUCED MODEL WITH SAND

Experimental Installation

The installation is dimensioned by similitude equations where the symbol * concerns the sand properties and the reduced installation

$$\begin{cases} a_s = \dfrac{1}{\pi}\left(\dfrac{2Kc}{\rho g h \sin(2\alpha)}\right)^2 \\ a_s^* = \dfrac{1}{\pi}\left(\dfrac{2Kc^*}{\rho^* g h^* \sin(2\alpha)}\right)^2 \end{cases} \Rightarrow \frac{h^*}{h} = \frac{\rho Kc^*}{\rho^* Kc}\sqrt{\frac{a_s}{a_s^*}}$$

$$\begin{cases} \rho = 400 kg/m^3 \\ \rho^* = 1400 kg/m^3 \end{cases} \Rightarrow \frac{\rho}{\rho^*} = 3.5 \quad \boxed{\frac{a_t}{a_t^*} = \frac{\sigma_f \rho^*}{\sigma_f^* \rho}}$$

For $h = 1m$**, and** $\alpha = 35°$**:** **We find :** $h^* = 2{,}723 mm$, $a_s^* = 4{,}62 cm$

•Around 30 experimental tests were performed in order to find the conditions of occurrence of the 2 modes of release

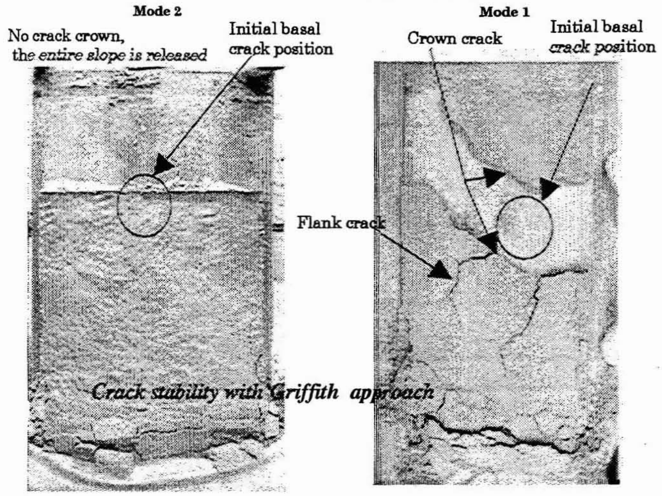
•The initial basal crack was made of a disk of polyethylene leave of 20 cm diameter.

•The slab layer is made of hand-compacted wet sand in order to get a non zero cohesion of the grain. (It is then inclined to find the slope which induced a release in mode 1 or in mode 2.

•The critical layer is layer of one mm of dry sand raining-like deposed.

As a result, there was not clearly a value of the slope angle which leads to one or other mode. Both were equally observed.

This may be due to the difficulty of realising repeatable samples (density, homogeneity...)



The measured α (between 40° to 60°) is always greater than the theoretical one

Dimensions l=64 cm, L=80 cm, h = 5 cm, properties : ρ_{sand}=1400k/m², σ_t=11N, K_c=9.8 Pa m$^{1/2}$,

CONCLUSION

•The static equilibrium approach can be used to predict which layers are potentially instable. It may predict the instability fo the crown crack and the flank one in the slab , around the shape of the basal crack . But it doesn't predict the expansion of this last one . It may be reproduced by the mode 1 experimental tests.
•The Griffith rupture mechanic approach may predict two different release modes : the splitting slab fragment one and the breaking wave slab one.
•The experiments performed seem to show the two types of release. But it doesn't present clear parameters of transition due to the difficulty of doing those tests with a good reproducibility.
•However it is important to continue this work to get more repeatable results , especially experimental ones, and find amelioration of the installation

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