CREEP INSTABILITY OF THE WEAK LAYER AND NATURAL SLAB AVALANCHE TRIGGERINGS

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ABSTRACT: The weak layer connecting the snow slab with the older snow substrate is modelled as an open cell ice foam made of an array of ice bonds. The comparison of bond rupture and rewelding rates under load show that the weak layer experiences stable flow for a wide range of loads, of bond brittleness and of bond rewelding rates. However, a critical point may be reached, corresponding to a finite shear rate of the weak layer, beyond which the shear rate increases catastrophically, leading to a natural avalanche release. This critical situation is equivalent to a ductile to britte transition, whose activation energy is one half of that of self diffusion in ice. A measurement of the exponent in the shear strain vs stress relation may give a hint on the imminence of the avalanche release. A similar calculation performed at the tip of a preexisting basal crack gives a simple criterion for crack stability and an analytical expression for snow fracture toughness.

KEYWORDS: brittle ductile transition, creep, ice, snow, avalanche, toughness.

1. INTRODUCTION

It is well known that slab avalanches result from a failure of the weak layer that connects the slab to an older snow substrate. More precisely, human triggered slab avalanches are most often initiated by the expansion in shearing mode of a "basal" crack in the weak layer (McClung 1981, Gubler 1992). The corresponding time scales are short enough to allow considering snow as a brittle medium, in which fast crack expansion is governed by Griffith's criterion (Louchet 1999, Michot and Kirchner 1999, Louchet 2000 a). By contrast, time scales involved in snow cover evolution that may initiate natural avalanches are by far much longer. The behaviour of the weak layer has to be treated in this case in terms of creep rather than in terms of brittle fracture. The simple model proposed in the present paper aims at giving a possible physical basis for analysing the weak layer creep stability and the conditions required for catastrophic flow initiation.

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2. CREEP MECHANISM

The system is schematised as shown in figure 1. The weak layer bonding the slab to the substrate is described as an open cell ice foam, as proposed for snow by Michot et al. (1999), i.e. as a network of ice bonds, with a porosity δ defined as the average bond separation.

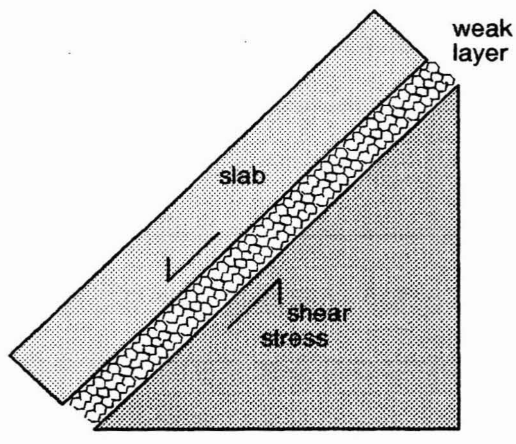

Figure 1: The weak layer is treated as an open cell foam made of a network of ice bonds. Under shear loading, it experiences creep flow through a balance between bond rupture and rewelding events.

These bonds are prone to break under stress, but also, since the material is close to its melting point, broken bonds may reconstruct through self diffusion welding if they are facing each other for some time. Creep is thus supposed to result from a balance between bond rupture and welding rates. For the sake of simplicity, we neglect here sintering under compressive loading. These starting assumptions are somewhat similar to those at the basis of a recent numerical simulation of diphasic flow in semi-solid alloys by Perez et al (2000), except that here, instead of computing a viscosity at constant applied shear rate, we aim at studying possible creep instabilities at imposed shear stress, through a derivation of bond breaking and rewelding kinetics.

3. RATE EQUATIONS

Let n be the proportion of unbroken bonds. The coresponding proportion of broken bonds is thus $(1 - n)$. If $n = 1$, all bonds are intact, and each of them experiences a shear stress τ. This stress is related to the macroscopic shear stress τ_0 by:

$$\tau = \tau_0 \left(\frac{\delta}{w}\right)^2 \qquad (1)$$

where δ is the bond separation, w the bond thickness, and $(w / \delta)^2$ the surface fraction of bonds. In the current case, n is comprised between 0 and 1, and each bond experiences a stress τ / n larger by a factor $1 / n$ than that in the case where $n = 1$. We consider that the bond breaking rate, i.e. the number of breaking events per second, is proportional to both the number of unbroken bonds n and this stress τ / n, which means that it is proportional to τ, with a proportionality factor α supposed to be independent of temperature.

Let us now consider the bond welding rate. Since two half bonds have to meet to achieve the welding process, the welding rate is proportional to the square $(1 - n)^2$ of the broken bond fraction $(1 - n)$. The welding rate is also taken proportional to the time during which the two broken bonds are facing each other, i.e. inversely proportional to the shear strain rate. It is also proportional to a factor $\beta(T)$ that accounts for ice welding kinetics, and varies exponentially with temperature T as does the diffusion of water molecules in ice, i.e. proportional to $exp(-Q / kT)$, where the activation energy Q is the self diffusion energy, and k is the Boltzman's constant.

It can be shown (Louchet 2000 b), that the net balance between breaking and welding kinetics varies as shown in fig. 2. The curve is shown for three different values of the load due to the snow cover. Let us consider the intermediate curve. In the positive parts of this curve, welding is faster than breaking, and conversely in the negative parts. The exact balance between breaking and welding rates is found at the two points where the curve intersects the horizontal axis, and correspond to steady state values of the fraction n of unbroken bonds. The point on the right corresponds to a stable steady state: if for some reason n decreases from its steady state value, the balance becomes positive, i.e. welding becomes faster than breaking, which restores n to its initial value. For the same reason, the point on the left corresponds to an unstable state, and will be disregarded in the following.

Each breaking event produces two half bonds that reweld somewhere in its vicinity, resulting in some local creep. The steady state creep rate $d\gamma / dt$ can be determined as the product of the breaking rate (proportional to τ) by the corresponding creep distance, i.e. the separation between unbroken bonds $1 / \tilde{n}$, where $\tilde{n}$ is the proportion of unbroken bonds at the stable steady state point. The steady state creep rate is therefore proportional to $\tau / \tilde{n}$.

4. CREEP INSTABILITY

Let us now discuss the influence of snow load (through shear stress τ) and weak layer properties on both creep rate and creep stability. It can be easily shown (Louchet 2000 b) that an increase in stress τ (due to snow or rain fall for instance) will shift the curve downwards, bringing the steady state value of unbroken bond fraction $\tilde{n}$ to a smaller value. Since the strain rate is proportional to $\tau / \tilde{n}$, the corresponding strain rate gradually increases. However, a further increase of snow load may bring the curve to the lower position shown in fig. 2, tangent to the horizontal axis, at which both stable and unstable steady states merge together. Beyond this stage, the balance becomes negative, which means that breaking starts becoming faster than welding; n continuously decreases with time, which makes the balance even more negative, rapidly leading to catastrophic flow, that may be further enhanced by local frictional melting. The conditions for natural avalanche release are thus met for a critical load τ^*, corresponding to a fraction of unbroken bonds of 1/3, as illustrated in fig. 2, which can be shown to be:

$$\tau^* = \frac{1}{\alpha}\sqrt{\frac{4\beta}{27}} \qquad (2)$$

or, in terms of the macroscopic load τ_0^*:

$$\tau_0^* = \tau^*\left(\frac{w}{\delta}\right)^2 = \frac{1}{\alpha}\left(\frac{w}{\delta}\right)^2\sqrt{\frac{4\beta}{27}} \qquad (3)$$

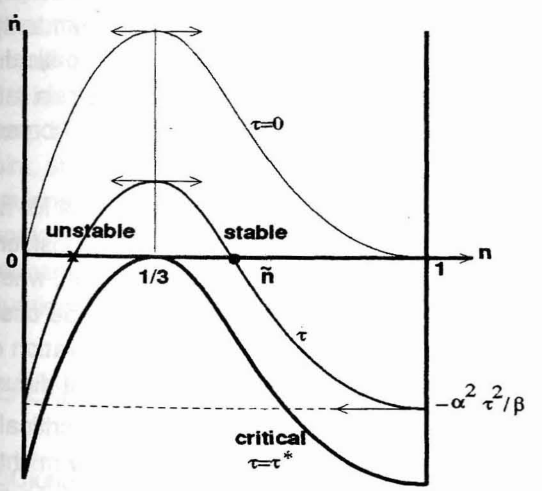

Figure 2: Schematic variations of the balance between welding and rupture rates as a function of the proportion of unbroken bonds *n*, at zero stress (upper curve), in the general case (intermediate curve), and in critical avalanche release conditions (lower curve).

In a same way, a decrease of weak layer temperature is expected to slow down the ice welding rate $\beta(T)$, and an evolution of the weak layer morphology (under temperature gradient for instance) may increase bond brittleness through bond thinning, both promoting avalanche release.

This critical situation may be considered as a ductile to brittle transition of the weak layer. It was recently suggested (Kirchner, Michot and Suzuki 2000) that the temperature dependence of the activation energy characteristic of the ductile to brittle transition of snow in tension should be the same (0.6 eV) as that for self diffusion in ice (Ramseier 1967). An interesting point is that in the case of shear examined here the critical stress τ^* can be shown to be proportional to $\sqrt{\beta(T)}$ (Louchet 2000 b), which means that the activation energy of the ductile to brittle transition in shear is expected to be $\frac{Q}{2} = 0.3\ eV$, i.e. one half of the value proposed by Kirchner et al. for tensile failure, based on a quite different mechanism.

It can also be shown that the constitutive law accounting for the steady state creep in the vicinity of the critical point can be written:

$$\frac{\tau_0 / \dot{\gamma}}{\tau_0^* / \dot{\gamma}^*} - 1 = \frac{2}{\sqrt{3}}\left[1 - \frac{\tau_0^{\,2}}{\tau_0^{*2}}\right]^{1/2} \qquad (4)$$

where τ_0 is the macroscopic shear load experienced by the weak layer, τ_0^* its value at the critical point corresponding to the avalanche release, $\dot{\gamma}$ the shear strain rate, and $\dot{\gamma}^*$ its value at the critical point. This law, and in particular the value of the exponent 1/2, are valid only in the vicinity of the critical point. A determination of the exponent of the constitutive equation, obtained from field measurements of both snow load and drift velocity, might perhaps give a hint on the imminence of unstable flow.

5. INFLUENCE OF A PREEXISTING CRACK

In the above calculation, the weak layer is considered as homogeneous at a scale larger than the porosity. Large scale defects, as preexisting stable basal cracks, should modify the above analysis through an enhancement of the creep rate due to stress concentration at the crack tip. A simple way to solve the problem is to multiply the shear stress in the balance equation by a concentration factor $\sqrt{\frac{a}{\delta / n}}$ where (δ / n) is the actual porosity in the flow direction, and a the size of the preexisting crack. The calculations, whose details are not given here for the sake of brevity, show that creep instability at the crack tip (corresponding to $\dot{\gamma} \rightarrow \infty$) is reached for a macroscopic critical shear stress:

$$\tau_0^* = \frac{1}{\alpha} \frac{w^2}{\delta^2} \sqrt{\frac{\beta\delta}{a}} \qquad (5)$$

Beyond this stress value, the breaking rate at the crack tip becomes larger than the welding rate, which results in a catastrophic growth of the basal crack size, and in avalanche release. This scenario is similar to a type B overcritical triggering described in (Louchet 2000 a) in which the tensile rupture of the slab takes place quasi-simultaneously along the whole crown crack line, after a fast expansion of the basal crack.

Since the crack size a is large as compared to the parallel porosity δ, the critical stress for avalanche release is significantly reduced as compared to the homogeneous case, as expected. Eq. (5) is equivalent to say that unstable basal crack propagation takes place when the stress concentration factor $\tau_0{}^*\sqrt{\pi a}$ reaches a critical value:

$$K_c = \frac{1}{\alpha}\left(\frac{w}{\delta}\right)^2 \sqrt{\pi\beta\delta} \qquad (6)$$

which can be considered (by definition) as the fracture toughness of the weak layer. This fracture toughness obviously increases with snow "compacity" w / δ. It is again proportional to $\sqrt{\beta}$, and the associated activation energy is expected to be one half of self diffusion in ice, as for the critical stress in the homogeneous case.

6. CONCLUSIONS AND PERSPECTIVES

This simple model shows that, under reasonable assumptions, downhill slow drift of a snow slab can be described through steady state creep of the weak layer. The physically based rate dependent constitutive equation obtained for the weak layer may also be used for other types of snow under shear loading, provided specific values are taken for bond brittleness α and rewelding rate β. Some of the quite recent measurements of ice on ice friction by Kennedy, Schulson and Jones (2000), qualitatively accounted for on rather similar ideas of making and breaking of bonds, may possibly be discussed in terms of the present calculation.

An increase of the weight of the snow cover, due to snow or rain fall, may gradually bring the system to a critical point at which natural avalanche release takes place through rapid strain rate softening. This critical situation can also be reached through either a decrease of weak layer rewelding rate (colder weather) or an increase of bond brittleness (e.g. hoar frost development resulting from temperature gradient metamorphism). The critical conditions correspond to a finite shear strain rate, i.e. a finite slab drift velocity, which increases with temperature.

These critical conditions for natural avalanche triggering can be considered as a sharp ductile to brittle transition, whose temperature dependence can be described by an Arrhenius law, with an activation energy equal to one half of that of self diffusion in ice.

A measurement of the critical exponent of the stress vs strain rate law might possibly be used to forecast the imminence of the avalanche release.

The stability of a preexisting basal crack can be studied in a similar way, through a correction of the shear stress at the crack tip by a stress concentration factor. Avalanche release takes place for a smaller stress than in the previous case, and an analytical expression can be obtained for snow toughness, with the same activation energy as above.

The present approach differs from McClung's (McClung 1979, McClung 1981) essentially in that it is based on kinetic evolution equations and not on static rupture criteria, even if time is introduced by McClung in the slab itself, through a superimposed energy release rate. To this respect, since we take into account rate-dependent healing in the weak layer, we are dealing here with strain rate softening rather than with strain softening.

The present model may be transposed to landslip problems. Though being relevant of granular rather than of cellular media, the mechanisms involved in wet landslips may indeed be treated in a fairly similar way, grains sliding over each other through a series of capillarity controlled debonding and rewelding events. The load is directly related to the amount of rain, whereas humidity due to either rain, snowmelt or springs may affect bond rupture and rewelding rates.

At this stage, the present approach is however fairly rough, in that it deals with an idealistic case (planar and infinite slope, without any boundary conditions, ...). A temperature dependence of ice brittleness (Gubler and Bader 1989), that may change the creep behaviour in the vicinity of the melting point, should be introduced. Future work, requiring the use of numerical simulations, should try to reproduce the present mechanisms, and explore more realistic situations.

REFERENCES

McClung D. 1979. Shear fracture precipitated by strain softening as a mechanism of dry slab avalanche release. J. Geophys. Research **84**, n° B7, 3519-3524.

Mc Clung D. 1981. Fracture Mechanical Models of Dry Slab Avalanche Release. J. Geophys. Research **86**, n° B11, 10783-10790.

Gubler H. 1992. Measurements and modelling to improve our understanding of avalanche formation. Universite Europeenne d'Ete 14-25 sept. 1992, Cemagref Editions; Chamonix, 89-97.

Gubler H. and Bader H.P. 1989. A model of initial failure in slab avalanche release. Annals of Glaciology **13**, 90-95

Kennedy F.E., Schulson E.M., and Jones D.E., 2000. The friction of ice on ice at low sliding velocities. Phil. Mag. A **80**, n°5, 1093-1110.

Kirchner H.O.K., Michot G., and Suzuki T., 2000. Fracture toughness of snow in tension, Phil. Mag. A **80**, n°5, 1265-1272.

Louchet F., 1999. Deux modes possibles de depart d'avalanches de plaque. Colloque Plasticite, March 29-31, Nancy (France).

Louchet F., 2000 a. A transition in dry snow slab avalanche triggering modes, Proc. Int. Symp. on snow, avalanches and impact on the forest cover, Innsbruck, 22-26 may 2000, Annals of Glaciology, in press.

Louchet F., 2000 b. Submitted to Cold Regions Science and Technology.

Michot G. and Kirchner H.O.K., 1999. Plasticite et rupture de la neige, la transition ductile-fragile. Colloque Plasticite, March 29-31, Nancy (France).

Perez M., Barbe J.C., Neda Z., Brechet Y., and Salvo L. 2000. Computer simulation of the microstructure and rheology of semi-solid alloys under shear, Acta Mater. **48**, n°14, 3773-3782

Ramseier R.O. 1967. Self diffusion of Tritium in natural and synthetic ice monocrystals. J. Appl. Phys. **38**, 2553.