PLACE OF SNOW RECRYSTALLIZATION IN MODELING THE SNOW COVER EVOLUTION

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ABSTRACT: The paper presents results of analysis of experimental data on activity of snow recrystallization under temperature gradient conditions. In addition to temperature and temperature gradient effects, recently accepted as the main factors influencing the snow recrystallization process, the effects of the water vapor flux and the volumetric mass production, related to the formed in snow heat- and mass-fluxes, were estimated. The results suggest that the reported discrepancies between the results of observation and the modeled by presently used empirical construction recrystallization rates can be related to neglected before parameters of the water vapor in the pore space of snow.

KEYWORDS: snow metamorphism, recrystallization rate, heat- and mass-transfer, volumetric mass production, acceleration of gravity

1. INTRODUCTION

Presently available data on observed snow recrystallization activity under different environmental conditions do not allow yet to produce a detailed combined description of the snow cover evolution on micro- and macro-scale. A physical description of the processes responsible for the snow recrystallization in physical models of snow cover has mainly a form of a formalized snow evolution classification in terms of bond, crystal size, sphericity, and dendricity of crystals, extracted from results of observations in a selected area (Brun et al. 1992), and based on empirical model of temperature- and temperature gradient-dependent crystal size change with time (Bartelt et al. 2000; Gubler 1998). The latter normally is an approximation of experimental data, corresponding to specific experimental conditions (Marbouty 1980; Baunach et al. 2001). Such formalization allows avoiding some uncertainties in the present physical understanding of the snow cover evolution, but can result in missing of some components, not important or not measured in the experiments used as a base for the process interpretation by Marbouty (1980) or Baunach et al. (2001). This can noticeably limit the range of these and other empirical models applications.

In present work, experimental data obtained in a wider than it was before range of applied temperatures and temperature gradients is analyzed by the same way of separate estimation of the measured components correlation with the recrystallization rates, as it was done by Delsol et al. (1978), Marbouty (1980), Pahaut and Marbouty (1981). However, in addition to temperature and temperature gradient effects, an attempt was made to estimate the recrystallization rate dependence on the water vapor flux characteristics. The “crystal size” and a “recrystallization (growth) rate” terms in this paper correspond to a “diameter of a circle with an area equal to area occupied by a crystal on an analyzed microphotograph” and “change of such areas with time” respectively.

2. BACKGROUND

The recrystallization rate is expected to depend on temperature \((T)\) and on the difference in the water vapor pressures near a crystal surface and in the surrounding environment of the pore space \((p_1\) and \(p_s\) respectively). The mass balance on the surface was accepted to have the form of the Knudsen–Langmuir equation (Sokratov et al. 2001), following (Colbeck 1982). If \(P\) is the maximal possible water vapor production or absorption in a unit of snow volume, \(\alpha\) is the evaporation/condensation coefficient of an ice matrix' surface, \(S\) is the specific surface area of snow, \(m\) is the mass of water molecule, and \(k\) is Boltzmann’s constant:

\[
P = \alpha(p_1 - p_s)S\sqrt{\frac{m}{2\pi k T}}.
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The accepted before reason for the water vapor pressure difference was the difference in the grains surface curvature (Brown et al. 1994; Colbeck 1980). The recently observed temperature fields around individual crystals growing in supercooled water (Braslavsky and Lipson 1998; Notcovich et al. 1999), when considered for a growing ice crystal in air (Fujino and Tsushima 1998), allow to expect a water vapor pressure difference in Equation 1 to be enhanced relative to the presently accepted curvature-related values by a temperature difference between the ice surface and the pore space. If such temperature difference, instead of the curvature effects, is accepted, the recrystallization rates of aged firm correlate linearly with the temperature-dependent $P$ (Sokratov et al. 2001).

Despite intensive theoretical analysis, almost no experimental results on the equilibrium metamorphism of "new" (either granular or depth hoar) snow were reported. The results cited by Yen (1981) after Yosida (1955) showed that the recrystallization rate of a fresh-fallen snow at $-20^\circ$C was approximately 60% of the recrystallization rate at $-6^\circ$C. Similar ratio was observed in the snow used in our recrystallization experiments under isothermal conditions (Kamata, personal communication, 2000). Such results do not allow accepting the Equation 1 for the description of the process of the "new" snow recrystallization. If that could be the case, the recrystallization rate of a "new" snow at $-20^\circ$C had to be approximately 30% of those at $-6^\circ$C. On the other hand, the data on the crystal size change in dependence on temperature presented by Marbouty (1980) gave the recrystallization rate at $-20^\circ$C to be 9% of those at $-6^\circ$C (if the squares of the grains' diameters are considered). However, those results were obtained under temperature gradient conditions.

The considerations of the temperature gradient (kinetic) recrystallization can not be separated from the description of the heat-flux in snow ($q$). The latter, under quasi-steady state conditions, is normally defined as the following (Colbeck 1982):

$$ q = -k_y \frac{\partial T}{\partial y} + LF, $$

where $k_y$ is the heat conductivity of the ice matrix, $y$ is the coordinate in the direction of flux, $L$ is the latent heat of sublimation, and $F$ is the water vapor flux. For the quasi-steady state conditions and without volumetric mass production the water vapor flux can be written as (Sokratov and Maeno 2000):

$$ F = -\phi \frac{D}{\tau} \frac{\partial C}{\partial T} \frac{\partial T}{\partial y}, $$

where $\phi$ is the snow porosity, $f$ is the gradient enhancement factor, $\tau$ is the tortuosity, $D$ is the water vapor diffusion coefficient in air, and $C$ is the water vapor concentration in the pore space of snow.

### 3. EXPERIMENTAL DATA

Table 1: Temperature gradient recrystallization rates ($P$) of the light compacted snow under temperature gradient conditions*

<table>
<thead>
<tr>
<th>$T$, °C</th>
<th>$\frac{\partial T}{\partial y}$, K m$^{-1}$</th>
<th>$F$, 10$^{-6}$ kg m$^{-2}$ s$^{-1}$</th>
<th>$\frac{\partial F}{\partial y}$, 10$^{-6}$ kg m$^{-3}$ s$^{-1}$</th>
<th>$P$, 10$^6$ kg m$^{-3}$ s$^{-1}$</th>
<th>$K$, 10$^{-14}$ m$^2$ s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>94 hr., downward heat-transfer</td>
<td>-3.1</td>
<td>-83</td>
<td>-0.30</td>
<td>-2.91</td>
<td>836</td>
</tr>
<tr>
<td></td>
<td>-5.5</td>
<td>-67</td>
<td>-0.22</td>
<td>-0.64</td>
<td>695</td>
</tr>
<tr>
<td></td>
<td>-7.8</td>
<td>-73</td>
<td>-0.18</td>
<td>-0.93</td>
<td>582</td>
</tr>
<tr>
<td>72 hr., upward heat-transfer</td>
<td>-7.1</td>
<td>-56</td>
<td>-0.16</td>
<td>5.16</td>
<td>613</td>
</tr>
<tr>
<td></td>
<td>-5.3</td>
<td>-60</td>
<td>-0.19</td>
<td>-1.65</td>
<td>709</td>
</tr>
<tr>
<td></td>
<td>-3.2</td>
<td>-64</td>
<td>-0.25</td>
<td>-1.27</td>
<td>829</td>
</tr>
<tr>
<td>72 hr., upward heat-transfer</td>
<td>-24.4</td>
<td>-277</td>
<td>-0.14</td>
<td>0.79</td>
<td>145</td>
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<tr>
<td></td>
<td>-18.9</td>
<td>-183</td>
<td>-0.20</td>
<td>-3.41</td>
<td>235</td>
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<tr>
<td></td>
<td>-13.7</td>
<td>-91</td>
<td>-0.16</td>
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</tr>
<tr>
<td>54 hr., upward heat-transfer</td>
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<td>-613</td>
<td>-0.02</td>
<td>-0.85</td>
<td>5.02</td>
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<td></td>
<td>-35.9</td>
<td>-711</td>
<td>-0.10</td>
<td>-3.77</td>
<td>49.3</td>
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<td>-408</td>
<td>-0.19</td>
<td>-12.11</td>
<td>185</td>
</tr>
<tr>
<td>56 hr., downward heat-transfer</td>
<td>-27.2</td>
<td>-497</td>
<td>-0.17</td>
<td>-4.15</td>
<td>113</td>
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<tr>
<td></td>
<td>-40.9</td>
<td>-336</td>
<td>-0.04</td>
<td>-0.83</td>
<td>29.8</td>
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<tr>
<td></td>
<td>-56.3</td>
<td>-497</td>
<td>-0.01</td>
<td>-0.87</td>
<td>5.36</td>
</tr>
<tr>
<td>156 hr., upward heat-transfer; the one,</td>
<td>-13.1</td>
<td>-3</td>
<td>-0.004</td>
<td>-0.209</td>
<td>286</td>
</tr>
<tr>
<td>corresponding to Sokratov and Maeno (2000)</td>
<td>-11.2</td>
<td>20</td>
<td>0.039</td>
<td>-7.69</td>
<td>333</td>
</tr>
<tr>
<td></td>
<td>-8.5</td>
<td>-90</td>
<td>-0.22</td>
<td>9.03</td>
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<tr>
<td></td>
<td>-7.5</td>
<td>15</td>
<td>0.040</td>
<td>-6.88</td>
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<td></td>
<td>-6.0</td>
<td>-40</td>
<td>-0.118</td>
<td>11.8</td>
<td>502</td>
</tr>
</tbody>
</table>

* The description of the experimental procedure can be found in Kamata and Sato (1998), Kamata et al. (1999a-b), Sokratov and Maeno (2000). Same type of snow and similar duration of the experimental runs allow neglecting a possible impact of an initial structure difference in the results of measurements.
4. DATA ANALYSIS

Figure 1 Observed recrystallization rates ($K$): ⋆— for the conditions of a prevail condensation and ○— for the conditions of a prevail evaporation, plotted against the observed quasi-steady-state temperatures ($T$).

The recrystallization data from Table 1, plotted against the temperature (Figure 1), shows decrease of the recrystallization rate with the temperature decrease. However, there are additional parameters involved into the recrystallization process, and the plotted in Figure 1 trendline can be hardly accepted as a real representation of the process.

4.1 Evaporation and condensation

Figure 2 Observed recrystallization rates ($K$) plotted against $\frac{\partial F}{\partial y}$, estimated from the experimental conditions by Equation 3.

The influence of $\frac{\partial F}{\partial y}$ is noticeable even for the “raw” data from Table 1 (Figure 2). The higher is the condensation caused by an incoming from outside water vapor ($\frac{\partial F}{\partial y} < 0$), the larger is the recrystallization rate. In case of prevail evaporation in a studied snow layer ($\frac{\partial F}{\partial y} > 0$), the recrystallization rates are the smaller, the higher is the evaporation.

The experimental data on the recrystallization rate can be adjusted to $\frac{\partial F}{\partial y} = 0$ according to the linear dependence shown by the trendline in Figure 2. However, as it was discussed in Sokratov et al. (2001), there had to be at least one more effect, related to the prevail evaporation or prevail condensation in a studied snow layer. For a “given” $\frac{\partial F}{\partial y}$, the ratio $(\frac{\partial F}{\partial y})/P$ can be different. And the latter, corresponding to the ratio between a part of the specific surface area subjected to condensation and a part of the specific surface area subjected to evaporation, has to influence the recrystallization rates observed (Figure 3).

Figure 3 Adjusted according to the trendline in Figure 2 recrystallization rates ($K$) ⋆— for the conditions of a prevail condensation ($K_c$) and ○— for the conditions of a prevail evaporation, plotted against $(\frac{\partial F}{\partial y})/P$, estimated by use of Equations 1 and 3 from the experimental conditions.

For a prevail evaporation ($K_e$), it can be suggested that the shape of the dependence will be opposite to those of $K_c$. However, almost no such conditions were formed during our experimental runs, and we have to exclude the $K_e$ data from any further adjustments.

4.2 Temperature gradient

When the adjusted according to the trendlines in Figures 2 and 3 recrystallization rates are plotted against the observed quasi-steady-state temperature gradient (Figure 4), the data can be clearly divided on the $K_e$ observed in the
samples with the downward heat- and mass-fluxes ($K_{+d}$) and the data from the samples with the heat- and mass-fluxes directed upward ($K_{-u}$).

![Graph](image)

Figure 4 Adjusted according to the trendlines in Figures 2–3 recrystallization rates ($K$) for the conditions of prevail condensation with ■—downward ($K_{+d}$) and ●—upward ($K_{-u}$) heat- and mass-fluxes, and for the conditions of prevail evaporation (○), plotted against the observed quasi-steady-state temperature gradients ($\partial T/\partial y$).

![Graph](image)

Figure 5 Adjusted according to the trendlines in Figures 2–4 recrystallization rates ($K$) for the conditions of prevail condensation with ■—downward ($K_{+d}$) and ●—upward ($K_{-u}$) heat- and mass-fluxes, and for the conditions of prevail evaporation (○), plotted against the water vapor flux ($F$), estimated by Equation 3 from the experimental conditions.

Under the upward heat-flux the growth rate increases with the temperature gradient increase up to some limiting value (Figure 4), with no temperature gradient dependence under higher temperature gradients. In the "downward" samples, the recrystallization rate decreased with the temperature gradient increase, up to approximately same as for $K_{+d}$ limit, and for higher temperature gradient the difference between $K_{+d}$ and $K_{-u}$ could not be seen in our data.

The water vapor flux seems to have opposed effects on the recrystallization rates in the upward and the downward snow samples (Figure 5).

4.3 Temperature

When the recrystallization data adjusted by the above steps is plotted against the corresponding temperatures (Figure 6), the dependencies are very different from those in Figure 1. Of course, we can not expect that the results of a detailed multidimensional data analysis will provide exactly the same as in Figure 6 shape of the temperature dependence. However, the plots of the components effects (Figure 1–5) have a similar form when the raw data (Table 1) is used, though are different numerically.

![Graph](image)

Figure 6 Adjusted according to the trendlines in Figures 2–5 recrystallization rates ($K$) for the conditions of prevail condensation with ■—downward and ●—upward heat- and mass-fluxes, and for the conditions of prevail evaporation (○), plotted against the observed quasi-steady-state temperatures ($T$).

5. CONCLUSIONS

The results of the present analysis suggest that the recrystallization rate relation only to temperature and temperature gradient cannot allow construct an accurate empirical model of the
recrystallization process in snow. Moreover, the relationships presented in Figure 6 can be interpreted as a possibility to explain almost all the wide range of the obtained experimental data (Table 1) by effects related to other, than presently accepted, components of the heat- and mass-transfer in snow.

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REFERENCES


