SNOWPACK FACTORS ASSOCIATED WITH STRENGTH CHANGES OF BURIED SURFACE HOAR LAYERS

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ABSTRACT: Every winter in North America, failures in layers of buried surface hoar (frost) release many slab avalanches, some of which kill recreationists. Some surface hoar layers stabilize within a week of burial and others require a month or more. Little is known about whether snowpack factors such as crystal size, snowpack depth, slab thickness, load, temperature and temperature gradient are associated with strength changes of these layers.

We tested buried surface hoar layers once or twice per week at study sites in the Columbia Mountains from 1994 to 1998, and measured over 300 changes in shear strength.

We assess the factors associated with the rate of strength change using rank correlations. The factors are ranked to identify which are most relevant for forecasting changes in strength. Useful predictors include the total snowpack depth, the maximum grain size, the strength at the start of the interval, the depth and temperature of the surface hoar layer, the air temperature and the load on the SH layer. The correlations between these predictors and the measured rate of strength change are discussed in terms of physical processes.

We illustrate the predictive potential of combined factors by comparing measured values of the rate of strength change with fitted values from a regression tree.

KEYWORDS: snow strength, avalanches, avalanche forecasting, snow metamorphism, snow physical properties, snow temperature

1. INTRODUCTION

In the Columbia Mountains of Western Canada, surface hoar (SH) crystals (Figure1) comprise the failure layers (Figure2) of fatal slab avalanches more often than any other grain type (Jamieson and Johnston, 1992). Yet little is known about the factors such as load, temperature and temperature gradient that may influence the rate at which the strength (and stability) of SH layers change over time.

There have been numerous studies of changes in the shape and size of snow crystals/grains over time (metamorphism). However, strength depends on bonding which is not directly related to grain shape and grain size. We focus on strength change and consider grain size as a possible predictor of the rate of strength change.

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Figure 1 Surface hoar crystals on a tree and on the snow surface. Once buried by subsequent snowfall, buried surface hoar layers often form persistent weak layers and potential failure layers for slab avalanches.

Figure 2 Fractured and unfractured portions of buried surface hoar layer. The fracture produced a "whumpf" sound.

The objectives of this paper are to:

- identify some merits and difficulties of measurements associated with strength changes, and
- determine which factors are associated with the rate of strength change of buried SH layers. We regard this as a first step towards developing a model for predicting strength changes based on measurements that are easier to make than repeatedly measuring the shear strength of the layers with the shear frame.

2. LITERATURE REVIEW

In a cold lab, de Quervain (1958) studied the effect of various temperature gradients and loads on the shear strength and hardness of snow that was initially "fresh". Also in a cold lab, Akitaya (1974, 1975) studied the interacting effect of temperature gradient and density on change in hardness. Both de Quervain and Akitaya observed

the development of depth hoar under certain conditions. Fierz (1998) made a detailed field study of a layer of faceted crystals and depth hoar that had formed just under a crust, focussing on the temperature, density and texture of the layer. However, none of these studies assessed surface hoar.

Lang and others (1985) monitored a SH layer in the field from formation through metamorphic and strength changes after burial. Strength increased from 25 to 390 Pa over about 44 days. They focussed on the micro-meteorological conditions that contributed to surface hoar formation and did not discuss the factors that contributed to the measured strength increase. Jamieson and Johnston (1995), Jamieson (1995) and Schweizer and others (1998) measured strength changes of buried SH layers. However, they focussed on the stability trends and associated avalanche activity. These authors did not assess the factors which may have influenced the measured strength changes.

3. FIELD METHODS

We conducted field studies in the Columbia Mountains of western Canada between December 1994 and March 1998. Of the nine study plots, five range in slope angle from 10° to 36° and four are level or almost level (Table 1). The total depth of the snowpack, *HS*, ranged from 36 cm to 490 cm. Table 1 also gives the aspect, elevation and number of strength changes measured at each site.

To measure strength changes in areas of relatively thin snowpack, two "air boxes" were constructed during the summer (Figure 3). These plywood

Table 1 Attributes of Study Sites

Study Site	Aspect	Elevation (m)	Slope (°)	<i>HS</i> (cm)	No. of strength changes	No. of strength changes with continuous temperature measurements
Mt. St. Anne	E	1900	0	175-375	67	13
Mt. St Anne Airbox	E	1900	0	68-153	42	24
St. Anne Cut Block	E	1600	23-32	127-255	9	7
Vermont	N	1600	0-6	66-189	82	19
Vermont Airbox	N	1600	0	36-96	21	12
Moose	NW	1900	20-34	111-217	40	23
Elk	N	2300	10-24	140-285	35	0
Crystalline N Fork	NE	2050	21-29	223-363	30	0
Rocky	NE	2100	27-36	263-490	11	0

Figure 3 End view of air box with accumulation of undisturbed snow. Early in the winter, snow that accumulates on top of the box is shovelled off until the adjacent snowpack is slightly above the height of the box. Subsequent snow layers are allowed to accumulate naturally for the rest of the winter. During early and mid-winter, stronger temperature gradients are expected in the shallow snowpack on the air box compared to the nearby full depth snowpack.

boxes were 14.5 m long, 2.5 m wide. One was 1 m high and placed in an area where the snowpack typically reaches 1.5 m. The other was 2 m high and placed in an area where the snowpack typically reaches 3 m. During early winter the snow was shovelled off the top of each box until the snowpack reached the height of the box. Subsequently, the snow was allowed to accumulate on top of the box. Once SH layers were buried in the snowpack on top of the box, they were tested in the same manner as in the adjacent full-depth snowpack, except that the distance between pits was reduced to 0.5 m (1.0 m standard) and the pits were carefully back-filled after the measurements were completed.

The air temperature, T_a , and depth of the SH layer, H , are measured as described in snow observation manuals (e.g. CAA, 1995).

3.1 Shear frame test

Andre Roch of Switzerland was instrumental in the development of the shear frame test (de Quervain, 1951; Roch, 1966). The avalanche forecasting program at Rogers Pass has used the shear frame test since 1962 (Schleiss and Schleiss, 1970). The test is also used by the avalanche safety programs at the Kootenay Pass Highway and some ski operations.

Before the shear frame test is performed, the weak SH layer is identified with a profile of snow layers, a tilt board test, a shovel test, or a rutschblock test (CAA, 1995). Overlying snow is removed, leaving approximately 40-45 mm of undis-

turbed snow above the weak layer (Figure 4). The shear frame is then gently inserted into the undisturbed snow so that the bottom of the frame is approximately 2-5 mm above the weak layer (Perla and Beck, 1983). For a detailed discussion of the shear frame test and frame placement, see Jamieson (1995). Shear strength is determined by dividing the maximum load on the force gauge by the area of the frame, usually 250 cm² in the present study. Coefficients of variation of shear frame measurements average 14% (Jamieson, 1995). Average shear strengths of weak layers were based on sets of 12 shear frame tests to reduce the standard error of shear strength. The shear strengths reported in this paper are adjusted for size effects (Sommerfeld, 1980; Föhn, 1987) and denoted Σ .

3.2 Grain size

The SH crystals are manually separated from the snowpack and observed under low magnification (e.g. 8x) on a crystal screen with 1-mm, 2-mm, 3-mm and 10-mm grids. The minimum size, E_{min} , and maximum size, E_{max} , of the characteristic crystals are observed and recorded.

3.3 Load

The load over a SH layer is measured in two ways. Most often, we sample the snow layers over the SH layer by pushing a tube vertically down

Figure 4 Shear frame test showing shear frame placed a few millimetres above a weak snowpack layer.

through the layers. The load is calculated as $Load = mg/A$ where m is the mass of the sample, g is the acceleration due to gravity and A is the cross-sectional area of the tube (28 cm² in our study). We also calculate the load from the density measurements and thickness of the layers overlying the SH layer

$$Load = g (\rho_1 h_1 + \rho_2 h_2 + \dots) / A \quad (1)$$

where ρ and h are the density and thickness of the i^{th} layer. When load over a particular SH layer is measured both ways, the values are averaged.

3.4 Thickness of layer

The thickness of a buried SH layer, L , is measured by placing a scale graduated in millimetres against a vertical wall of the snow pit. This measurement was made for 175 changes in the last two winters. L and related predictors are used for the rank correlations but not for the multi-variate regression since this would have limited the data to 175 of the 337 changes.

3.5 Manual temperature measurements

On each test day, the temperatures are measured 5 cm above and below the SH layers with hand-held digital thermistors with 20 cm-long probes. The temperature of the SH layer, T_{wl} , is taken to be the average of the temperatures 5 cm above and below the SH layer. The temperature gradient, TG , is calculated from the two temperatures. The average temperature of the SH layer over the 1-8 day interval, $T_{wl_{avg}}$, is calculated from the average of T_{wl} at the start and end of the interval. The average temperature gradient across the SH layer over the 1-8 day interval, TG_{avg} , is calculated by averaging TG values at the start and end of the interval. $T_{wl_{avg}}$ and TG_{avg} are available for all 337 changes analyzed in this paper.

3.6 Continuous temperature measurements

We used thermistors connected to a datalogger to measure snowpack temperatures continuously. The thermistors were individually calibrated in slush to achieve an accuracy of approximately $\pm 0.1^\circ\text{C}$. However, there are larger errors associated with the placement of the thermistors. We excavate a square pit in undisturbed snow near where the shear frame tests are made. Into the vertical wall aligned down the slope, we make 25 cm-deep horizontal holes into the undisturbed snow, 5 cm above and below the SH layer, with a rod the same diameter (8 mm) as the thermistors in their protective sheaths. We insert the thermis-

ters into these holes (Figure 5) and backfill the pit, being careful not to disturb the thermistor cables. Typically 7-14 days later we carefully dig out the thermistors, noting the distance they are above and below the SH layer. About half the time we find the thermistors are 7-9 cm apart, implying that the thermistors were initially 10 cm apart and the snow between the thermistors compressed by 1-3 cm. However, we have occasionally excavated thermistors and found them to be more than 10 cm apart or less than 4 cm apart, indicating that the thermistors were placed inaccurately. We minimize this source of error by placing two pairs of thermistors across most SH layers, and using the results from the best-placed pair for calculating the temperature of the SH layer and temperature gradient across the layer.

The thermistors readings are recorded hourly by a datalogger. The average temperature of the SH layer based on thermistor measurements over the 1-8 day interval is denoted by T_{th} . The average temperature gradient calculated from the hourly thermistor measurements over the 1-8 day interval is denoted by TG_{th} . Not all SH layers we tested with shear frames were monitored with thermistors. Consequently, T_{th} and TG_{th} are available for 98 of the 337 changes analyzed in this paper. T_{th} and TG_{th} are used for the rank correlations but not for the multi-variate regression since this would have limited the data to 98 of the 337 changes.

3.7 Comparison of continuous and manual temperature measurements

To compare manually measured temperatures with continuously measured temperatures, we plotted the difference, $T_{wl_{avg}} - T_{th}$, in Figure 6a. The difference decreases with depth. Below 30 cm, the two measurements differ by less than 1°C in most cases. We attribute the improved agree-

Figure 5 Thermistors in wall of snow pit, 5 cm above and below the surface hoar layer.

Figure 6 a. Difference between manual and continuously measured temperatures. b. Difference between manual and continuously measured temperature gradients. The magnitude of the differences reduces with depth.

ment with depth to decreased diurnal temperature fluctuations with depth that make T_{th} less sensitive to inaccurate placement of the thermistors.

The difference between manually and continuously measured temperature gradients, $TG_{avg} - TG_{th}$ is plotted in Figure 6b. The greatest differences occur near the surface. Below 30 cm, most values differ by less than $5^{\circ}\text{C}/\text{m}$. Based on this comparison, we expect manual and continuous measurements to give similar results below 30 cm.

4. PREDICTOR VARIABLES

A predictor variable is a measurement that might be useful for predicting the rate of strength change and is easier to measure than shear strength. These are listed in Table 2. Some predictors are measurements such as snowpack depth or air temperature. Others are calculated values such as strength or load. We also used some elaborated variables such as TG/T_{wl} , TG_{th}/T_{th} , T_a/HS and $E_{max}-E_{min}$ to assess their merit as possible predictors of the rate of strength change. We use subscripts to distinguish between the measurement at the start of a 1-8 day interval, e.g. H_0 , and at the end of the interval, e.g. H_1 . From these two values we calculate H_{avg} which is the average of the values at the start and end of the interval, and $\Delta H/\Delta t$ which is the average rate of change during the interval, Δt .

We consider the shear strength at the start of the interval, Σ_0 , as a possible predictor for the rate of change during the interval.

5. RESPONSE VARIABLES

In the following analyses we seek associations between the predictors and the response variable, $\Delta\Sigma/\Delta t$, which is the average rate of change of shear strength during the 1-8 day interval. To assess whether the correlations were better for

Table 2 Predictor Variables

H	Thickness of the slab, measured vertically from the surface of the snowpack to the top of the surface hoar layer (cm)
T_a	Air temperature ($^{\circ}\text{C}$)
HS	Total depth of snowpack, measured vertically (cm)
L	Thickness of surface hoar layer (mm)
$Load$	Weight of overlying snow (kPa)
T_{wl}	Temperature of SH layer measured with thermometer ($^{\circ}\text{C}$)
T_{th}	Temperature of SH layer, average of hourly thermistor measurements over 1-8 day interval ($^{\circ}\text{C}$). Each hourly temperature is the average of the temperature from thermistors initially placed 5 cm above the SH layer and 5 cm below the SH layer.
TG	Temperature gradient, $(T_{+5} - T_{-5})/0.1$ where T_{+5} and T_{-5} are the temperature measured manually with thermometers placed 5 cm above and 5 cm below the SH layer, respectively ($^{\circ}\text{C m}^{-1}$)
TG_{th}	Temperature gradient of SH layer, average of hourly thermistor measurements over 1-8 day interval ($^{\circ}\text{C m}^{-1}$). Each hourly temperature gradient is the average of the temperature gradient from thermistors initially placed 5 cm above the SH layer and 5 cm below the SH layer and adjusted for settlement of the snow between the layers.
Σ	Strength of surface hoar layer measured with shear frame and adjusted for size effects (Sommerfeld, 1980; Föhn, 1987)
Age	Number of days since the surface hoar layer was buried (d)
E_{min}	Minimum length of characteristic crystals (mm)
E_{max}	Maximum length of characteristic crystals (mm)

Table 3 Response Variables

Depth at start of interval	1-8 day intervals		1-4 day intervals		5-8 day intervals	
	Variable	N	Variable	N	Variable	N
any	$\Delta\Sigma/\Delta t$	337	$\Delta\Sigma/\Delta t_{1-4}$	162	$\Delta\Sigma/\Delta t_{5-8}$	175
< 30 cm	$\Delta\Sigma/\Delta t_{<30}$	52	-	-	-	-
≥ 30 cm	$\Delta\Sigma/\Delta t_{\geq 30}$	282	-	-	-	-

deeper or shallow layers, we also calculated $\Delta\Sigma/\Delta t_{<30}$ and $\Delta\Sigma/\Delta t_{\geq 30}$ which are, respectively, the rate of strength change for SH layers less than 30 cm below the surface, and for layers at least 30 cm below the surface at the start of the 1-8 day interval. To assess whether the correlations were better for shorter or longer intervals, we calculated $\Delta\Sigma/\Delta t_{1-4}$ and $\Delta\Sigma/\Delta t_{5-8}$ for 1-4 and 5-8 day intervals, respectively.

These response variables and the number of data for each are summarized in Table 3.

Measures of correlation and regression fit such as R^2 can be over-estimated if the data are serially correlated. However, this concern is reduced since the 337 data are from 61 different series.

The 337 values of $\Delta\Sigma/\Delta t$ are partitioned into 100 Pa intervals and plotted along with the expected normal curve in Figure 7. The fit to the normal curve appears poor. The hypothesis of normality is rejected by the Kolmogorov-Smirnov test ($d=0.125$, $p < 0.01$) and the Chi-squared test ($X^2 = 79$, $df = 6$, $p < 10^{-7}$). For the Chi-squared test, intervals with less than 5 data were combined. The non-normality of the response variable, $\Delta\Sigma/\Delta t$, precludes the use of statistical techniques such as Pearson correlations.

6. RANK CORRELATIONS

Rank correlations between the predictor variables and the five response variables are listed in Table 4. We used Kendall tau rank correlations since $\Delta\Sigma/\Delta t$ is not normally distributed as shown in Figure 7.

6.1 Effect of the depth of the surface hoar layer on correlations

We assess whether the correlations are better for deeper or shallow layers by considering the correlations of $\Delta\Sigma/\Delta t_{<30}$ and $\Delta\Sigma/\Delta t_{\geq 30}$ with the predictors in Table 4. There are 19 significant correlations ($p < 0.05$) between $\Delta\Sigma/\Delta t_{\geq 30}$ and the predictors ($N \leq 285$) whereas there are only 10

Figure 7 Distribution of $\Delta\Sigma/\Delta t$ and expected normal curve.

significant correlations between $\Delta\Sigma/\Delta t_{<30}$ and the predictors ($N \leq 52$). Also, only 4 of the predictors correlate significantly with both $\Delta\Sigma/\Delta t_{<30}$ and $\Delta\Sigma/\Delta t_{\geq 30}$. Since $\Delta H S/\Delta t$, $\Delta H/\Delta t$, $\Delta \text{Load}/\Delta t$ correlate significantly with $\Delta\Sigma/\Delta t_{<30}$ but not with $\Delta\Sigma/\Delta t_{\geq 30}$, it suggests that shallow layers are more sensitive to changes in load and depth than deeper layers. Nevertheless, we expect more significant correlations for the deeper layers because there are more data and because of the more accurate temperature measurements for deeper SH layers as shown in Figure 6a and 6b.

All the predictors that correlate significantly with $\Delta\Sigma/\Delta t_{\geq 30}$ also correlate significantly, and with the same sign, with $\Delta\Sigma/\Delta t$ ($N = 337$, all depths). Consequently, in subsequent analyses we do not partition the data according to the depth of the surface hoar layer.

6.2 Effect of interval length on correlations

We usually tested SH layers buried less than 75 cm below the surface every 3-4 days, although there are a few changes for 1-2 day intervals. We measured 162 changes over 1-4 intervals (response variable $\Delta\Sigma/\Delta t_{1-4}$). We measured 175 changes over 5-8 day intervals and denote the response variable for these generally deeper SH layers as $\Delta\Sigma/\Delta t_{5-8}$. Combining these data sets gives 337 changes over 1-8 days for layers of various depths (response variable $\Delta\Sigma/\Delta t$).

Table 4 Kendall Tau Rank Correlations

	$\Delta\Sigma/\Delta t$			$\Delta\Sigma/\Delta t_{1-4}$			$\Delta\Sigma/\Delta t_{5-8}$			$\Delta\Sigma/\Delta t_{<30}$			$\Delta\Sigma/\Delta t_{\geq 30}$		
	N	τ	p	N	τ	p	N	τ	p	N	τ	p	N	τ	p
Ta_0	332	-0.04	0.251	160	-0.07	0.207	172	-0.04	0.458	50	-0.04	0.661	282	-0.03	0.426
Ta_{avg}	331	-0.11	0.004	160	-0.15	0.004	171	-0.09	0.084	50	-0.01	0.953	281	-0.11	0.005
$\Delta Ta/\Delta t$	331	-0.05	0.207	160	-0.10	0.067	171	0.01	0.79	50	0.01	0.94	281	-0.06	0.156
HS_0	333	0.25	2E-11	158	0.24	1E-5	175	0.27	1E-7	51	0.14	0.151	282	0.24	2E-9
HS_{avg}	333	0.25	2E-11	158	0.24	6E-6	175	0.27	9E-8	51	0.17	0.072	282	0.24	3E-9
$\Delta HS/\Delta t$	333	-0.04	0.343	158	0.00	0.929	175	-0.06	0.247	51	0.25	0.01	282	-0.06	0.128
H_0	337	0.17	4E-6	162	0.18	5E-4	175	0.19	1E-4	52	0.02	0.804	285	0.13	0.001
H_{avg}	337	0.16	1E-5	162	0.18	9E-4	175	0.18	3E-4	52	0.17	0.082	285	0.11	0.004
$\Delta H/\Delta t$	337	0.07	0.048	162	0.03	0.623	175	0.12	0.02	52	-0.20	0.034	285	0.10	0.012
$Load_0$	336	0.14	2E-4	161	0.17	0.002	175	0.15	0.003	51	-0.02	0.814	285	0.10	0.014
$Load_{avg}$	336	0.14	1E-4	161	0.16	0.003	175	0.17	7E-4	51	0.17	0.078	285	0.10	0.015
$\Delta Load/\Delta t$	336	0.03	0.362	161	0.01	0.905	175	0.07	0.189	51	0.31	0.002	285	-0.01	0.847
Twl_0	337	-0.15	3E-5	162	-0.12	0.019	175	-0.17	0.001	52	-0.14	0.151	285	-0.17	2E-5
Twl_{avg}	337	-0.15	3E-5	162	-0.16	0.003	175	-0.14	0.006	52	-0.09	0.368	285	-0.18	3E-6
$\Delta Twl/\Delta t$	337	-0.04	0.337	162	-0.12	0.025	175	0.05	0.281	52	0.15	0.125	285	-0.06	0.112
TG_0	337	0.09	0.01	162	0.06	0.241	175	0.12	0.015	52	0.17	0.07	285	0.09	0.031
TG_{avg}	337	0.07	0.04	162	0.01	0.795	175	0.15	0.003	52	0.20	0.036	285	0.05	0.212
$\Delta TG/\Delta t$	337	-0.07	0.045	162	-0.11	0.044	175	-0.02	0.735	52	-0.06	0.535	285	-0.09	0.022
TG_{avg}/Twl_{avg}	337	-0.16	2E-5	162	-0.06	0.268	175	-0.26	3E-7	52	-0.21	0.028	285	-0.15	2E-4
L_0	175	0.001	0.978	77	-0.01	0.907	98	0.02	0.816	32	-0.18	0.141	143	0.03	0.557
L_{avg}	175	-0.01	0.806	77	-0.03	0.722	98	0.00	0.99	32	-0.19	0.135	143	0.02	0.714
$\Delta L/\Delta t$	175	-0.02	0.732	77	-0.04	0.615	98	0.00	0.963	32	0.05	0.699	143	-0.02	0.673
$Emin_0$	337	-0.14	1E-4	162	-0.12	0.019	175	-0.15	0.003	52	-0.05	0.629	285	-0.12	0.002
$Emin_{avg}$	337	-0.15	4E-5	162	-0.14	0.008	175	-0.15	0.003	52	-0.09	0.36	285	-0.14	0.001
$\Delta Emin/\Delta t$	337	-0.04	0.254	162	-0.06	0.234	175	-0.02	0.761	52	-0.11	0.232	285	-0.05	0.172
$Emax_0$	328	-0.18	1E-6	160	-0.11	0.037	168	-0.26	7E-7	51	-0.28	0.004	277	-0.14	4E-4
$Emax_{avg}$	321	-0.17	3E-6	157	-0.11	0.039	164	-0.24	4E-6	48	-0.27	0.007	273	-0.13	0.001
$\Delta Emax/\Delta t$	321	0.04	0.317	157	0.00	0.987	164	0.08	0.117	48	0.09	0.376	273	0.04	0.334
$Emax_{avg}-Emin_{avg}$	321	-0.14	2E-4	157	-0.03	0.517	164	-0.24	5E-6	48	-0.31	0.002	273	-0.09	0.020
Ta_{avg}/HS_{avg}	327	0.01	0.73	156	-0.04	0.442	171	0.06	0.272	49	0.07	0.464	278	0.00	0.961
Σ_0	337	0.12	8E-4	162	0.15	0.004	175	0.13	0.012	52	-0.06	0.507	285	0.09	0.023
Age_0	337	0.01	0.735	162	0.01	0.896	175	0.04	0.406	52	-0.19	0.047	285	-0.02	0.657
Tth_{avg}	98	0.15	0.024	47	0.25	0.012	51	0.07	0.49	24	0.35	0.017	74	0.10	0.214
$TGth$	94	0.15	0.039	45	0.04	0.732	49	0.27	0.006	23	0.37	0.014	71	0.06	0.450
$TGth/Tth$	94	-0.09	0.178	45	0.08	0.451	49	-0.29	0.003	23	-0.22	0.146	71	-0.04	0.634

Rank correlations (Kendall tau) between the response variables $\Delta\Sigma/\Delta t_{1-4}$, $\Delta\Sigma/\Delta t_{5-8}$ and $\Delta\Sigma/\Delta t$ and the predictors are shown in Table 4. $\Delta\Sigma/\Delta t_{1-4}$ is significantly correlated with 17 predictors ($p < 0.05$) and $\Delta\Sigma/\Delta t$ is significantly correlated with 22 predictors. The only improved correlation for the 1-4 day strength changes of shallower layers compared to the 1-8 day strength changes for layers of various

depths is $\Delta Twl/\Delta t$ which is only significant at the 5% level.

$\Delta\Sigma/\Delta t_{5-8}$ is significantly correlated with 20 predictors ($p < 0.05$) compared to $\Delta\Sigma/\Delta t$ which correlates significantly with 22 predictors. Only $TGth/Tth$ is better correlated with the strength changes of deeper layers over 5-8 day intervals than for layers of various depths over 1-8 day intervals.

Since the larger data sets for strength changes of layers of various depths over 1-8 day intervals are better correlated with the predictors than changes for shallower layers over 1-4 day intervals and for deeper layers over 5-8 day intervals, the combined data set is used in subsequent analyses.

7. RANKING PREDICTOR VARIABLES

The top 10 predictors in order of decreasing significance level of their rank correlations are: HS_{avg} , $E_{max_{avg}}$, H_{avg} , TG_{avg}/Twl_{avg} , Twl_{avg} , $E_{min_{avg}}$, $Load_{avg}$, $E_{max_{avg}} - E_{min_{avg}}$, Σ_0 , Ta_{avg} . These are discussed subsequently in terms of their correlations with other predictors and the underlying physical processes.

8. REGRESSION TREE

To assess the combined predictive potential of the variables associated with the rate of strength change we use a regression tree (Breiman and others, 1984). This technique detects associations between a response variable with interval properties and predictors. Tree regressions do not require normalizing transformations of the predictors, and allow for non-monotonic and complex relationships between the predictors and the response variable (Davis and Elder, 1995). Regression trees recursively split the data into two groups based on critical values of the predictors. These critical values are chosen to minimize the node deviance, which is the measure of fit (Breiman and others, 1984). A deviance of zero indicates a perfect fit.

Potentially, the data could be partitioned until there is only one datum in each node. However, while the initial splits reflect structure in the data (which is important), splitting into very small subgroups results in fitting a tree to individual data (which is not relevant to most problems). For the analysis of $\Delta\Sigma/\Delta t$ and its predictors, we allowed trees to continue to grow if there were at least 10 data at a node and stopped the growth if there were less than 5 data at a node.

For the regression tree we use Twl_{avg} and TG_{avg} ($N = 337$) in preference to Tth and $TGth$ ($N = 94$) because there are 3.5 times more data and because the rank correlations (Table 4) are more significant. Except for S_0 and Age_0 , we excluded the initial values Ta_0 , HS_0 , H_0 , $Load_0$, Twl_0 , TG_0 , E_{min_0} and E_{max_0} and used their respective average values which in most cases correlate as well or better with $\Delta\Sigma/\Delta t$ (Table 4) than the initial values. We also excluded the predictors involving the thickness of the weak layer, L_0 , L_{avg} and $\Delta L/\Delta t$ because the thickness was only measured for 175 of the 337 changes (last two winters).

Using the recursive binary partitioning algorithm for regression trees, we regressed $\Delta\Sigma/\Delta t$ on Ta_{avg} , $\Delta Ta/\Delta t$, HS_{avg} , $\Delta HS/\Delta t$, H_{avg} , $\Delta H/\Delta t$, $Load_{avg}$, $\Delta Load/\Delta t$, Twl_{avg} , $\Delta Twl/\Delta t$, TG_{avg} , $\Delta TG/\Delta t$, TG_{avg}/Twl_{avg} , L_{avg} , $\Delta L/\Delta t$, $E_{min_{avg}}$, $\Delta E_{min}/\Delta t$, $E_{max_{avg}}$, $\Delta E_{max}/\Delta t$, $E_{max_{avg}} - E_{min_{avg}}$, Ta_{avg}/HS_{avg} , Age_0 , and Σ_0 . The upper part of the tree is shown in Figure 8.

Figure 8 Upper portion of regression tree showing splits involving important predictors. For each split, the left branch is for the observations for which the "less than" condition is true.

The preferred way to assess the predictive merit of a model is to use some data to develop the model and other data to compare predicted values with measured values. However, we are reluctant to split our data because regression trees require large data sets. Consequently, we plot the *fitted* values against the measured values of $\Delta\Sigma/\Delta t$ in Figure 9. This gives an optimistic indication of the predictive potential of the predictors. The majority of the fitted data are within ± 100 Pa/d of the measured values, showing that the predictors have the potential to distinguish between rapid and slow strength gains of buried SH layers.

9. DISCUSSION

In this section we discuss the highest ranked predictors in terms of their correlations *with other predictors* and the underlying physical processes that are probably affecting the rate of strength change of buried SH layers. Relevant correlations between predictors are noted in the following discussion but the complete matrix is not given.

9.1 Snowpack depth HS

Deeper snowpack correlates positively with $\Delta\Sigma/\Delta t$ ($p = 2 \times 10^{-11}$). This is not surprising since HS is negatively correlated with $|TG|$ ($p < 10^{-6}$) [associated with bonding] and positively correlated with $Load$ ($p < 10^{-6}$) and strength at the start of an interval ($p < 10^{-6}$).

9.2 Maximum crystal size E_{max}

$\Delta\Sigma/\Delta t$ is negatively correlated with E_{max} ($p = 10^{-6}$), indicating that larger grains are slower to gain strength. This is consistent with reports of field

workers, and supports the idea that bigger umbrellas (Davis and others, 1997) are associated with bigger pore spaces and fewer bonds per unit area. Larger crystals are also negatively correlated with snowpack depth ($p = 10^{-2}$), load ($p = 10^{-4}$), slab thickness ($p = 10^{-3}$), and initial strength ($p < 10^{-6}$), which are likely associated with lower elevation study sites. E_{min} has similar correlations as E_{max} but generally slightly lower significance levels.

9.3 Slab depth H

Deeper slabs are positively correlated with $\Delta\Sigma/\Delta t$ ($p < 10^{-6}$). This is expected since deeper slabs are associated with greater load ($p < 10^{-6}$) which causes pressure on the bonds, lower magnitude of temperature gradient ($p < 10^{-5}$) generally associated with increased bonding, and greater insulation between the SH layer and the colder air.

9.4 Load

The rate of strength change, $\Delta\Sigma/\Delta t$, is positively correlated with $Load$ ($p = 10^{-4}$) since strength increases faster when greater loads are applied. Increased loads probably push surface hoar crystals into adjacent layers, thereby contributing to bonding and strength.

9.5 Range of crystal size $E_{max}-E_{min}$

The rate of strength change, $\Delta\Sigma/\Delta t$, is negatively correlated with the range of crystal size, $E_{max}-E_{min}$, ($p = 10^{-4}$). A large range of crystal size is only likely for large E_{max} . Consequently, large values of $E_{max}-E_{min}$ are associated with slow strength gain because large grains are slow to gain strength.

9.6 Temperature of SH layer T_{th}

Surprisingly, the temperature of the SH layer is negatively correlated with the rate of strength change ($p = 3 \times 10^{-5}$). However, the temperature of the SH layer is negatively correlated with snow depth ($p < 10^{-6}$), so the negative correlation of $\Delta\Sigma/\Delta t$ with temperature is probably a result of slower strength gains in shallower snowpack areas. The correlation with temperature of the SH layer measured with thermistors, T_{th} , is positive as expected (but less significant).

9.7 Strength at the start of an interval Σ_0

The rate of strength change, $\Delta\Sigma/\Delta t$, is positively correlated with the strength of the layer at the start of the interval ($p = 10^{-4}$), suggesting that stronger

Figure 9 Fitted values of $\Delta\Sigma/\Delta t$ from regression tree plotted against measured values.

layers are faster to gain strength. This is surprising since strength is roughly proportional to density squared (e.g. Perla and others, 1982) and densification slows over time (e.g. Armstrong, 1980). The positive correlation between $\Delta\Sigma/\Delta t$ and strength is probably due to the fact that initial strength is positively correlated with HS ($p < 10^{-6}$), $Load$ ($p < 10^{-6}$), slab thickness ($p < 10^{-6}$) and Twl ($p = 10^{-5}$). Strong layers are usually found under heavy loads in areas of deep snowpack and they tend to continue to gain strength during the changes we observed.

9.8 Air temperature T_a

The rate of strength change, $\Delta\Sigma/\Delta t$, is negatively correlated with the air temperature, T_a , during the interval. This suggests that strength increases faster at colder temperatures. However, T_a is negatively correlated with HS ($p = 10^{-4}$), $Load$ ($p < 10^{-3}$) and slab thickness ($p = 10^{-4}$) suggesting that the fastest strength increases took place at higher elevations where the temperature was generally colder, the snowpack deeper, the slabs thicker and the loads greater.

9.9 TG/Twl

Except for some shallow SH layers and some measurements in late March, most values of TG are negative (cooler toward the surface). Since Twl is never positive in any field study of dry snow and negative in our data set, TG/Twl is usually positive. TG/Twl is negatively correlated with TG ($p < 10^{-6}$) but not with Twl ($p = 0.17$). Consequently, TG/Twl decreases as the magnitude of TG decreases (and the value of TG increases).

The rate of strength change, $\Delta\Sigma/\Delta t$, is negatively correlated with TG/Twl ($p = 10^{-5}$) presumably because higher values of TG/Twl are associated with shallow snowpack ($p < 10^{-6}$) and thinner slabs ($p = 10^{-4}$) and decreased magnitudes of TG ($p < 10^{-6}$).

9.10 Temperature gradient across the surface hoar layer TG

The rate of strength change, $\Delta\Sigma/\Delta t$, is weakly but positively correlated with the temperature gradient, TG ($p = 0.04$), suggesting that the fastest strength gains occurred for the higher values of TG and lower magnitudes of TG . This is consistent with increased bonding when the magnitude of the temperature gradient decreases.

9.11 Thickness of the surface hoar layer L

The rate of strength change, $\Delta\Sigma/\Delta t$, did not correlate with the thickness of the SH layer ($p = 0.8$). We interpret this as a result of the competing effect of grain size (E_{max} is positively correlated with L , $p < 10^{-6}$, and negatively correlated with $\Delta\Sigma/\Delta t$, $p = 10^{-6}$) and initial strength (Σ_0 is negatively correlated with L , $p = 10^{-6}$ and positively correlated with $\Delta\Sigma/\Delta t$, $p = 10^{-3}$).

9.12 Average temperature gradient of the snowpack Ta/HS

The rate of strength change, $\Delta\Sigma/\Delta t$, did not correlate with Ta/HS ($p = 0.7$). This suggests that the strong correlation between $\Delta\Sigma/\Delta t$ and HS may be more related to greater $Load$ where HS is greater than to the average temperature gradient of the snowpack as estimated by Ta/HS .

10. SUMMARY

The air temperature, T_a , and the temperature of the SH layer, Twl , gave correlations with $\Delta\Sigma/\Delta t$ with unexpected signs. However, these are explained in terms of strong correlations between these temperature variables and the predictors $Load$, H and HS .

The three variables that correlate most significantly ($p < 10^{-4}$) with $\Delta\Sigma/\Delta t$ are HS , E_{max} and H . The effect of each of these variables on the rate of strength change can be interpreted physically. In summary, $\Delta\Sigma/\Delta t$ increases with HS and H and decreases with E_{max} . We believe:

- The thickness of the slab, H , contributes to increased strength of the underlying SH layer through its association with load and because thicker slabs contribute to warmer SH layers with lower $|TG|$ across the SH layer by insulating the layer from the usually colder air.
- The snowpack thickness, HS , contributes to increased strength through its correlations with $Load$ (pressure on the bonds) and H . Also, areas of deeper snowpack generally have lower $|TG|$, thereby contributing to bonding.
- SH layers consisting of larger crystals are generally slow to gain strength probably because they reduce the number of grains that fall between the SH crystals and bond (Davis and others, 1997).

The strong correlations of HS , E_{max} and H with the rate of strength change suggest that micro-

structure and bond-stress may be important to understanding and modelling strength changes of buried SH layers.

As expected, $\Delta\Sigma/\Delta t$ is positively correlated with TG (and negatively correlated with $|TG|$) since bond growth is associated with lower $|TG|$ and rounding. However, TG is a weaker predictor than HS , E_{max} , H and $Load$. $\Delta\Sigma/\Delta t$ correlates better with TG/Twl than with TG .

In future studies we plan to:

- improve the accuracy of continuously measured temperature and temperature gradient at the weak layers,
- increase the number of observations with continuously measured temperature and temperature gradient, and
- average the *magnitude* of the temperature gradient to improve correlations for shallow layers where temperature gradients often vary from positive to negative within 24 hours.

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