Avalanche modeling in France – Theory and applications

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ABSTRACT

First, this paper succinctly describes our numerical powder snow avalanche model. Then, an application of this model is presented in the frame of an avalanche zoning. This application is an example of a practical use of the numerical model in avalanche risk engineering. For each study the model was used to simulate different avalanche scenarios. It allows to give a coherence between input data (production conditions) and the effects of the avalanche. The use of the model can be made according to the following process :

- First it begins with searching in the archives for the most important powder snow avalanche having taken place on the site. This search concerns the conditions of production as well as the effects of the avalanche in its flowing and run-out zones. For this purpose, different indicators can be used. All caused damages "indicate" that the dynamic pressure developed by the avalanche was superior to the threshold stress of the damaged objects (tree, pylon, etc.). Effects are mapped in terms of pressure fields.
- Secondly we use the numerical model to simulate an avalanche scenarios whose entry conditions take into account all the available observations concerning the production factors (meteorology, snow, starting zone etc.). It is important to note here that such type of data is very often sketchy for several technical reasons (access to starting zone ...). Several numerical simulations allow to specify and complete the model input conditions. The purpose is to define a "reference avalanche" which produces effects similar to the observed one (pressure field, velocities, height etc.).
- Last we define the "observed-extreme-avalanche" and simulate it. The model gives access to a large spectra of scenarios by increasing or reducing production conditions or by modifying the topography of the flowing or run-out zones. Therefore, we can build a database of avalanches that can be used in defining the protection for the desired site.

1. INTRODUCTION

An analysis of the meteorological conditions causing the main powder snow avalanches taking place in French Alps shows that this type of avalanches occurs most often after a strong snow fall by cold weather (T < -5°C during 24 hours preceding the release). The powder snow avalanches are characterized by a low density, high velocities and by the structure of its particular flow. Whirlwinds are one of the remarkable characteristics that differentiate it from the dense avalanche. The intense turbulence induces a strong incorporation of air and causes a huge increase of the volume. This kind of avalanches is here considered as the flow of a heavy fluid (particles + air) in a light one (the air). The powder snow avalanche is similar to the gravity current from the visual point of view and also in term of physical process.

The equations governing this flow are fluid mechanics conservation laws. The strong turbulence requires the use of a closure model that allows to calculate both its intensity and scale. The interaction between the avalanche and the ground is taken into account throughout a model of erosion / deposition which allows to incorporate or to deposit the snow according to the concentration in particles and the intensity of the turbulence.

In order to numerically solve the equations system, and considering the complexity of the topography, the area of the flow is decomposed horizontally and vertically by means of a finite elements mesh. The equations are written under their conservative form and integrated on each cell. The resolution is performed by a Godunov numerical scheme.

2 MODELING

In this paper the powder snow avalanche is considered as a diphasic flow of a heavy fluid (air + snow) in a light fluid (air). The gravity coupled to the difference of density between these fluids generates the flow. This flow is therefore studied as a gravity current. The proposed modelling assumes that air is a Newtonian perfect gas, and that the particles in suspension do not modify neither the behavior of the fluid nor the turbulence of the flow. The equations are written in a Galilean axis system formed by three axes; Ox and Oy are in the horizontal plan and Oz is the vertical axis. In this co-ordinates system, the gravity vector is given by $g=(0,0-g)^t$. The other variables are: ρ_a the density of the air, ρ_s density of the snow, $u=(u, v, w)^t$ the velocity vector, p the pressure, c the particles volumetric concentration and $\rho_m=\rho_a+c\rho_s$ the average density.

2.1 BASIC EQUATIONS

Equations governing this kind of flow are the fluid mechanics conservation laws. They are the conservation of the air mass, the conservation of the particles mass and the conservation of the momentum. The introduction of the Reynold's decomposition followed by a statistical processing enables to obtain the average equations of the mean flow. In these equations some new variables appear. They correspond to the second order correlation resulting from the non-linearity of the Navier Stokes equations. We determine them using the Boussinesq law (so called eddy viscosity) which belongs to the class of models using the turbulent viscosity concept. Equations of motion can be written as follows:

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho\\ \rho u\\ \rho v\\ \rho w\\ c\\ k\end{bmatrix}+\mathrm{div}\begin{bmatrix}\rho u & \rho v & \rho w\\ \rho u^2+p & \rho uv & \rho uw\\ \rho uv & \rho v^2+p & \rho vw\\ \rho uw & \rho vw & \rho w^2+p\\ cu & cv & cw\\ ku & kv & kw\end{bmatrix}=\mathrm{div}\left(\upsilon_t\nabla\begin{bmatrix}0\\ \rho u\\ \rho v\\ \rho w\\ c\\ k\end{bmatrix}\right)+\begin{bmatrix}0\\ 0\\ 0\\ \rho g\\ 0\\ P_r-E\end{bmatrix}$$

where $p = k\rho_g^{\gamma}$, w_g is the particle fall-velocity, the turbulent viscosity is linked to the kinetic turbulent energy k by the following formula : $\upsilon_t = c_1 l k^{1/2}$

the turbulant energy production is given by: $e = \frac{0.078}{l} k^{3/2}$

kinetic turbulent energy production is given by,

$$P_r = \left[\upsilon t \left(\frac{\partial ui}{\partial xj} + \frac{\partial uj}{\partial xj} \right) - \frac{2}{3} k \delta_{ij} \right] \frac{\partial uj}{\partial xj}$$

2.2 NUMERICAL RESOLUTION

The calculation area is decomposed using a finite elements mesh into cells of variable dimensions. The equations system can be written under the following vectorial form :

$$\frac{\partial U}{\partial t} + \mathrm{div}[F(U)] = \mathrm{div}\left[u H \nabla(U) \right] + G(U)$$

The equations system is written in the conservative form for its numerical resolution. The scheme is realized in finite volumes. That allows a good adaptation to an area of variable shape (relief of an avalanche path). The scheme is of second order accuracy in space and first order accuracy in time. The system is then integrated on each cell between two consecutive dates. The scheme is obtained in two steps: the projection step followed by the integration step. The projection step consists in allocating to each cell the average value of the vector U, obtained by integrating U on the cell volume :

$U_{ie}^{n} = \frac{1}{V} \int_s U \, dv$ (V is the volume of ie)

and the integration step consists in integrating the system on each cell between times t(n) and t(n+1) :

$$V\left(U_{ie}^{n+1} - U_{ie}^{n}\right) + \int_{t(n)}^{t(n+1)} \int_S \left[div\, F(U) - div\left(\upsilon_t\, \vec{\nabla}(U)\right) - G(U) \right] dv\, dt = 0$$

The Ostrogradsky theorem allows to transform the volumetric integral in surface integral. It is then transformed into a sum on the facets of the numeric fluxes :

$$U_{ie}^{n+1} = U_{ie}^{n} - \frac{\Delta t}{V} \int_{\Gamma} \left[F(U^n)\vec{n} - \upsilon_t\, \vec{\nabla}(U^n)\vec{n} \right] dS + \Delta t\, G(U_{ie}^{n})$$

$$U_{ie}^{n+1} = U_{ie}^{n} - \frac{\Delta t}{V} \sum_{if=1}^{4} \left[F(U_{if}^{n})\vec{n}_{if} \right] S_{ia} + \frac{\Delta t}{V} \sum_{ia=1}^{4} \left[\upsilon_t\, \vec{\nabla}(U_{if}^{n})\vec{n}_{if} \right] S_{if} + \Delta t\, G(U_{ie}^{n})$$

where n is the normal to the surface of ie.

Using the average values of neighbor cells, we build the gradient of U in order to obtain a linear approximation of U. The three components of the local gradient vector are obtained by minimizing the following function :

$$\psi = \sum_{ieve \in K(ie)} \left[U_{ieve}^{n} - \left(U_{ie}^{n} + \frac{\partial U}{\partial x}(x_{ieve} - x_c) + \frac{\partial U}{\partial y}(y_{ieve} - y_c) + \frac{\partial U}{\partial z}(z_{ieve} - z_c) \right) \right]^2$$

where (K (ie)) is the set of the ie neighbours. In the calculation of the diffusion terms the U gradient is obtained by this minimization. On the other hand for the calculation of the hyperbolic part of the equations system and in order to preserve the stability of the scheme, it is necessary to limit the gradient value of U on each element in order to avoid new local extrema. To obtain the contribution of the hyperbolic part of the equation system to the numerical flux term, we proceed as follow :

- Using the property of invariance by rotation of the conservation equations, we write the system in the local axis system formed by the tangent plan and the normal to the facet. In order to reduce the calculation of the flux to a one dimensional case we suppose that the variation in the tangent direction is negligible.
- The problem becoming mono-dimensional, we can then use a Riemann solver on each interface in order to determine the numerical flux of the facet. The Riemann problem can be put as follows :

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x_n} = 0.,\ U(x,0) = \begin{cases} U_g & if \quad x_n < 0 \\ Ud & if \quad x_n < 0 \end{cases} /$$

$$\begin{Bmatrix} U_g = U_{ieg} + (\vec{\nabla} U)_{ieg} \cdot (\vec{x}_{interface} - \vec{x}_{ieg}) \\ U_d = U_{ied} + (\vec{\nabla} U)_{ied} \cdot (\vec{x}_{interface} - \vec{x}_{ied}) \end{Bmatrix}$$

where i_{eg} is the left cell and i_{ed} is the right cell of the facet. The resolution of the Riemann problem allows to determine the numerical flux through the interface. It is important to notice here the limitation of the time step introduced by the use of this explicit scheme.

2.3 BOUNDARY CONDITIONS

The model needs a set of boundary conditions near the ground. These conditions are the momentum and the mass exchanges. Concerning the first one, near the ground the flow is considered as a turbulent boundary layer defined by friction velocity and roughness. The snow erosion or deposition is taken into account using a mass flux eroded or deposited by the avalanche. It depends on :

threshold velocity : corresponding to the friction velocity when the erosion starts

threshold concentration : corresponding to the maximum concentration that the flow can carry along, It depends on the turbulence intensity of the flow. This volumetric concentration is less than (10 %) because above this value the behavior of the fluid becomes non-Newtonian.

the fall velocity of the snow particles, this parameter controls the deposit in the run-out zone of the avalanche. It has another role : it reduces the vertical diffusion of the snow particles.

2.4- VALIDATION

In this section the aim is to verify the validity of the model, comparing its results to those obtained in experiments on density currents realized in a water flume. In these experiments were studied the flow produced by a finite volume of heavy fluid in the flume containing the ambient fluid. The effects of the slope, the volume and the density of the gravity current are explored. To measure the flow parameters, the dense fluid is generally colored. The positions of the front and the dimensions of the gravity current (length and height) are determined. Several experiments have been used to test the validity of the numerical model. All these experiments were realized by Pierre Beghin in the case of bi-dimensional density currents.

For the established phase as well as for the phase of deceleration, measurements show that the results of the model are globally rather close to the experimental results. For the acceleration phase, the dynamic is only shown by numerical simulations because we have no experimental measures in this stage. The qualitative study of numerical results shows that the model correctly predicts the height and velocity evolution of the front of the bi-dimensional gravity current. The numerical model reproduces different characteristics of the gravity currents. Quantitative comparisons are more difficult to do because the experimental results come from a unique experience. However we can conclude that the proposed model describes globally the different phases of the flow. Indeed, for the height of the front and its speed the results given by the numerical model are close to the experiences results.

3. APPLICATION FRAME AND EXAMPLE

The aim of this section is to show practical examples of what the proposed model can bring in term of assistance in analysing and mapping the avalanche risk. We present

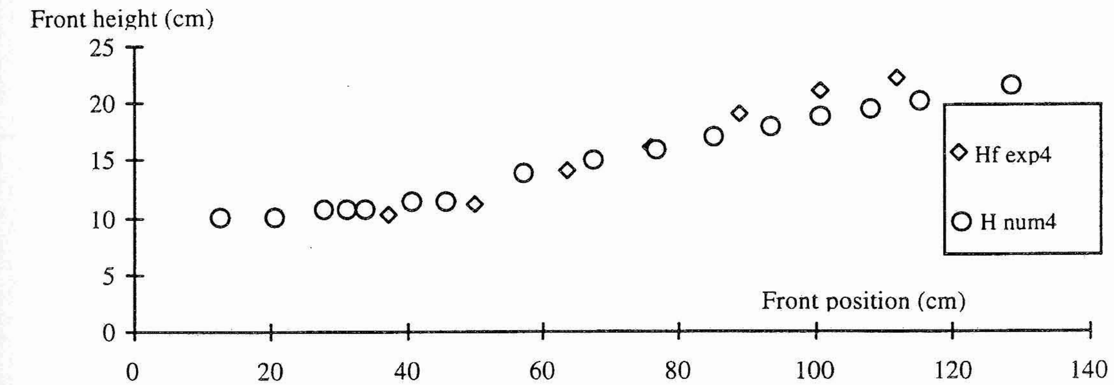

Figure 1 : Example : front height as a function of the front position

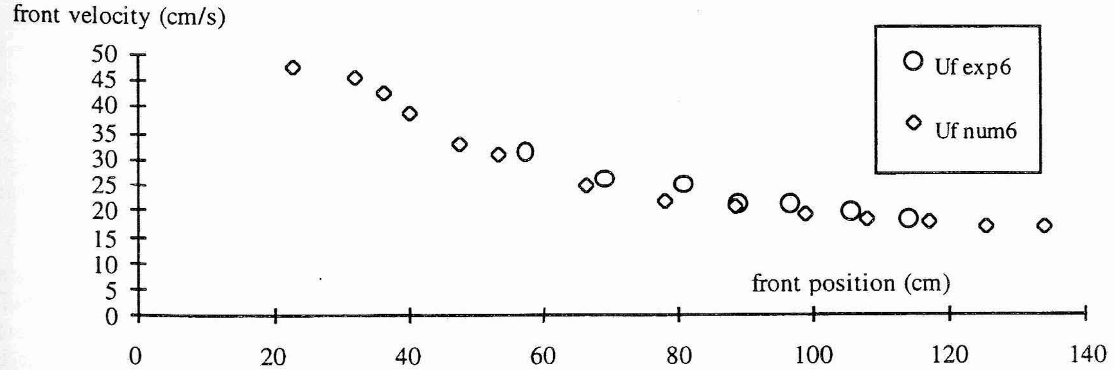

Figure 2 : Example : front velocity as a function of the front position

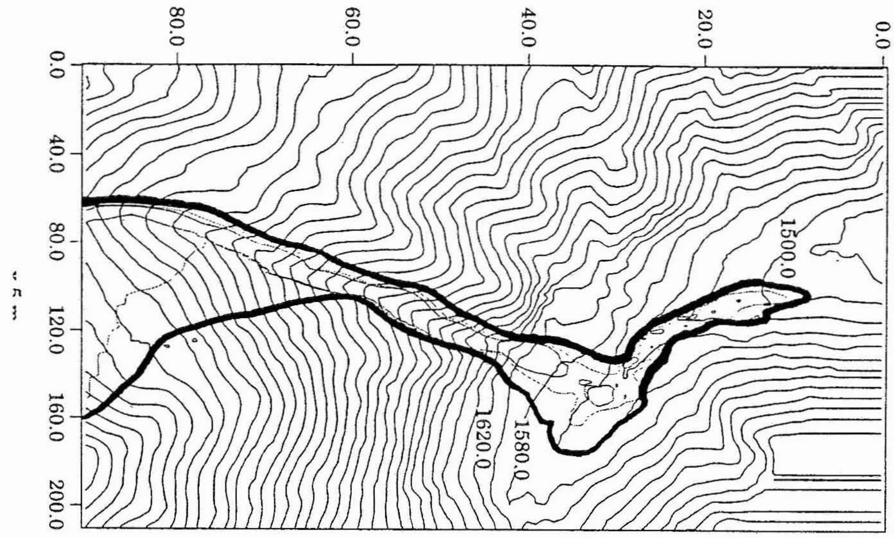

Figure 3 : Reference avalanche simulated by the model

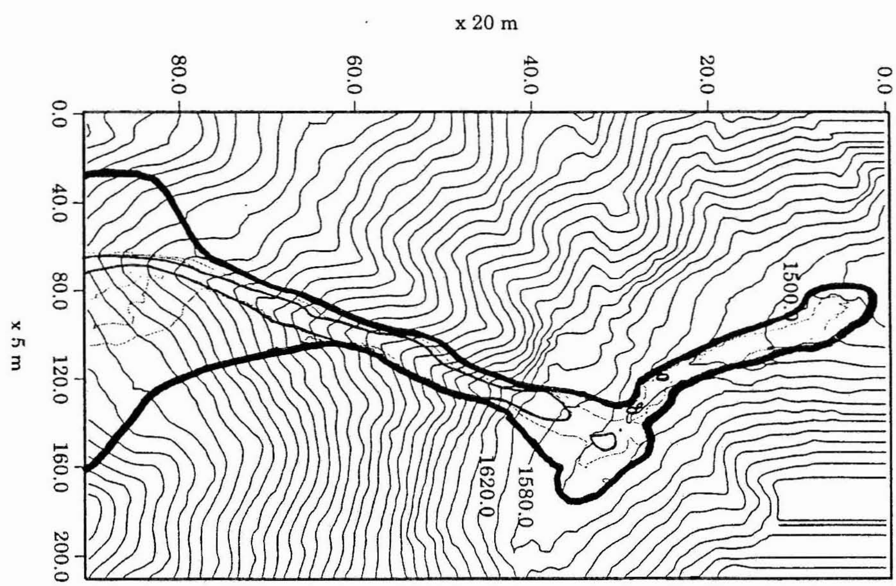

Figure 4 : avalanche obtained using the same input data than the reference avalanche except the snow height is increased of 30%

here an example concerning the site of Arinsal (Andorra) for which numerical investigations were performed. The numerical model allows to find an equivalent to the powder snow avalanche which occurred in February 1996. For this case the purpose was to determine an equivalent-avalanche in terms of damages, using the available data and the digital elevation model. The model was used following the process below :

First, concerning the effects of the real avalanche, a map of the damages was made by the French Mountain Forest Service (RTM -*Restauration des Terrains de Montagne*). For the production conditions, the limit of the starting zone was mapped and a ram test was realised near the starting zone. It was shown that the thickness of the light-snow layer was about 1 m and the density in this layer was about 100 kg/m3. Different searches in archives showed that this avalanche was probably the most important that ever occurred on the site.

Secondly we use the numerical model to simulate an avalanche scenarioso whose input conditions take into account all available observations in term of production factors (meteorology, snow, starting zone, etc.). Several numerical simulations allow to specify and complete the numerical model input conditions. Thus a reference avalanche with effects and production condition (pressure field, velocities, height) similar to the real case is defined, and then simulated.

Afterwards we study the influence of a modification of the input conditions and of the topography of the flowing or run-out zones. The maximum pressure field is mapped in order to evaluate the effect of these modifications on the extension of the avalanche. Thus we improve the knowledge of the avalanche phenomenon on the considered site, which can be used to determine the characteristics of the avalanche protection to be realized.

The following figures feature different pressure fields obtained in the cases of (three curves corresponding to 100 daN/m2, 500 dan/m2 and 1000 daN/m2):the reference avalanche (figure 3).

- the avalanche obtained using the same input data than the reference avalanche except the snow height is increased of 30% (figure 4).
- the avalanche obtained using the same input data than the reference avalanche but with an extension of the starting zone (figure 5)

In this example are to be found all the steps considered as necessary, depending from French experience, to set up a comprehensive dialogue between the scientist, the consultant and the client :

- investigating the conditions of the event, i.e. initial conditions and destruction effects (consultant);
- doing historical research on the same site (consultant);
- using the model as a tool find one or several possible scenarios relating the two sets of conditions mentioned in the first item (scientist);

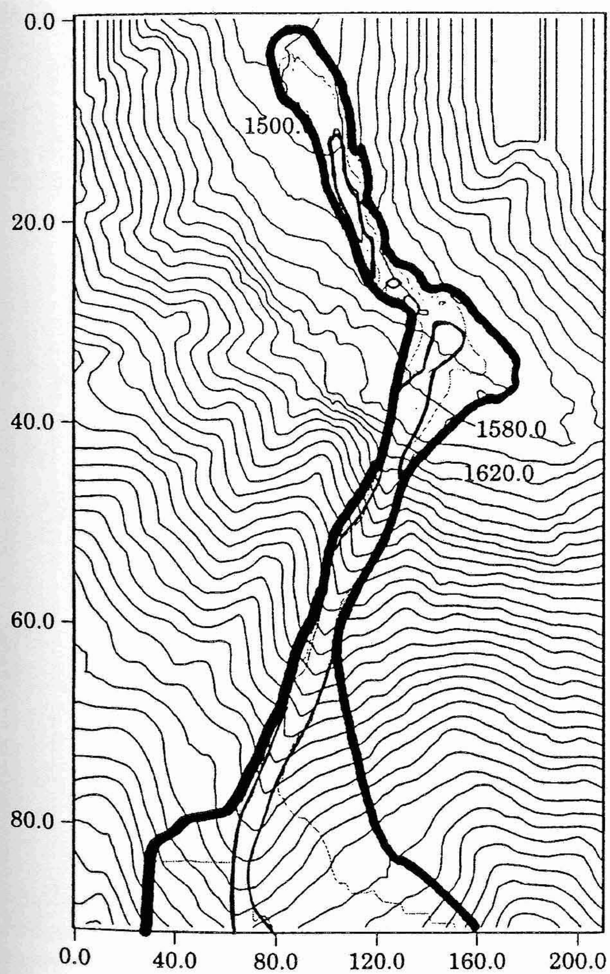

Figure 5 : avalanche obtained using the same input data than the reference avalanche but with an extension of the starting zone

- choose the most probable scenarios (consultant, with the help of the scientist) ;
- the client decides, with the help of the consultant, of a scenario of initial conditions that is selected depending first on historical data, second on the level of safety he/she is willing to buy ;
- using again the model, in order to determine the effects of this avalanche in the run-out zone.Figure 6 : avalanche obtained using the same input data than the reference avalanche but with an extension of the starting zone

4. CONCLUSION

The different steps of the development of a numerical model were shown. We have succinctly recalled the principles and the physical laws that were used. The different numerical resolution steps and the phase of validation using laboratory measures were detailed. Finally we have presented an application case in which the model allow to determine the extensions of a powder snow avalanche. A suitable use of this type of model can allow to build a plausible image of an avalanche. It also offers the possibility to present a better coherence between the production conditions and the effects produced by the resulting avalanche.

REFERENCES

Beghin P., 1979, *Etude des bouffées bidimensionnelles de densité en écoulement sur pente avec application aux avalanches de neige poudreuse* - Grenoble : Université Joseph Fourier, (Thèse de doctorat).

Naaim M., 1995, *Modélisation numérique des avalanches aérosols*, Houille Blanche, Revue Internationale de l'eau, N:5/6 1995, pp:56-62.

Naaim M, Martinez H, 1996, *Simulacion fisica y numerica de las avalanchas de nieve en Francia*, Geogaceta, 20 (6), pp: 1375-1376.