STATISTICAL MODELLING OF SNOW COVER STABILITY IN MOUNTAIN SLOPES

Pavel A. Chernouss
Antiavalanche service of production association "Apatit"
50th anniversary of October st., 33"a",
Kirovsk, Murmansk region,184230, Russia.
Fax +4778914010-124, telex 126161 chorr su,
email master@apatit.murmansk.su

New class of snow cover stability models are suggested. The base of such models are determined models of snow cover mechanical equilibrium and parameters of the models are stochastic functions. As an example of this class consider the model that is used for evaluation snowstorm snow stability.

As a determined model was chosen simplified Boginsky's model [1] in which comparison tensile stress with tensile strength is used for evaluation snow cover stability. For practical using the model was simplified by Boginsky [2] additionally and instead of tensile stress was used snow mass of instability zone and instead of tensile strength - value of critical mass. The algorithm of the modelling is described briefly below.

The profile of the slope is divided on segments with equal length in projection on horizontal plane.

For each segment inclination α_k and length Δl_k are calculated.

For each segment snow thickness h_k, snow density ρ_k and shear strength c_k are set.

For each segment critical snow thickness is calculated with approximate formula (1).

$$h_{k*} = c_k/\rho_k g(\sin\alpha_k - f\cos\alpha_k) \tag{1}$$

For each segment snow thickness h_k is compared with critical one h_{k*}.

Zones in profile where snow thickness more then critical one are selected.

Snow masses in selected zones are calculated.

Calculated snow masses M are compared with critical one M* for each zone.

It is considered, that snow is in unstable conditions in zones where snow masses more then critical one.

In accordance with [2] a friction coefficient f and a critical mass M* are effective constants which are determined by inverse calculation with data on avalanche releases.

There is a big problem with data of snow parameters for each segment. Usually these parameters are determined in few points of avalanche starting zone only. As our studies showed [6], two tens of measurements are enough usually for reliable definition of average characteristics only. Spatial changeableness of these parameters is doing interpolation between points of measurements senseless. Statistical modelling method for obtaining of input data for determined models are suggested.

For realization above mentioned algorithm, the parameters' distribution along profile (obtaining input data for each segment) was simulated as multidimensional normal vector ξ. Our studies [3,6] showed that distribution of used parameters are very close to normal. Such distribution is determined by vector of mathematical expectations $\mathbf{m}$ and covariance matrix $\mathbf{R}$. It is very easy to get vector with such distribution by linear transformation (2) of vector η which components are random values with mathematical expectations equal zero and dispersions equal one.

$$\xi = \mathbf{A}\eta + \mathbf{m} \tag{2}$$

Coefficients of transformation matrix are determined with a special procedure [4] on base of covariance coefficients. For case with constant mathematical expectation and dispersion (stationary process), distances between segments l and autocorrelation function r(l)determine covariance coefficients entirely (3).

$$R_{ij} = \sigma^2 r(l_{ij}) \tag{3}$$

So if we have mathematical expectations, dispersions and autocorrelation functions above mentioned parameters we can produce realizations theirs distributions along profile by Monte Carlo method. When realizations have been got enough, evaluations of two kinds of probabilities P_l and P_k are calculated as ratios of definite kind outcomes numbers to total outcomes number.

P_l is probability of unstable zone formation in a slope with a given profile (probability of avalanche occurrence)

P_k is probability of that the snow cover in segment k are in zone of initial displacement.

Autocorrelation functions of snowstorm snow thickness $r_h(l)$, density $r_\rho(l)$ and shear strength $r_c(l)$ were got on base more then 10000 measurements in Khibini Mountains on slopes with inclinations 25-30 degrees. Autocorrelation functions have the same view and were approximated by the next expressions*:

$$r_h(x)=\exp[-0.09x^{1.50}] \tag{4}$$

$$r_\rho(x)=\exp[-0.15x^{0.83}] \tag{5}$$

$$r_c(x)=\exp[-0.16x^{1.02}] \tag{6}$$

Since value of critical mass for determined model was obtained by inverse calculation for one avalanche site only [4], in stochastic model it is chosen by Brier's [5] criteria minimization (7).

$$E = 1/N \sum_{j=1}^{2} \sum_{i=1}^{N} (P_{ij} - E_{ij}) \tag{7}$$

Where j=1,2 - numbers of classes, j=1 - avalanche situations, j=2 - non-avalanche situations, i - number of situation, P_{ij} - calculated probability of that i-situation relate to j-class, E_{ij} - have a value 1 or 0 depending on relate or no i-situation to j-class, N -.total number of situations

In practice, if we have a big avalanche starting zone, calculations for a few profiles could be make. The maps of unstable snow cover conditions received by results of such calculations (fig.1) give rough visual picture of avalanche dangerous zones that are useful for using at artificial avalanche releasing.

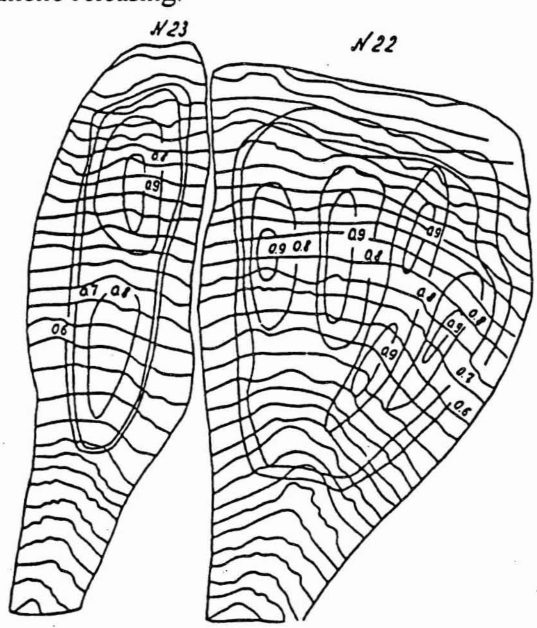


Fig.1. An example of definition of places with different P_k probabilities. Equal probability lines have been outlined on the map of avalanche starting zones.

* Formulas (4) - (6) are not autocorrelation functions for real process f(x) (we have one-dimensional model), but for process $\varphi(x) = 1/a \int_{x-a/2}^{x+a/2} f(x)\,dx$, where a = 5m.

Advantage of such modelling are adequateness of the results to input data. Uncertainty in initial information is reflected in probabilistic conclusion about snow stability. Evidently it is no sense to use complicated determined models if it is impossible to supply its by required data. With using of statistical modelling it is easy to explain differences in quality of avalanche forecasts which were made in different geographical conditions. For example, it is enough to consider spatial autocorrelation function of fresh snow shear strength in the Khibini Mountains (Arctic region) and Tien Shan (Middle Asia) (fig.2) for understanding these differences.

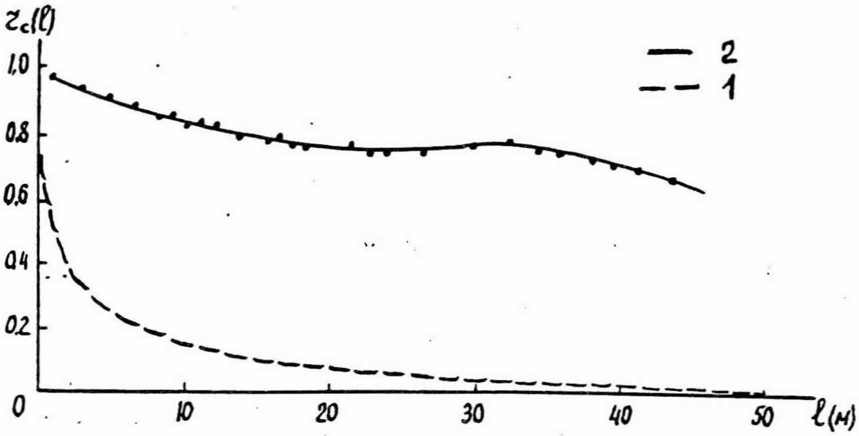


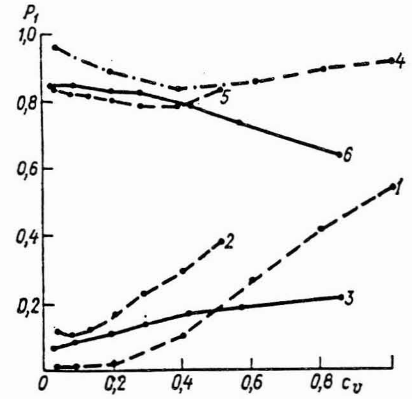
Fig.2. Spatial autocorrelation function of fresh snow shear strength:
1 - for the Khibini Mountains, 2 - for Tien Shan.

Some numerical experiments were carried out for study of parameters h, ρ and c variability influence on zone instability formation. The results of these experiments are presented on fig.3-5. The real profile with average inclination 34 degrees and horizontal projection 170m and ideal smooth one with such projection's length with inclination 40 degrees were chosen for experiments.

In first experiment dependence P_l from c_v - coefficients of variability of h, o, c parameters were studied. At modelling mean values h_a, ρ_a, c_a were constant , dispersions of two from three parameters were constant too and third was changed. Obviously, the concrete dependence are determined by profile, values h_a , ρ_a , c_a , σ_h , σ_ρ , σ_c , f , M* . On the whole, it should be noted, that values P_l rise with c_v rising up to definite angle of slope, but at big angles values P_l drop at c_v rising (fig.3). So gentle slopes non-avalanche dangerous in regions with low spatial variability snow properties can be avalanche dangerous in regions with high variability of these properties.

The length of slope influence on P_l are shown on fig.4. Probability P_l strongly depends on length of slope. It is easy to see that minimum length of slope exists on which avalanche release is possible. Avalanche formation is impossible on the more short slope. The minimum length is determined by set of values h_a , ρ_a , c_a , σ_h , σ_ρ , σ_c , f , M*. This conclusion have important practical application. It is possible to calculate maximum permissible length of slope between terraces at antiavalanche terracing on a base of long-term measurements h, ρ, c parameters in given geographical region.

The influence of slope longitudinal curvature on probability P_l is shown on fig.5. From fig.5 it could be seen the influence of average slope inclination on probability P_l too. At small average angles of slope the longitudinal curvature of profile increases the probability avalanche release P_l considerably. The probability of instability zones formation on convex and concave slopes of



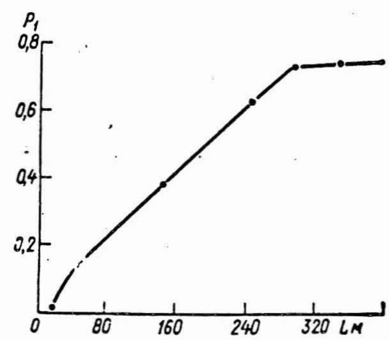


Fig.3. An example of dependence P_1 from c_v of h, ρ, c - parameters for real profile with average inclination 34° (curves 1,2,3 respectively) and for ideal even profile with inclination 40° (curves 4,5,6 respectively).

Fig.4. An example of slope length influence on avalanche release probability P_1. The slope is smooth and has an inclination of 40°.

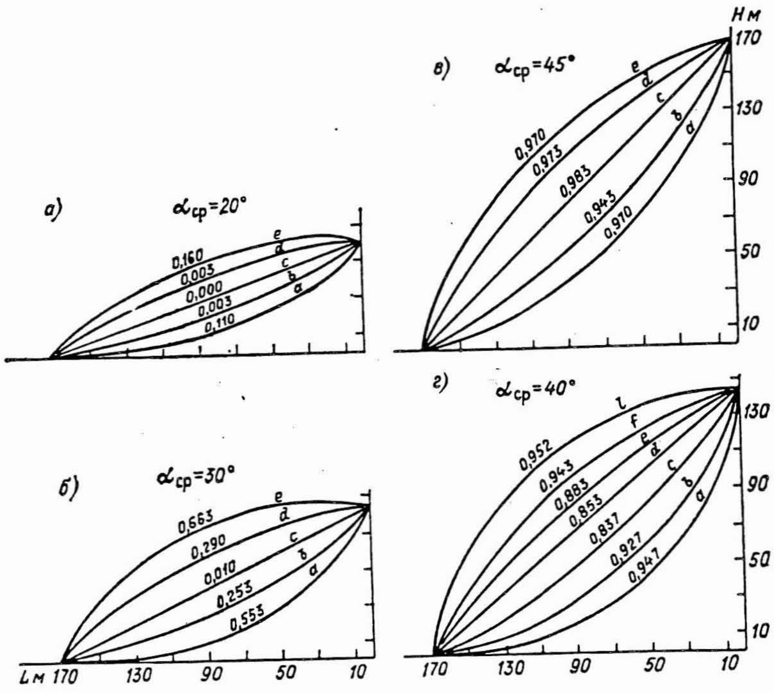


Fig.5. The influence of longitude profile curvature on formation of snowstorm snow instability zones. Numbers near the profiles are probabilities P_1.

identical curvature and with identical average inclination are approximately equal. However on convex slopes the maximal values P_k , corresponded to places of the most frequent avalanche releasing, are on their bottom parts and on concave - on top ones.

The considered method of avalanche danger diagnostic with statistical modelling is, in point of fact, detailed large scale avalanche forecast with range of forecast period equal zero. If to use as entrance parameters of the method forecasted values of snow parameters or extrapolate in time the results of diagnostic it is easily to transform the method in forecast in time.

Required range of forecast period at enough developed system of antiavalanche measures, as P.A."Apatit" experience has shown, can be enough short, usually a few hours.

At present, besides that on the basis of statistical modelling the entirely formalized methods of avalanche danger forecast can be developed, this approach can render the considerable help at interpretation snow-avalanche information by means of the most modern determined numerical models, lefts, however, acceptance of final decision about wording of forecast for experienced forecaster.

REFERENCES

1. Boginsky A.N. , Grigorian S.S. On snow cover equilibrium in mountain slopes. (In Russian) - Data of glaciological studies., M., 1978, N 34, p.p. 101 - 107.
2. Boginsky A.N. Unstability of natural ice and snow masses in mountain slopes. (In Russian) - The resuts of science and technics, glaciology., M, 1980, v. 2, VINITI, 122 p.
3. Boginsky A.N., Chernouss P.A. Probability model of snow stability on mountain slopes. (In Russian) - Data of glaciological studies., M., 1986, N 55, p.p. 53 - 61.
4. Yermakov S.M., Mickhailov G.A. Statistical modeling. (In Russian), M., 1982, Nauka, 296 p.
5. Panovsky H.A., Brier G.W. Some applications of statistic to meteorology. Pennsilvania, 1958, University Park, 209 p.
6. Chernouss P.A., Khristoyev Y.V. Accuracy evaluations of the data on snow thikness in avalanche catchments. (In Russian) - Data of glaciological studies., M., 1986, N 55, p.p. 201 - 206.