1. Introduction

The formation of weak layers in the snowpack of slopes causes avalanches. One of the most important weak layers is known as depth hoar due to vapor transport between snow grains. Vapor transport is produced by temperature gradient in the snow pack. Even in the case of a uniform temperature gradients throughout the snow pack, a layer of poor thermal conductivity makes stronger temperature gradient. Therefore this layer may quickly become depth hoar as a result of active transport of water vapor.

Generally, empirical models of the effective thermal conductivity have concentrated on snow density as the determining factor, since the density is surely a dominant parameter and is easily measured (Mellor, 1977). To explain measured scatter of thermal conductivity of snow, the influence of microstructure should be considered. We have begun to study the transport mechanisms of heat and water vapor with concerning microstructure, both theoretically and experimentally.

2. Thermal conductivity of snow

a) Idealized snow cover:

The idealized thermal conductivity model which has been developed assumes a collection of uniformly packed ice spheres (Adams and Sato, 1993). When regular uniform packing is assumed, every sphere will, theoretically, contact N other spheres. The coordination, N, will depend on the type of packing and the ice-volume fraction, $f_i$ (the fraction of the volume occupied by the ice).
In the case of a dry snow cover, this may be represented by $f_1 = \frac{\rho_s}{\rho_i}$, where $\rho_s$ and $\rho_i$ are the density of snow and ice, respectively. The relation of the coordination $N$ and snow density is shown as Figure 1. We see our model snow has slightly higher coordination number than natural snow pack, but the tendency is acceptable.

![COORDINATION NUMBERS](image)

Fig. 1 The relation of snow density and coordination numbers of ice grains. Black squares show the relation for the case of well-known regular packing, and the circles represent a curve fit for this data. Solid line indicates observed relation for natural snow pack (Edens and Brown, 1991).

In general for a solid material the thermal conductivity, $k$, is defined in terms of Fourier's Law for two temperatures, $T_1$ and $T_2$, separated by a distance $l$ as

$$q = \frac{kA}{l} (T_1 - T_2)$$

where $q$ is the heat transfer rate, $A$ is area, $T$ is temperature.

Heat transfer in a moist granular or porous material such as snow, however, does not take place by pure conduction. The material is composed of ice, air and water vapor which each contribute to the overall transfer rate. As shown in Fig. 2 heat may be transferred by

1) Conduction through the ice network, 2) Water vapor flux across the pore, 3) Conduction in interstitial space. In addition to these, convection and radiation may occur but are not considered to be significant in the present case.
Fig. 2 Ice sphere model for effective heat conductivity. Heat flow consists of three components; ice network, vapor series and pore space.

We calculated thermal resistance for the above heat transport, that is defined as

\[ R = \frac{1}{kA}. \]

The result for the total thermal resistance \( R_t \) is,

\[ \frac{1}{R_t} = \frac{1}{R_n} + \frac{1}{R_{ip}} + \frac{1}{R_p} \]

where \( R_n \) is the thermal resistance for the heat flow due to the necks, \( R_{ip} \) due to the vapor between ice grain and pore in series, and \( R_p \) due to the pore.

b) Results

The effective thermal conductivity for the idealized snow cover as developed above is calculated to reveal the detailed properties (Adams and Sato, 1993, Sato and Adams 1995). First, the importance of ratio of contact radius and grain radius is shown in Figure 3. The ratio means bonding thickness between ice grains. With increasing the ratio, conductivity due to ice network increases significantly and accounts for most of the total snow conductivity. However, if the ratio of contact radius and grain radius is small, say less than 0.04 in the Figure, conductivity due to vapor series becomes more important.
Fig. 3 The relation of effective conductivity and the ratio of contact radius and grain radius. Circles is a effective conductivity as a sum of three components.

For more detail on the relative role of conduction due to ice network and vapor series, calculations were made to find the dominant mechanism for different bonding ratios, densities, and temperatures.

Fig. 4 Diagram for dominant factor of conductivity in snow. For bigger bonding ratio, Ice network is dominant, and for smaller ratios, water vapor series become dominant. As the temperature increases, the boundary between these dominant mechanisms shows a trend toward lower bonding ratios.

In the diagram the region above curved line are the region where conduction due to ice network is dominant, on the other hand the region below the line the conduction due to
vapor series is dominant. With decreasing temperature, the area of dominance of conduction due to ice network increases. This is explained by the increase of conductivity with decreasing temperature while the conduction due to vapor series decreases.

**Effective Conductivity Diagram**

![Diagram](image)

Fig. 5 The diagram for an effective conductivity of snow.

The ratio of contact radius and grain radius as well as snow density define the effective conductivity of snow.

Finally, we show the increase of the effective conductivity with increase of both of density and ratio rc/rg as seen as Figure 5. We can see how the bonding ratio is as important as the density, which has been recognized well. A ratio of the neck radius to ice-sphere radius of 0.2, at a temperature of -5°C, yields a variation in which the effective thermal conductivity increases with density in a manner very similar to the empirical fit of Yen (1981).

### 3. Sintering

a) Physical model

Metamorphism is also studied as a result of water vapor transport in an idealized snow consisting of spherical ice particles (Brown, Edens and Sato, 1994). As a first stage, equitemperature metamorphism was investigated. The analysis models (1) the exchange of mass between grains of differing surface curvature and (2) the process of intergranular sintering. For the first process, it was assumed that mass exchange took place primarily by vapor transport between neighboring grains (Fig. 6).
The principles of mass balance, momentum balance and energy balance were used to evaluate time and spatial variations in temperature, vapor velocity, vapor pressure and mass exchange between the two grains. The calculation showed that grain size and temperature have an effect on the rate of metamorphism. Figure 7 illustrates the effect of grain-a radius on the rate of mass transfer. By the time the grain-a has been increased to a value of 0.1 mm it requires on the order of 1500 hours to be reduced to ten percent of its original radius. This is due to the vapor pressure being reduced by increasing curvature and to the increased mass involved as the radius is increased. In fact, this latter effect is probably the dominant effect in this case, since the mass increases with the cube of the grain radius.
For the second process of intergranular sintering, mass exchange was also assumed to be dominated by vapor flow, but in this case from the grain surface to the neck surface.

![Fig.8. Schematic of neck/grain geometry for calculating flux of vapor mass from the grain surface to the neck surface. The relative size of the grain and neck is not drawn to scale.](image)

Results obtained show that small grains will slowly lose mass to the larger grains, and this rate is affected by the pore size, relative size of the ice grains and the temperature. The rate of sintering, as determined by the rate of vapor deposition on the neck, is governed by temperature, grain curvature, and neck curvature. The movements of vapor along the grain surface to the neck surface is not a uniformly distributed process. Rather it is determined by the details of the surface curvature of the grain and neck.

b) Experiment

Along with the above theoretical physical modeling, a "standard snow" was produced by a newly developed machine designed for this study. In this study the snow consisted of fine-grained spherical ice particles with a mean grain size of about 30 micrometer with a standard deviation of 15 micrometer. As soon as the samples were made, the aging process started. One sample was immediately filled with aniline solution and frozen at -20 °C. This sample was then denoted with the initial time \( t = 0 \). The other samples were stored at -5 °C in a sealed container packed with snow to guarantee a vapor saturated environment. Samples were filled with aniline solution at times of 30 minutes, one hour, two hours, six hours, 12 hours, one day, three days, one week, two weeks, one month and two months and frozen at -20 °C to stop the metamorphism. The samples were then used to make surface sections to analyze the microstructure.

The surface sections were made by using water blue powder to stain the ice particles that were exposed on the section surface. Figure 9 shows a typical surface section. As can
be seen, the particles have a predominantly spherical shape, and the existence of bonds with adjoining necks can clearly be seen.

(a) Time = 0 hrs  
(b) Time = 3 days

Fig. 9 Digitized surface sections of model snow samples showing the evolution of grain size and intergranular bonding with time under equitemperature conditions. Scale shown is 300 μm.

For those digitized surface sections, two dimensional image analysis was conducted. The mean diameter of the ice grains indeed increased with time, as can be seen as Fig. 10. Of interest is the decrease of minimum diameter with time, which may imply that the small grains are sacrificed as the large grains acquire mass from the smaller grains.

Fig. 10 Grain growth analyzed from the surface sections.
Conclusions

This study consists of two parts, the effective thermal conductivity of idealized snow consisting of spherical ice particles, and the sintering modeling of snow with spherical ice particles. Both a theoretical and experimental study was conducted.

Calculated results for the effective thermal conductivity of idealized snow show a strong dependence on snow density. However, the ratio (rc/rg) of the neck radius to grain radius also plays a very significant role, and a variation with temperature is also observed. The effective thermal conductivity increases as rc/rg increases. Except at low ratio, rc/rg, conduction through the ice network is the predominant mechanism of heat transfer.

The sintering study was undertaken to determine the details of the process of equitemperature metamorphism. Physical models were formulated to calculate the movement of vapor and mass between ice grains and from ice grain surfaces to the bonds connecting the grains. Results show that the microstructure of the material does change slowly with time. Small grains will slowly lose mass to the larger grains, and this rate is affected by the pore size, relative size of the ice grains and the temperature. More work needs to be done to determine the temporal evolution of the grain/neck geometry during the sintering process.

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Reference