SNOW AVALANCHE DYNAMICS AS A GRANULAR FLUID PHENOMENON

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ABSTRACT

Theory and experiment are combined to produce a picture of how stresses in idealized granular materials depend upon the flow kinematics and material properties of an idealized granular material. The theory is based upon simple mechanical interaction of the discrete particles making up a rapidly shearing flow. Although still a long way from determining a general constitutive law for the motion of nonuniform particles such as snow and ice, the results obtained so far are indicative of the behavior that has been observed in avalanches for many years and give a mechanical foundation for many of the dynamic theories that have been used to predict avalanche speed and runout distance.

INTRODUCTION

Snow avalanche dynamics since the time of Voellmy (1955) has depended upon equations of motion which contain several empirical friction parameters. These parameters can not be measured directly, but are found by fitting the dynamic model of the motion to real data on avalanche runout. General relationships may then be inferred about how these parameters depend upon snow conditions, avalanche size, and path geometry, but each new avalanche represents a unique problem with unknown friction coefficients. To use the dynamic models effectively, previous experience with them is required to determine the appropriate values of the friction coefficients.

In the last 10 years there has been much progress made in the understanding of the rapid flow of granular materials. The work is based upon analyzing the interaction between discrete particles making up the flowing mass. The mechanics of individual particle collisions and the frictional rubbing between grains is extended over the entire flow domain to determine a general flow law for the granular material. This flow law is a function of the flow geometry and the measurable properties of the grains making up the flow, properties such as the size, shape, and coefficient of restitution of the particles. Since snow is made up of individual snow and ice grains, the dynamical theory of grain flow has application to the dynamics of avalanches.

In particular, if a restriction is made to flowing dry snow avalanches, many of the results of granular fluid flow are applicable directly. In this type of avalanche the interaction of the snow particles with interstitial water and air can neglected. Typically this type of avalanche starts as a slab of snow and is broken up as it travels down the avalanche path. To simplify the analysis, consider the flow after it has become well developed and is flowing on a smooth snow surface. Experiments by Dent and Lang (1982) have shown that the avalanche mass is moving as a slowly deforming body on top of a rapidly shearing layer of snow. This conclusion is reinforced by the description of the relatively smooth ride people have had when caught in an avalanche. Furthermore, evidence from rock avalanches has shown that large rocks on the surface of the avalanche are not significantly deformed, displaced relative to other rocks, or even rotated during the slide. For this situation the speed and runout of the avalanche is governed by the mechanics of the

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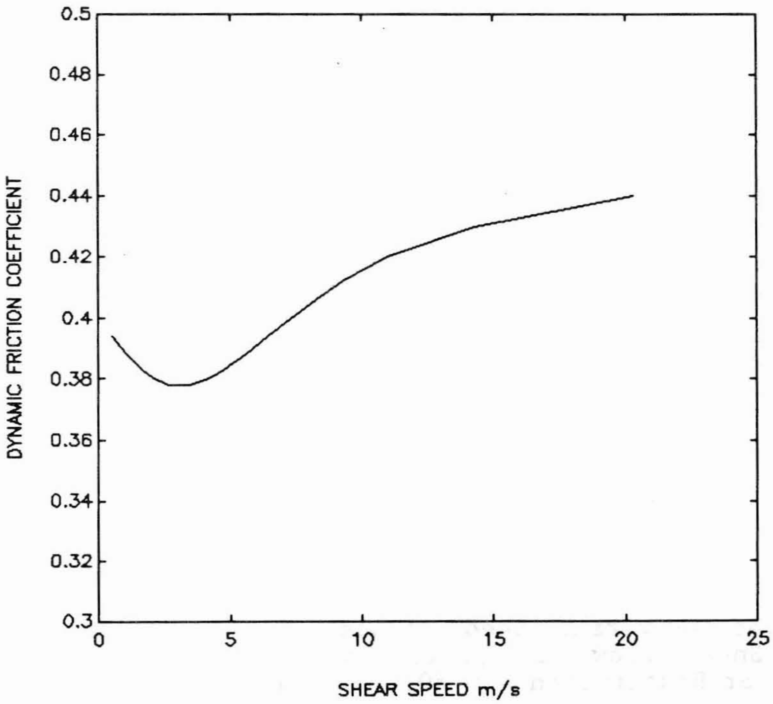
rapidly shearing layer of snow its base. At the base of the avalanche, the weight and rapid shearing causes the particles that make up the shear layer to be ground into their smallest constituent size, which for a snow avalanche is the size of the snow grains making up the snow fabric. Also, the shape of the particles will tend to be rounded or nearly spherical. To describe the bulk motion of the avalanche, it thus becomes necessary to determine the mechanics of this granular layer of snow.

GRANULAR FLOW MODEL

Granular flow models have only progressed to the point that descriptions of the mechanics are based upon the interaction of idealized particles. The analysis has been simplified by assuming that the particles are uniform, spherical, cohesionless, and frictionless. The mechanics of the inelastic particle collisions and intergranular friction are extended by suitable averaging techniques to include the entire flow domain of the rapidly shearing particles. This process is similar to the derivation of gas laws from the kinetic theory of gases. The results are a constitutive law and an equation of state for this simple granular material. To apply the results to the problem of snow avalanche motion, the balance equations of mass, momentum, and energy can then be solved for the two-dimensional shear flow of steady, uniform, gravity free grains.

Based upon the work of Jenkins and Richman (1985,1986), and Richman and Chou (1988), a boundary value problem is solved for the dynamic friction coefficient for the shearing of smooth particles between rough boundaries. This coefficient, which is the ratio of the shear stress to normal stress in the shear layer, is found as a function of the speed of the avalanche, the depth of the avalanche, and the material properties of the snow grains. A typical solution is plotted in figure 1. The normal stress of 1000 Pa corresponds to an avalanche 0.33 m deep, and the coefficient of restitution is a little larger than what has been measured for ice.



Figrure 1: Granular Flow Theory Calculation of Friction versus Speed for Simple Shear Flow (Normal Stress - 1000 N/m^2, Coefficient of Restitution - 0.80, Particle Diameter - 0.001 m)

GRANULAR SHEAR FLOW - NUMERICAL SIMULATION

The equations that result from the granular fluid models are quite complex, even in the simplest case of the uniform shear of smooth particles. Solutions can only be obtained by numerically solving the set of nonlinear differential equations that result. Another approach to this problem is to construct a numerical model of the interacting particles. In this model the interactions of all the particles are kept track of and used to determine the motion of each particle in the flow as a function of time. Simple models of collision and friction are used to determine the forces between particles. Forces then determine the accelerations of the particles, and the accelerations can be integrated to find the velocities and positions. Flow properties may then be averaged over all the particles to determine quantities like the stresses and velocity gradients in the flow. This technique allows the constraints on particle uniformity, sphericity, and the absence of cohesion to be relaxed. The limiting factor for this technique becomes computing power, the ability to keep track of all the particles and the forces between them. Typically, on a newer microcomputer and most minicomputers, arrays of 25 to 1000 particles can be followed in reasonable lengths of time (hours). Using so-called periodic boundary conditions, where the array of particles is assumed to be reproduced over and over again to form a long flow of shearing particles, an array of 25 particles, 5 deep by 5 wide, can be made to simulate a two dimensional shear flow 5 particles deep and infinitely long. This technique has been used to calculate the average dynamic friction coefficient for the rapid shear of uniform, cohesionless spheres. Again the friction is calculated as a function of the shear speed, normal stress applied to the flow (corresponding to the depth of the avalanche), and the properties of the particles making up the flow. A typical result is shown in figure 2.

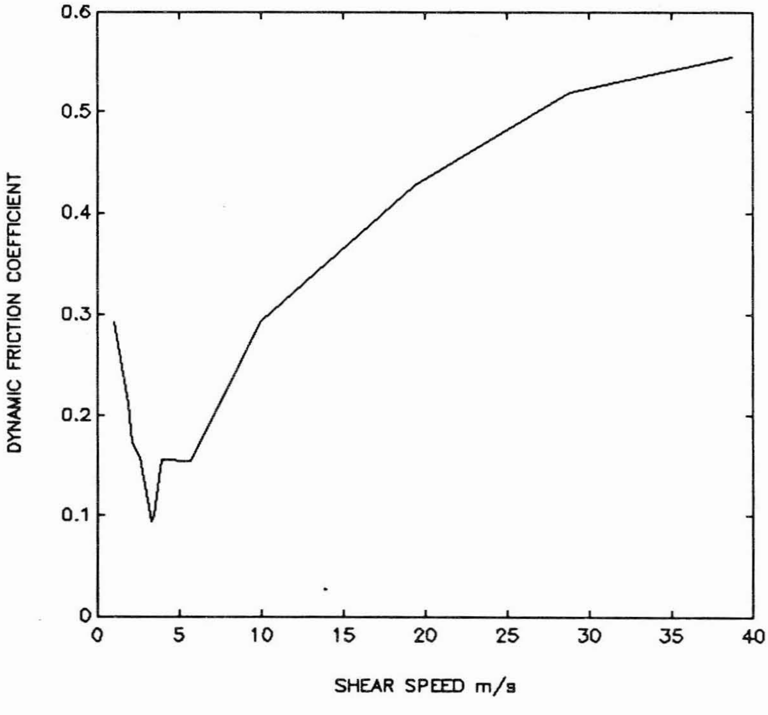


Figure 2: Friction versus Speed Calculation from the Numerical Modeling of Simple Shear Flow (24 particles, Normal Stress - 1000 N/m^2, Coefficient of Restitution - 0.80, Particle Diameter - 0.008 m)

GRANULAR SHEAR FLOW EXPERIMENTS

To confirm the results of the granular fluid theory and the numerical simulation of two-dimensional shear flow, and to gain a better understanding of the mechanics of granular flow, an apparatus was constructed to produce a two-dimensional shear flow. This instrument, which is called an annular shear cell, was used to produce two dimensional shearing of uniform 8 mm acetate beads. This apparatus consisted of two concentric cylinders of approximately 0.5 m diameter separated by a gap just large enough to accommodate one bead. The annular region between these two cylinders was then filled with beads to any desired depth up to 0.15 m. These cylinders were then spun together as a unit at a known speed by an electric motor. Another weighted skinny cylinder was lowered into the annular region between the first two cylinders until it came in contact with the top surface of the beads. This cylinder was kept from spinning by a mount that was instrumented to record the torque transmitted to it. Since the bottom beads were in contact with a surface that was rotating and the top beads were in contact with a surface that was stationary a shearing of the beads took placed. To aid in the establishment of the shear flow, the cylinder surfaces in contact with the top layer of beads and the bottom layer of beads were roughened by gluing a row of beads to each of the surfaces. The apparatus then produced a simple two-dimensional shear flow consisting of a vertical cylinder of beads, 1 bead in diameter, and as high as the number of beads that were put into the annulus. From the measured torque transmitted to the top surface from the spinning bottom surface through the cylinder of beads, the shear stress can be calculated as function of the rate at which the system is spun and the amount of weight applied by the top cylinder. That is, the dynamic friction coefficient can be found as a function of the normal stress and the shear speed as was done in the analytical model and the numerical simulation before. Typical results are shown in figure 3. These results correspond to the same parameters as were modeled in the results shown in figure 2.

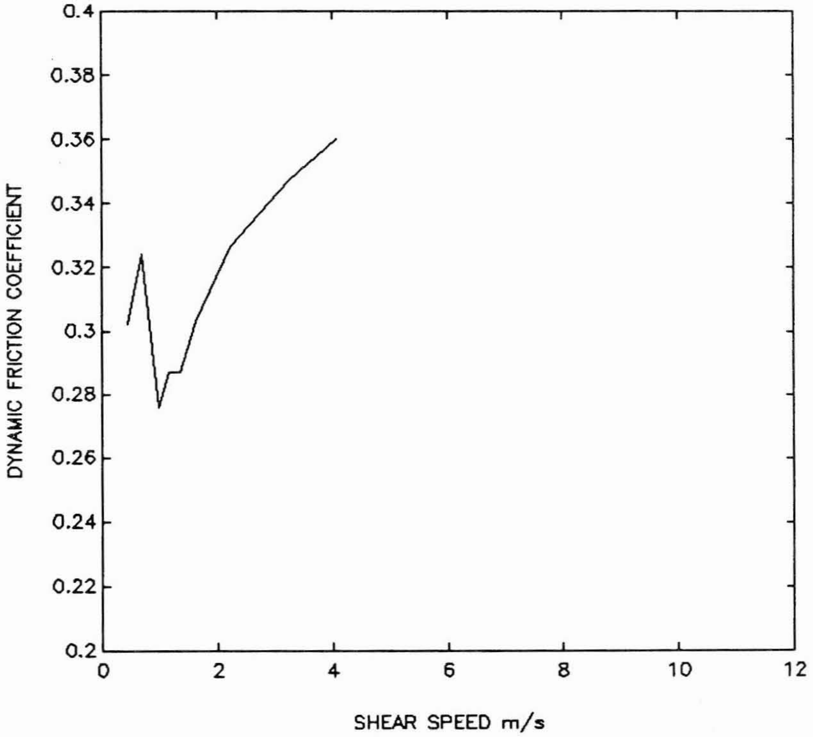


Figure 3: Friction versus Speed for the Annular Shear Cell Experiment (Normal Stress - 3094 N/m^2, Coefficient of Restitution for 8 mm Acetate Beads - 0.82)

The results shown in figure 3 do not match precisely the results from the numerical model as shown in figure 2. This is due in part to a calibration error in the annular shear cell force measuring device. The error was inadvertently introduced by the method in which the load cell that was used to measure the force transmitted to the stationary top cylinder was mounted. The results are still correct qualitatively, but it is thought that a correction of this problem would increase the quantitative agreement as well between figures 2 and 3. A redesign of the load cell mount is in progress.

DISCUSSION

Significant differences can be seen between the theoretical result shown in figure 1 and the modeling and experimental results shown in figures 2 and 3. This difference is due to more than just the different particle diameter used in the two cases studies. It is a reflection of the simplifications that have been made in the analytical model. Friction between particles is neglected in the theory but not in the simulation or experiment. Further, collisions between only two particles at a time are considered in the theory. This particularly influences the results at low speeds where particles are squeezed together and often contact several of their neighboring particles. It is believed that this is primarily responsible for the smaller relative minimum observed in the plot in figure 1 when compared to the plots in figures 2 and 3. In any case, as can be seen from the 3 figures, the results of the analytical modeling, the computer modeling, and the physical experiment all give curves of approximately the same shape for the dynamic friction as a function of speed. The dynamic friction coefficient decreases from some initial value to a minimum and then increases as the shear speed is increased. These curves indicate that for a fully developed shear flow, the dynamic friction is a definite function of the shear speed, contrary to the results published by Bagnold in 1956.

Further study has also shown that the friction coefficient in all three studies is a decreasing function of normal stress when the speed is held constant. Figure 4 shows the result for one set of annular shear cell experiments. Similar results are found in the numerical simulation and the analytic model.

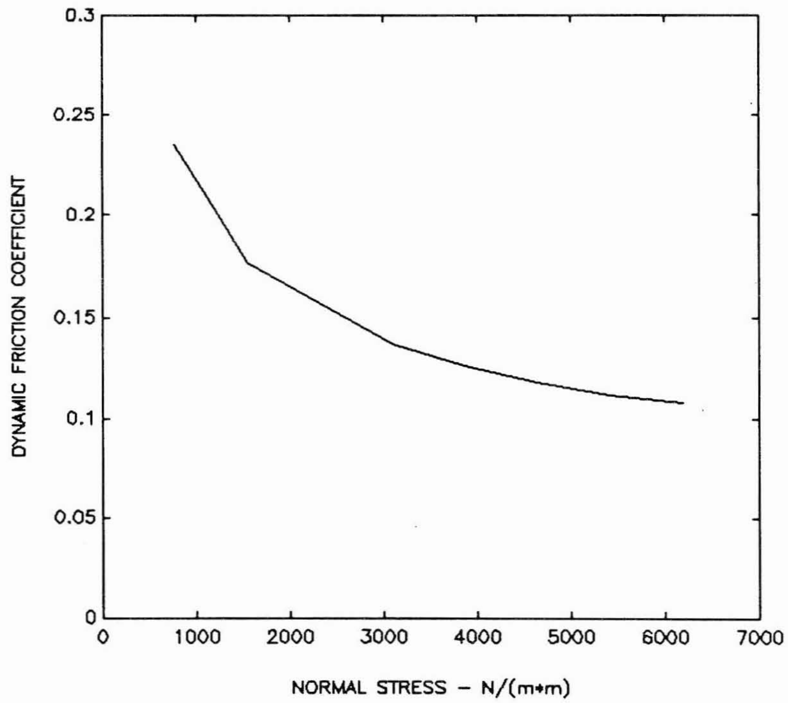


Figure 4: Friction versus Normal Stress for Annular Shear Cell Experiment (Shear Speed - 2.34 m/s, Coefficient of Restitution for 8 mm Acetate Beads - 0.82)

Relating the given results to snow avalanches, if it is assumed, as discussed earlier, that avalanche motion is governed by a rapidly shearing layer of granular snow at the base of the avalanche then that layer should behave similar to the models of granular shear flow. The dynamic friction coefficient for snow avalanche motion should initially decrease and then increase with the speed of the avalanche. Additionally the dynamic friction should decrease with the increasing size of the avalanche.

Certainly the initial decrease of the friction coefficient is to be expected or the avalanche would find it difficult to get started. The static friction angle, measured by the angle of repose of a pile of grains, even for a frictionless granular material, is greater than the slope angle of the typical avalanche starting zone. So in order for the avalanche to start there must be a drop in the friction from this static value.

The increase in the dynamic friction after the minimum at some speed is shown to be a non-linear function of the increasing speed of the avalanche. These results are qualitatively in agreement with all the previously proposed models of avalanche motion, all of which are based upon empirically derived flow laws and have been used for many years to model avalanche motion. As an example consider Voellmy's equation for the speed of an avalanche. In addition, incorporate the suggestion of Schaerer (1975), that the static friction coefficient in this equation be dependent upon the velocity of the avalanche in an inverse manner. The equation that results for the speed of an avalanche can be inverted and solved for the equivalent dynamic friction coefficient as a function of the speed of the avalanche. This result is plotted in figure 5.

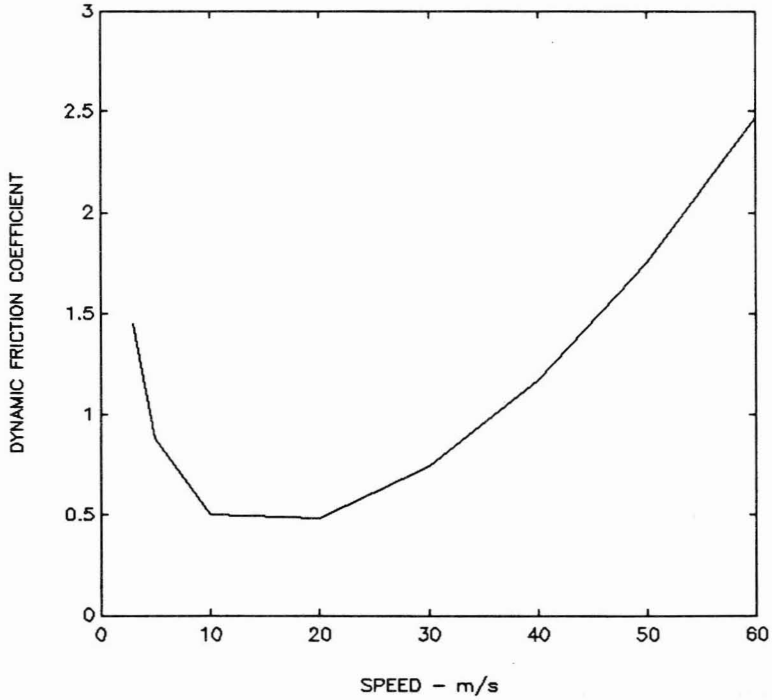


Figure 5: Friction versus Speed from the Voellmy Model of Snow Avalanche Motion (slope - 30 degrees, depth of avalanche - 1 m, turbulent friction coefficient - 1500, static friction - 5/speed)

Again the shape of the curve is similar to the shapes derived from the granular flow models, a decrease in dynamic friction with speed and then an increase. Care should be exercised here by noting that the curve given by figure 5 is not the only flow law that can be used to model avalanche motion, accurate predictions of avalanche runout can be obtained by other models. Still it is obviously that the model plotted in figure 5 differs greatly in the magnitude of the friction predicted by the granular flow models. This indicates that there must be other mechanisms beside the simple shearing of granular snow at the base of the avalanche that must be considered when modeling avalanches. Possible mechanisms include cohesion and the locking of the granular snow into a solid matrix at very low speeds. At high speeds, air drag, snow entrainment, and most importantly the effects of terrain variations on the formation and collapse of the granular shear layer would all increase the dynamic friction. A general constitutive law for flowing granular material coupled with a numerical solution of the flow field corresponding to terrain geometry would solve this last problem. The other mechanisms mentioned occur as boundary conditions in this model. The equations that describe granular shear flow have not progressed to the point that this kind of model is possible, but progress is being made in that direction.

Finally note should be made of the relationship between the size and the speed of an avalanche predicted by the granular models. The decrease in the friction coefficient as the normal stress on the shear layer is increased, provides a mechanism to account for the high speed and long runouts observed in the largest dry snow avalanches. The larger the avalanche, the greater will be the normal stress on the basal shear layer. This will result in lowering the effective dynamic friction on the base of the avalanche, which will allow higher speeds to be achieved.

It should be emphasized again that the current granular fluid model is valid at the base of the avalanche only in the case where terrain allows the shear layer to develop. Furthermore there are several additional effects such as entrainment, that may be as important or more important in determining avalanche motion at certain speeds. The granular fluid models presented are but a first step in the quest for a rationally based model of avalanche motion. The models are extremely simple in concept, but extremely difficult to work with, even with all the simplifications. However, progress is being made, more complicated grain configurations and more complicated flow geometries are being considered. It is hoped that eventually a model of the basal friction force on an avalanche can be determined. One that is still based upon easily measured parameters such as the size and shape of the snow grains and their coefficient of restitution.

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