TIME SERIES MODELS FOR AVALANCHE HAZARD EVALUATION

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Nomenclature

Y	Index of avalanche activity (the total number of medium and large avalanches in any twelve-hour period).
$\hat{Y}$	Predicted value of Y
R	Simple or multiple correlation coefficient
N	Twelve-hour snowfall (cm)
W	Twelve-hour precipitation (mm)
S	Depth of snowpack (cm)
H	Relative humidity (%)
V	Wind speed (km/h)
t	time (subscript)
e_t	Random uncorrelated residual error at time, t

Introduction

Avalanche hazard evaluation by field analysts mainly involves the recognition of particular combinations of meteorological factors which have resulted in certain levels of avalanche activity in the past. The success of this approach depends to a large extent on the interpretive skill and experience of the analyst in his own particular area. Most of the past data, which forms the basis of the analyst's experience, is of either a numerical nature or can be coded numerically. In principle, it should, therefore, be possible to quantify the field approach by the development of mathematical models, based on this past data. Such models could then be used to predict levels of avalanche activity with an accuracy comparable to that of the field analyst.

Avalanche Activity and Meteorological Variables

An index of avalanche activity, Y, for the Rogers Pass area is defined, for the purpose of this study, as the total number of medium and large avalanche occurrences recorded once every 12 hours (12-hourly data). Medium and large avalanches include only avalanches that enter runout zones. Sluffs and small releases confined to starting zones are excluded from this study.

The identification of suitable meteorological factors, which can be used to describe the various levels of avalanche activity found in past data, is most easily achieved by performing a simple correlation analysis for each of the variables in turn.

Simple terms involving snowfall and precipitation amounts, air temperatures, wind velocities and relative humidities were studied together with more complex terms, mainly cross-products and quotients of the simple terms (Salway, 1976).

Simple Regression Models

New snowfall (N, measured in cm) might be a good indicator of avalanche activity and the simplest model that can be produced with one independent variable is, of course, a simple linear regression.

Using 12-hourly data from Rogers Pass for the years 1965-73, and omitting spring avalanching, avalanche activity indices were computed for all avalanche sites over the entire area, and the following model was obtained:

[1] $\hat{Y} = 0.522\ N, \qquad R^2 = 0.310$

The R^2-value (correlation coefficient squared) provides an indication of the amount of variation explained by the regression. In terms of percentages, the model explains 31% of the variation in avalanche activity. This value, although quite significant for the 1522 data cases, is not spectacular and equation [1] could not be employed as a reliable forecasting tool.

A better result is obtained if precipitation (W, measured in mm) is used instead of new snowfall, N. The model then becomes;

[2] $\hat{Y} = 0.703\ W, \qquad R^2 = 0.334$

indicating that precipitation is a better predictor of avalanche activity for Rogers Pass than is new snowfall.

However, to make a really significant gain, the precipitation term, W, should be multiplied by the depth of snowpack (S, measured in cm), in which case:

[3] $\hat{Y} = 0.00507\ SW, \qquad R^2 = 0.446$

It is quite apparent that the level of avalanche activity for a given amount of precipitation is influenced by the depth of snowpack, as measured at the Rogers Pass study plot. This result was obtained in spite of the fact that data obtained prior to the date of minimum snowpack depth required for the start of the avalanche season were not included in the analysis. In other words, small depths of snowpack, in conjunction with zero avalanche activity, were not permitted to contribute to the R^2-value.

A further small but significant gain can be realized if relative humidity, H, is used as another modifier. The model is the:

[4] $\hat{Y} = 0.0000613\ SWH, \qquad R^2 = 0.453$

The subtle influence of H may be real or merely the result of random correlations in the particular set of data studied. Further investigations will have to be made before it can be said with any degree of certainty that relative humidity really is an important factor.

The model described in equation [4] would do quite well in an actual forecasting situation.

Multi-Linear Regression Models

At this stage, it might now be felt that a multi-linear regression or discriminant approach would lead to even better models. However, the usual backwards, forwards or stepwise selection procedures, employed in normal least-squares regression and discriminant analysis, break down if strong intercorrelations exist among the independent variables. As Judson and Erikson (1973), Bois et al (1974) and Bovis (1974, 1976) have discovered, such conventional approaches can lead to complicated but relatively weak models, peculiar to the particular data sets analyzed, consisting of large numbers of interrelated meteorological terms, many of which are only just significant. The non-inclusion of time lag decay terms, autocorrelations in the data, insufficient variation in the dependent variable and sampling difficulties further combine to weaken the

discriminant approach (Salway, 1976). These problems and the nature of the phenomenon suggest that a time-series approach is required.

The Time Series Approach

Time-series analyses (Box and Jenkins, 1970) are designed to operate on observations which are dependent and for situations in which the data are both inter- and auto-correlated. In order to illustrate the time-series procedure, the development of a simple model will now be discussed.

Firstly, a suitable independent (or input) time-series is selected such as the precipitation-series. The best model describing this series in terms of its past values is identified and estimated. For the 1965-73 data the model is:

[5] $$W_t = 0.383\ W_{t-1} + e_t,$$

Current 12-hourly values of precipitation are described in terms of previous 12-hourly values. This model is used to reduce the series to a more nearly random, uncorrelated one in order to facilitate the identification and estimation of the transfer function (or relationship) between the Y-series and the W-series. For the data in question, this is:

[6] $$\hat{Y}_t = 0.609\ W_t + 0.097\ W_{t-1}$$

Stochastic noise (or correlated noise) is now added and the model re-estimated using a least-squares approach to give the final result:

[7] $$\hat{Y}_t = 0.633\ W_t + 0.173\ Y_{t-1} + 0.074\ Y_{t-2}$$

As can be seen, this model involves previous values of avalanche activity which are to influence the current activity. The importance of including such factors has long been recognized by field analysts. The contribution from these lagged terms is illustrated by the increased R^2-value, compared to that for equation [2] employing W alone, which was 0.334.

Moreover, the univariate models (equations [1] - [4]) indicated that SWH is much more highly correlated with avalanche activity than is W alone. Therefore, as expected, a far better time-series model can be produced by replacing

the W-series by the SWH-series. Furthermore, if a secondary input series, such as the V^2-series, (wind speed squared; wind measured in km/hr) is introduced along with the primary SWH-series, the best model obtained so far using 12-hourly data can be produced. After application of appropriate time-series procedures, this model can be written:

$$[8] \qquad \hat{Y}_t = 0.0000536\ SWH_t + 0.000633\ V_t^2 + 0.115\ Y_{t-1} - 0.000421\ V_{t-1}^2$$

$$R^2 = 0.477$$

Time Profile Comparisons

Model performances can be compared by plotting actual values of avalanche activity, in the form of a time profile, along with predicted values using each of the models in turn. A section of such a plot, for the 1971-72 season, is shown in Fig. 1, where it can be immediately seen that the (SWH, V^2)- model predictions, (equation [8], $R^2 = 0.477$), agree quite closely with the actual values. The N-model predictions, (equation [1], $R^2 = 0.310$), have been included in order to illustrate how much more powerful the time-series model is than the simple regression based on N.

Interpretation of the "Best" Model

Field analysts, possibly intimidated by the appearance of equation [8], will undoubtedly question the practical application of such a model in a field situation. However, these doubts are easily dispelled when it is realized that the model can be simply interpreted in terms of popular "rules of thumb" of the type commonly used by such workers. Table I contains sets of minimum meteorological conditions which, using equation [8], would give rise to a prediction of three medium and/or large avalanches. This might be regarded as a prediction of moderate hazard which could result in a decision to perform artillery control or close an area to ski-tourers. It must be remembered throughout the following discussion that the model describes average, and not unusual, conditions that give rise to avalanching. As can be seen from Table I, 8.8 mm of precipitation in any 12-hour period, in combination with a snowpack depth of 100 cm and average humidity of 70%, in the absence of current wind, previous wind and avalanching, gives rise to significant avalanching in the Rogers Pass area. Half this precipitation, 4.4 mm, produces the same amount of avalanching if the snowpack depth is doubled to

200 cm. An increase in humidity from 70% to 90% significantly lowers the amount of precipitation required to 3.4 mm. Returning to a snowpack depth of 100 cm, and relative humidity of 70%, a 12-hour average wind speed of 24 km/hr lowers the amount of precipitation required for moderate hazard from 8.8 mm to 7.8 mm. A wind speed of 48 km/hr decreases the required precipitation amount to 4.8 mm and 64 km/h to 1.8 mm. If the average 12-hour wind speed is 72 km/hr, a moderate hazard exists with zero new precipitation. The V^2 terms in equation [8] can be regarded primarily as measures of the amount of snow drifting into the accumulation areas of the avalanche sites.

Finally, referring to the last line of Table I, it can be seen that a previous 12-hour average wind speed of 48 km/hr in combination with previous avalanching of 3, causes an increase from 4.8 to 6.5 mm of the amount of precipitation required for moderate hazard when snowpack depth is 100 cm, relative humidity is 70% and current 12-hour windspeed is 48 km/hr. This is an expected result since sites which have unloaded during a previous 12-hour period are less likely to avalanche during the current period.

Table I can be expanded to any length and degree of complexity by substituting further sets of values for the terms in equation [8]. However, since the practical applicability of the model has now been demonstrated, it is far easier to apply it directly in an actual forecasting situation by substituting real values for the meteorological terms.

Model and Field Analyst Compared in Terms of Shoot Performance

In order to ascertain whether the mathematical model would perform as well or better than an experienced avalanche analyst in the field, it is obviously necessary to make some kind of operational comparison. Perhaps the most important function of a field analyst, certainly with respect to the Rogers Pass, is that he correctly identify optimum periods for the implementation of artillery control measures.

It is possible to compare the model with the field analyst by devising a success score based on the proportion of 12-hourly periods during which a particular hazard level, e.g., $Y \geq 3$ was correctly estimated. Studying the 1522 12-hourly periods for the years 1965-73, it was found that the analyst called for artillery control during 248 out of the 1522 periods. Assuming that artillery control is scored successful when the resulting total avalanche index is 3 or greater ($Y \geq 3$), then the analyst scored 155/248 or 67% for the 1963-73 period.

Applying the model given by equation [8] to the same periods, it is found that [8] predicts $\hat{Y} \geq 3$ for 297 periods. Within these 297 periods, $Y \geq 3$ during 162 periods, for a score of 162/297, or 55%.

Although the field analyst outperforms the numerical model, it is encouraging that the scores are comparable. Moreover, if the scores were based on predicting a higher intensity of activity (e.g., $Y \geq 4$ or $Y \geq 5$), then the model's score would approach quite closely the field analyst's score.

Conclusion

In view of the promising results obtained so far, work will continue on the development of more sophisticated time-series models for avalanche hazard evaluation. Ultimately, as the data base is expanded, and more frequent observations of avalanche occurrences and meteorological parameters become available, it should be possible to evaluate the specific hazard for individual avalanche sites, as well as overall hazard, on perhaps a two-hourly, rather than a 12-hourly basis.

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Discussion

WILLIAMS: Humidity appears as a contributory factor in your model. Can you give a physical reason why humidity could be important?

SALWAY: This is a controversial topic. There are several theories on the subject. For example, during wind transport, high humidity would suppress sublimation, leaving more snow to be redistributed in starting zones. Then there is Seligman's idea that snow sinters more readily into a slab-like texture when the humidity is high.

KOEDT: There appears to be a strong similarity, except for a Y-displacement, between the N-model predictions and the (SWH,V^2)- model predictions for February 1972, as depicted in Fig. 1.

SALWAY: This merely reflects the importance of the primary term SWH_t in the (SWH,V^2)-model and the fact that precipitation and new snowfall are closely related, and that snowpack depth does not vary radically during this particular month. Much greater differences between the N-model and the (SWH,V^2)-model would be apparent if time profiles for December and January had also been plotted along with February, but limitations of space did not permit this. A thorough comparison would demand that time profiles for all eight years from 1965-73 be examined, in which case it would be clear that the (SWH,V^2)-model (R^2=0.477) is far better than the simple N-model (R^2=0.310).

PERLA: The R^2-values of your model seem quite low for practical application.

SALWAY: I presented R^2-values at this workshop because they are the simplest numbers to illustrate progressive optimization of the model. Other statistical checks verify that the model has the capability to predict results of artillery shooting at a level comparable to the subjective forecast of the field analyst.

TESCHE: Your regression equation includes a wind term with a negative coefficient which indicates that wind is negatively correlated with avalanches. This seems to be contrary to the general idea that avalanche activity should increase with wind speed. What is your physical interpretation of the negative wind term?

SALWAY: If you examine the equation, you will note that avalanche activity is positively correlated with current wind speed, but negatively correlated with wind during the previous twelve hours. Perhaps the physical interpretation is that a slab condition is produced by a stiff over a weak layering. You would tend to have this if you had a strong wind during the current twelve-hour period, which followed windless conditions of the previous twelve hours. Conversely, if you had strong winds in the past, you would tend to build a stronger, more supportive base. Now, if you look again at the equation, you will see that the current wind speed term dominates. Thus, if you have the combination of strong current winds, and strong winds during the previous twelve hours, there will still be an increased probability of avalanches, as one would expect from experience.

Another possible interpretation is that high winds, during the previous 12-hr period, would tend to produce significant avalanching at a number of sites, thereby decreasing the probability of avalanching during the current twelve-hour period.

TABLE 1

Combinations of Minimum Meteorological Conditions Giving Rise to a Moderate Avalanche Hazard, $Y_t = 3$, Using Best Model Equation [8]

Depth of Snowpack cm S_t	12-Hour Precipitation mm W_t	Average 12-Hour Relative Humidity H_t	Average 12-Hour Wind Speed km/hr V_t	Number of Medium & Large Avalanches During Previous 12 hours Y_{t-1}	Average 12-Hour Wind Speed During Previous 12 hours mph V_{t-1}
100	8.8	70	0	0	0
200	4.4	70	0	0	0
200	3.4	90	0	0	0
100	7.8	70	24	0	0
100	4.8	70	48	0	0
100	1.8	70	64	0	0
Any	0	any	72	0	0
100	6.5	70	48	3	48

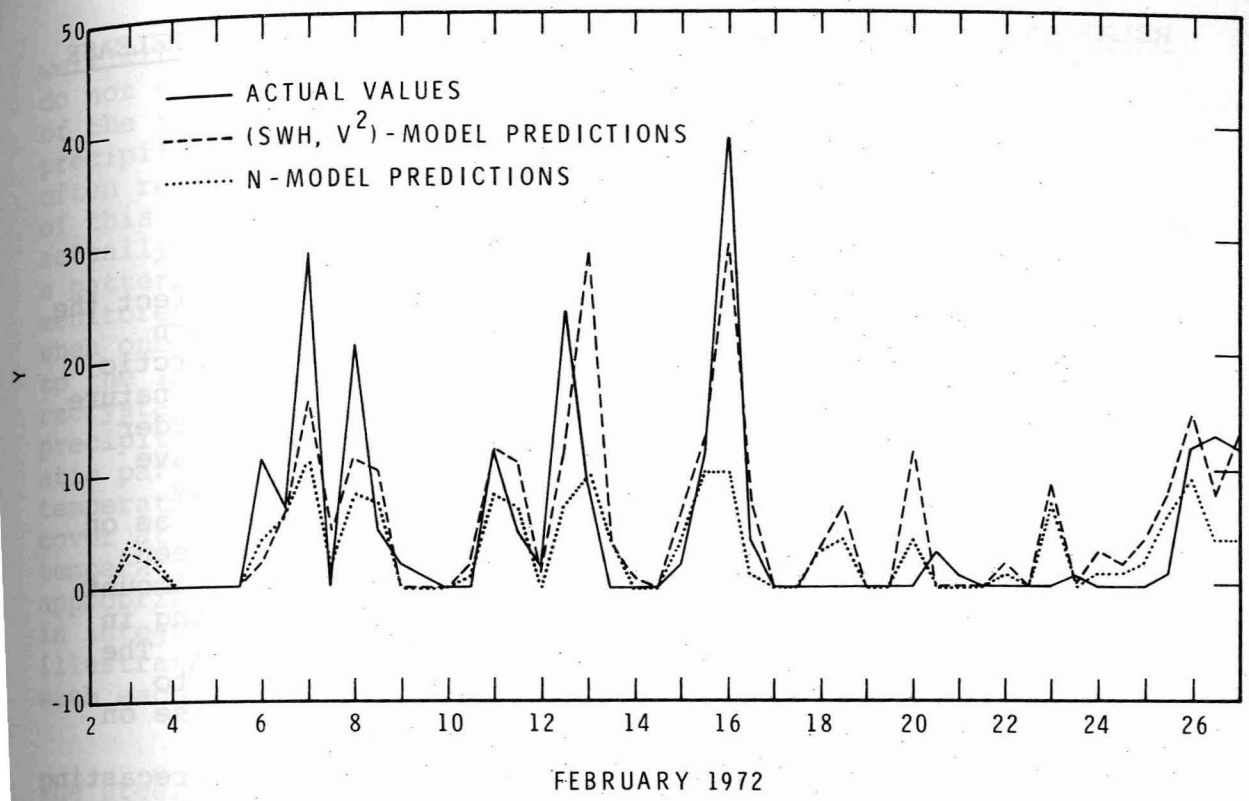

FIGURE 1 MODEL PREDICTIONS COMPARED TO ACTUAL VALUES OF AVALANCHE ACTIVITY INDEX FOR FEBRUARY, 1972