NEW INSIGHTS INTO SKIER-TRIGGERING OF SLAB AVALANCHES

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ABSTRACT: Most skiers trigger the slab avalanche in which they are caught. Preventing those accidents necessitates a better understanding of the factors contributing to the failure of the snowpack under the action of a skier. Our contribution gives a number of insights into a newly developed mathematical model of skier triggering based on the principles of mixed-mode anticracking. We give various examples of how the direction of the applied force and the penetration depth of the skis influence the chances of triggering fracture in the weak layer, and we investigate how the skier’s stance influences that risk. We also ask how the critical loads for triggering fracture depend on slope angle in general. We find that, for weak layers prone to anticracking, fracture is not easier to trigger on steep slopes, but is equally difficult or marginally more difficult to trigger the steeper the slope. We carried out extensive field experimentation to test this proposition using the extended column test method, the detailed results of which are given in a companion paper. As usual, our formulation includes simple shear cracking as a limiting case, so that the anticrack model is well-suited to emphasize the differences between the two fracture mechanisms. The results emphasize that the anticrack mechanism for fracture in snow requires scientists and practitioners to rethink previously accepted - and practically relevant - paradigms.

1. INTRODUCTION

Traditionally, the approach to skier triggering of snow slab avalanches has been based on the principle that the snowpack is about to fail when the ratio of shear strength to shear load falls below one, at least in principle (Föhner, 1987, “stability index”). However attractive the idea of a well-defined stress limit for fracture initiation may appear, it does not usually stand the test of reality. Fracture mechanicians abandoned this concept very early (Griffith, 1920). In snow science, the simple approach of the stability index was more influential and the concept is still widely in use. However, as we shall see, the usually large fluctuations of snow mask some pitfalls of the principle. Further progress must come from an improved understanding the mechanisms of skier triggering.

In order to take the next step, the authors propose a physical model of skier triggering based on the principles of fracture mechanics and anticracking (Heierli et al., submitted). Anticracks, as opposed to simple shear cracks, allow for mechanical work to be done by the compressive component of the load since the collapse of the weak layer provides a small but sufficient room for slope-normal displace-
2. MODEL CALCULATION

Consider the simple situation shown in Fig. 1. A uniform slab of constant thickness \( h \) and density \( \rho \) rests on a persistent weak layer with fracture energy \( \mu_0 \). We assume that the slab material is linear-elastic and deforms in plane strain, and that the slab and the substrate have comparable stiffness \( E \) and Poisson’s ratio \( v \). A skier with instant position \( x_0 \), is assumed to act as a line load \( p \) applied at a depth \( a \) of the snowpack. This depth corresponds to the penetration depth of the skis (Jamieson, 1995).

The slope-parallel and slope-normal components of the load are denoted by \( p_x \) and \( p_y \), respectively. The static load \( p_0 \) of a motionless skier is typically around 400 N/m (e.g. a skier of mass 75 kg on 1.80 m skies). Due to the accelerated motion while skiing, the instant magnitude \( p \) can be larger than \( p_0 \), and its direction \( f \) can be arbitrary (see Fig. 1). The stress field exerted by the skier on the weak layer can be estimated analytically for a linear isotropic material (Melan, 1932). We call \( \sigma(x) \) the slope-normal component of the stress induced by the skier in the weak layer plane at a point \( x \) and \( \tau(x) \) the shear component of the same. We note that \( p_y \) induces both a shear component and a compressive component of stress in the weak layer plane, and so does \( p_x \).

The two main assumptions of the physical model are i) brittle failure and ii) crack propagation by mixed-mode anticracking of the instant crack tip. With these two ingredients, the evaluation of the critical crack size \( l \) for a given load \( p \) and/or the evaluation of the critical applied load \( p \) given flaws of a certain size \( l \), reduces to a mathematical task. For the stress intensity factors we find, somewhat simplifying:

\[
\begin{align*}
K_{II}(x_0) &\equiv K_{II}^0 + \sqrt{\pi \tau} \left( \sigma_0 + \frac{1}{2} \sigma_1, \tau_0 \pm \frac{1}{2} \tau_1 \right), \\
K_{III}(x_0) &\equiv K_{III}^0 + \sqrt{\pi \sigma} \left( \sigma_0 \pm \frac{1}{2} \sigma_1, \tau_0 \right),
\end{align*}
\]

where \( \sigma_n(x_0) = \partial^n \sigma(x)/\partial x^n \big|_{x=0}, \quad \tau_n(x_0) = \partial^n \tau(x)/\partial x^n \big|_{x=0} \) for \( n = 0, 1, \quad r = \ell/2 \), and \( K_{II}^0, K_{III}^0 \) are the stress intensity factors in absence of the skier. Using the standard methods of fracture mechanics and with the additional directive of finding the worst-case position \( x_0 \) of the skier, the critical crack size is thus established. For further mathematical results, see Heierli et al. (submitted). Yet, this answer is not usually enough in practice. Even if the critical applied load can be estimated, we are still left with the question of how often the skier may hit such a flaw (and trigger fracture if hit with the necessary force), and how often he or she will not (and trigger nothing).

To outline the answer, we consider two scenarios: one with critical crack size \( l_1 \), the other with critical crack size \( l_2 \). If equal loads are applied, which scenario is riskier and approximately by how much? Assuming the bonds in the weak layer to be positioned randomly with an average density of \( \lambda \) bonds per unit length (e.g. \( \lambda = 150 \text{ m}^{-1} \) for buried surface hoar or large depth hoar), the average distance \( d(l) \) to be covered by the skier to hit a defect of length \( l \) or more is a priori \( d(l) = \exp(2l)/\lambda \). The relative risk of scenario 2 with respect to scenario 1 can be written in terms of \( m = d(l_2)/d(l_1) \) (Heierli et al., submitted),

\[
m = \exp\left[ \lambda(l_2 - l_1) \right].
\]

This simple relation says that scenario 2 is \( m \)-times riskier than scenario 1, since on average \( m \)-times less ground must be covered to trigger fracture. This gives a rough estimate of the order of magnitude of relative risk based on random bond spacing. More materials research is required for a more accurate estimation of relative risk.

3. RESULTS AND DISCUSSION

The stress field exerted by the skier on the weak layer can be used to calculate the strain energy density on the interface to the weak layer for a typical skier triggering scenario (Fig. 2). The direction of the displacements in the region of the crack tip determines which proportion of the available energy feeds the fracture process. If the displacements are purely in shear (simple shear mode), only terms containing \( \tau \) (dash-dotted lines in Fig. 2) feed the fracture process. If the displacements are purely slope-normal (simple anticrack mode), only terms containing \( \sigma \) (continuous lines in Fig. 2) feed the fracture process. In mixed-mode, all the available strain energy goes into fracture and terms containing \( \tau \) and \( \sigma \) contribute.

The lesson from Fig. 2 is that much more energy is available to drive fracture if the crack tip is loaded in anticrack mode than if loaded in simple shear mode. In our example of skier triggering, the ratio is of the order of 4:1 for slopes between 30° and 40° (approximately the ratio of the maxima of continuous lines compared to maxima of dash-dotted lines of the same color). For mixed-mode anticracking, the ratio goes up to 5:1.

Next, we assume that the skier passes over a flaw in the weak layer of, say, \( l = 0.05 \text{ m} \) in size. The model can be directly used to estimate the critical skier loads in terms of slope angle (Fig. 3). For simplicity we assume that the thickness of the slab at different points of the snowpack is proportional to the cosine of the slope angle, \( h \propto \cos \theta \). This corresponds to an idealized snowpack in which the snow is deposited homogeneously without local drift snow accumulations. The two cases of mixed-mode anticracking (continuous curves in Fig. 3) and simple shear cracking (dash-dotted curves in Fig. 3) are distinguished for comparison. For simple shear cracking, we see that the computed critical load to
trigger fracture is several tenfold the static skier load in the 30° to 45° window, and that the critical load rapidly decreases with increasing slope angle. On the contrary, for mixed-mode anticracking the computed critical load to trigger fracture is much lower (two to three times the static load) and virtually independent of slope angle up to about 60°. Up to that angle, the critical load slightly increases, but the increase is marginal and not practically relevant. Thus we infer from the model that weak layer fracture is not more difficult to trigger on gentle slopes than on steep slopes. Since this proposition relates to safe travel in avalanche terrain, we tested it in field experiments using the extended column test (ECT) method to trigger fracture (Simenhois and Birkeland, 2006).

The ECT experiments were conducted on selected slopes in Colorado, Montana and Alaska, in which gentle changes in slope angle or rollovers allowed for sampling a variety of slope angles with minimal changes in the snow structure, thus reducing the masking effect of snowpack variability. The experimental results confirm those of the model and show that triggering fracture in a weak layer is equally difficult (or marginally more difficult) on steep slopes in comparison with gentler slopes. These field data are discussed in more detail in the companion paper (Birkeland et al., 2010). Our results, especially those of the field experimentation, were obtained on persistent weak layers. Their validity for non-persistent weak layers is plausible, but not yet ascertained.

The main lesson to be learnt by this research is to realize that fracture can be triggered as easily on gentle slopes as on steeper ones, despite previous research having implied the contrary for lower angle slopes as on steeper ones, despite realizing that fracture can be triggered as easily on steep slopes. Due to the model that weak layer fracture is equally difficult or more difficult to trigger on gentle slopes than on steep slopes. Since this proposition relates to safe travel in avalanche terrain, we tested it in field experiments using the extended column test (ECT) method to trigger fracture (Simenhois and Birkeland, 2006).

From the example given in Fig. 5, we can see that the width of the stance and the load distribution on the skis significantly affects the critical crack width. The left panel Fig. 5a shows the scenario of a skier alternately weighting fully on one ski and then on the other, e.g. during telemarking or when travelling uphill with climbing skins. The right panel Fig. 5b shows the scenario of a skier weighting both skis equally and taking a rather wide stance of 50 cm (1.6 ft). We see that both the peak loads (in shear as well as in compression) and the critical half-widths (figure next to crack symbol) computed by the model are considerably smaller in the second scenario than in the first. If we use eq. (2) with \( \lambda = 150 \text{ m}^{-1} \) to estimate the relative risk \( m \), we find that the first scenario is roughly 30 to 40 times riskier than the second. The figures above are obtained for equal penetration depth. If this effect is also taken into account, the relative risk is increased again.

The lesson, according to the model, is that fully weighting on one ski represents the worst loading case in the sense that the critical width is the smallest when the load is entirely on one ski. Thus, by spreading the skis half a meter apart (or slightly more if the skier is comfortable with it) and paying attention to loading both skis evenly, the risk of triggering fracture can be substantially decreased. Nevertheless, we emphasize that, depending on the skier’s aptitude, taking a wider stance can increase the difficulty of skiing steep slopes. If using the technique is likely to result in a fall, its use would be counter-productive.

On a final note, we remark that when the slab is debonded from the substrate by mixed-mode anticracking of the weak layer in between, the fracture process does not necessarily lead to the release of an avalanche. For this to happen, the friction in the freshly formed fracture plane must be overcome. If this is not the case, a whumpf results (Heierli et al., 2008a; van Herwijnen and Heierli, 2009). This sup-
plementary condition does not exist if the weak layer fails by shear cracking.

4. CONCLUSION

Besides the simplifying assumptions of a uniform slab material, plane-strain deformation and a linear material model, the important assumptions of the physical model are i) brittle failure and ii) crack propagation by mixed-mode anticracking of the instant crack tip. In addition, in order to obtain an estimate of the relative risk between two scenarios, we needed to assume that the bonds linking the weak layer to the slab or to the weak layer itself are randomly spaced.

We conclude that:

1. For skier-triggering of slab avalanches, most of the mechanical work that drives fracture has to be attributed to the transient decompression in the anticracked area, and only a small part to the transient reduction in shear stress. The exact figure depends on the snowpack configuration and slope angle.

2. For persistent weak layers prone to anticracking, we find that fracture is not more difficult to trigger on gentle slopes than on steep slopes. A slight trend for increasing triggering difficulty with increasing slope angle is detectable but not practically relevant. For persistent weak layers prone to simple shear cracking, we find that fracture is considerably more difficult to trigger on gentle slopes than on steep slopes. Extensive field experimentation based on the ECT method were carried out on slopes with gentle rollovers to test the dependence on slope angle. These field data confirm the theoretical expectation for anticracking. The detailed procedure and more results of the experimentation are given in a companion paper.

3. Shear-based models of skier triggering such as the stability index predict substantially decreasing stability with increasing slope angle. This prediction is not compatible with the field data. We infer that these approaches are unsuitable in general for understanding the failure of persistent weak layers.

4. A practical lesson to be learnt from this research is to realize that fracture can be triggered as easily on lower angle slopes as on steeper ones, despite previous research having implied the contrary for many years. While avalanche practitioners know that fracture can be triggered on lower-angle slopes and anticipate the consequences, the point here is to understand that triggering fracture is equally easy on a 30° slope as on a 45° slope.

5. The compressive component of the load applied by the skier plays an important role in the triggering process. Up to the recent past, only shear loads were assumed to contribute to skier triggering. According to the present model, the most dangerous loading direction is close to slope-normal, and the least effective directions are the slope-parallel directions (uphill and downhill).

6. Fully leaning on one ski represents the worst loading case in the sense that the critical width is the smallest when the weight is entirely on one ski. By increasing the stance to half a meter apart (or slightly more if the skier is comfortable with it) and paying attention to loading both skis evenly, the risk of triggering fracture can be decreased very substantially.

The results emphasize that the anticrack mechanism for fracture in snow requires scientists and practitioners to rethink previously accepted and practically relevant paradigms, such as the suitability of the stability index to properly describe skier triggering, the perhaps somewhat counterintuitive slope-dependence of the triggering risk of fracture, and the primordial (instead of negligible) role of slope-normal loading.

5. ACKNOWLEDGEMENTS

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Figure 1. A skier with instant position \( x_0 \) sinks into a slab of thickness \( h \) by an amount \( a < h \). Under plain strain conditions, he/she acts as a line load with slope-parallel component \( p_x \) and slope-normal component \( p_y \). The load causes an additional compressive stress \( \sigma(x) \) and shear stress \( t(x) \) at an arbitrary point in the weak layer plane. The direction \( f \) of the applied load is arbitrary (not necessarily parallel to the acceleration of gravity \( g \)).

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Table 1. Skier-triggering scenario for illustration of the skier triggering model.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$h$</th>
<th>$E$</th>
<th>Poisson's ratio</th>
<th>$\gamma_0$</th>
<th>$p_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 kg/m$^3$</td>
<td>0.5 m</td>
<td>4.0 MPa</td>
<td>0.25</td>
<td>0.1 J/m$^2$</td>
<td>400 N/m</td>
</tr>
</tbody>
</table>

Figure 2. Partition of strain energy density on the interface to the weak layer resulting from normal stress component $\sigma$ (full lines) and from the shear stress component $\tau$ (dash-dotted lines). Black represents no ski penetration ($\alpha = 0$), red represents ski penetration half-way through the slab ($\alpha = \frac{1}{2} h$). The size and position of the critical anticrack is shown as thick coloured segment on the bottom. The thin segment indicates the critical size in the absence of the skier. The juxtaposed figure indicates the critical half-width in meters. a) slope angle $30^\circ$; b) $40^\circ$. Data: Table 1.

Figure 3. Critical skier load $p$ in units of a typical load $p_0 = 400$ N/m, in terms of slope angle $\theta$ and for a flaw size of 0.05 m. The critical loads for mixed-mode anticracking (full lines) are directly obtained from eq. 1. The critical loads for simple shear cracking (dash-dotted lines) are obtained from eq. 1 and forcing $K_\text{anti-I} = 0$, which then is not available. In the scenario in black (no penetration) fracture is triggered if the skier triples the load (e.g. by skiing very sharply or jumping). In the scenario in red (50 % penetration) fracture is already triggered if the skier only doubles the load. For this calculation, the load was assumed to act vertically. Data: Table 1.
Figure 4. Critical skier load in units of a typical load \( p_0 = 400 \text{ N/m} \), in terms of load direction \( f \). The slope angle \( \theta \) is 40°. The colors and figures indicate the relative penetration depth \( a/h \). The graph indicates that the triggering of fracture is distinctly easier under compressive loads (\( f = 0 \)) than under shear loads (\( f = \pm 90 \)). Data as in Fig.3.

Figure 5. Influence of spreading the skis apart and loading them evenly. a) Loading all weight on one ski, b) spreading the skis 0.5 m apart and loading them evenly. The result is shown for \( \theta = 40° \) and \( a/h = 0 \), but remains qualitatively valid for all slope angles and deeper penetration. The dots indicate the \( x \)-position of the loads. Data: see Table 1. The size and position of the critical anticrack is shown as thick segment on the bottom. The juxtaposed figure indicates the critical half-width in meters.

6. REFERENCES


