ABSTRACT: This paper presents two methods within the fuzzy or subjective probability framework in order to merge different decision strategies (particularly: Snowcard, Munter and our own empirical approach).

Keywords: Avalanche danger, decision strategy, fuzzy.

1. INTRODUCTION

In the Alps, most fatal snow avalanche accidents are caused by skiers or snowboarders. As a consequence several decision strategies for backcountry skiers have been established in the last two decades in order to prevent avalanche accidents or avalanche fatalities. The most important among them are: Munter’s method of reduction, Snowcard method (DAV), Stop or Go (OeAV) and an empirically driven decision strategy (Plattner (2001), Pfeifer (2009)) where the fundamental input variables are \( x = \) “danger level of avalanche”, \( y = \) “incline of the slope”. However, the skier’s or expert’s perception of danger level, incline/aspect of the slope and the strategies itself are more or less subjective and imprecise (or fuzzy which for instance was taken into account in a recent paper using a rule-based fuzzy approach (Pourraz et al. (2017))). Our goal is to put these strategies finally together embedding them in an imprecise probabilistic framework.

In a first approach we apply a rule-based fuzzification to each original decision strategy similar to Pfeifer and Fetz (2015). With the aid of a training data set as a result of different strategies we calculate an all over fuzzy system using the adaptive Neuro-Fuzzy algorithm ANFIS (Jang (1993)). In a second approach we use limit state functions \( g(x,y) \) which are less or equal to zero in cases where backcountry skiers trigger an avalanche and positive otherwise.

2. RULE-BASED FUZZY APPROACH

If we consider the Snowcard approach, see the e.g. the case with unfavorable exposition in Figure 1, the decision strategies could be seen as decision Matrix with green (go) yellow (be cautious) and red (stop) areas.

At first, we define within the fuzzy framework triangular membership functions for danger level (low (1), moderate (2), considerable(3), high (4); extreme (5) not considered) – which could be read,
value is equal to 3, but it could also be able for some probability that “Considerable” is smaller than 3 (e.g. 2.5) due to skier’s misinterpretation or underestimation – and trapezoidal membership functions for incline of the slope (flat (< 30), moderate (30-33), steep (33-36), very steep(36-39), extreme (> 39))

ANFIS (Jang (1993)) calculating an all over fuzzy system.

The result of defuzzification in our case is (see Figure 5):

Figure 5: Surface Merging

3. THE LIMIT STATE APPROACH

We denote \( g(x = (x_1, x_2)) \) as limit state function \( (x_1 \) danger level and \( x_2 \) incline of slope, for instance) or \( h(x, z) \) as parameterized limit state function according to an additional parameter \( z \) (Fetz (2012)). Values of \( g(x), h(x, z) \) mean failure if \( g(x), h(x, z) \leq 0 \)

Furthermore, the probability of failure is:

\[
p_f = \int_{X \subseteq Z} \chi(h(x, z) \leq 0) f_Z(z) d(z) f_X(x) d(x)
\]

where \( \chi(x) \) denotes the indicator function, \( f_Z(z) \) the conditional probability measure or density relating to \( z \) and \( f_X(x) \) the probability measure or density relating to \( x \).

In our case we consider the set \( Q \)

\[
Q = \left\{ q_s, q_M, q_E \right\}
\]

of imprecise failure regions (‘unsafe regions’), where \( q_s, q_M \) and \( q_E \) is referring to the Snowcard, Munter and empirical approach,

and the set of probability measures \( M_X \)

Using this sets of probability measures and precise failure regions, we are finally able to calculate specific probabilities according to:

2.1. Merging the different decision strategies

After fusing (and defusing) further decision strategies (Plattner (2001), Pfeifer (2009)) we generate a training data set as a result of the different decision strategies for the Neuro-Fuzzy algorithm

In a next step we do a fusion with fusion rules such as:

If Neig5 is flat and lws is 1 (low) then decision is Go!

with the help of fuzzy toolbox available in the numerical computing package Matlab using the Sugeno fuzzy model or Sugeno fuzzy inference system (step 1: fuzzify inputs; step 2: apply fuzzy operations, step 3: apply implication method). About 20 rules are defined in case of Snowcard, unfavorable exposition.

We further carry on to defussificate the fuzzy system resulting in the surface plot such as:

Figure 4: Surface SnowCard

Figure 3: Membership functions Neig

3. THE LIMIT STATE APPROACH

We denote \( g(x = (x_1, x_2)) \) as limit state function \( (x_1 \) danger level and \( x_2 \) incline of slope, for instance) or \( h(x, z) \) as parameterized limit state function according to an additional parameter \( z \) (Fetz (2012)). Values of \( g(x), h(x, z) \) mean failure if \( g(x), h(x, z) \leq 0 \)

Furthermore, the probability of failure is:

\[
p_f = \int_{X \subseteq Z} \chi(h(x, z) \leq 0) f_Z(z) d(z) f_X(x) d(x)
\]

where \( \chi(x) \) denotes the indicator function, \( f_Z(z) \) the conditional probability measure or density relating to \( z \) and \( f_X(x) \) the probability measure or density relating to \( x \).

In our case we consider the set \( Q \)

\[
Q = \left\{ q_s, q_M, q_E \right\}
\]

of imprecise failure regions (‘unsafe regions’), where \( q_s, q_M \) and \( q_E \) is referring to the Snowcard, Munter and empirical approach,

and the set of probability measures \( M_X \)

Using this sets of probability measures and precise failure regions, we are finally able to calculate specific probabilities according to:
\[
\overline{\mathcal{P}}_f = \sup_{P_X \in \mathcal{M}_X} \int \mathcal{X} q(x) dP_X(x) \\
= \sup_{P_X \in \mathcal{M}_X} \int \mathcal{X} \overline{q}(x) dP_X(x)
\]

where \( \overline{\mathcal{P}} \) denotes the upper envelope of the set \( Q \) of uncertain failure regions; \( \overline{\mathcal{P}} \) can be seen as membership function of the union of the fuzzy sets described by all \( q \in Q \).

### 3.1. Example

Uncertainty of \( x = (x_1, x_2) \) is modelled by a set \( A \) such as (see Figure 6):

- level of danger ‘moderate’ means \( x_1 \in [1.5, 2.5] \)
- incline of slope \( \leq 34^\circ \) means \( x_2 \in [0^\circ, 34^\circ] \)

which results in a set of Dirac probability measures

\[
\mathcal{M}_X = \{ \delta_a : a \in A \} \\
A = [1.5, 2.5] \times [0, 34]
\]

Further calculation leads to:

\[
\overline{\mathcal{P}}_f = \sup_{P_X \in \mathcal{M}_X} \int \mathcal{X} q(x) dP_X(x) = \max_{a \in A} \int \mathcal{X} q(x) d\delta_a(x) \\
= \max_{a \in A} q(a) = 0.72
\]

### REFERENCES


