SNOW ENTRAINMENT: AVALANCHE INTERACTION WITH AN ERODIBLE SUBSTRATE

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ABSTRACT: In this paper we physically describe how snow entrainment enhances the formation of fluidized flow states in the avalanche core, leading to low dissipative and long runout avalanches. The theory is based on two physical-mechanical descriptions. The first is to describe the avalanche interaction with the snow cover as an elastic-plastic collision. The collisional description implies that there must be a jump condition between the initial, pre-collisional and final, post-collisional avalanche flow velocity. These velocities can be determined by the application of both momentum and energy balances. The second mechanical description is to treat the avalanche core as a particle ensemble capable of assuming different statistical mechanical configurations. The configurations model different avalanche flow regimes and change according to energy fluxes produced during the entrainment process. There is therefore a direct link between the energy fluxes induced by the collisional interaction with the snow cover and flow regime transitions, particularly the formation of powder snow avalanches. Flow configurations can be amplified (fluidization) or damped (densification), according to the thermomechanical properties of the snow cover. Avalanche interaction with the snow cover therefore produces strong streamwise variations in velocity fluctuations and structure of the avalanche core. The description indicates that the avalanche interaction with the snow cover is a mechanically non-smooth process producing sudden changes in avalanche velocity. The velocity changes are both directional (blow-outs, frontal splashing) and non-directional (random particle fluctuations, turbulence). The magnitude of the energy fluxes associated with the directional and non-directional velocities we parameterize as a function of snow quality including snow temperature, water content and microstructure.

KEYWORDS: avalanche, snowcover, entrainment, splashing, ploughing, flow regime.

1. INTRODUCTION

The goal of this paper is to show how avalanche interaction with the erodible snowcover can produce energy fluxes that change the flow structure of the avalanche core. Snow avalanches contain dense flow structures of closely packed snow particles as well as disperse flow structures involving few inter-particle interactions (Fig.1). Dense particle ensembles are associated with frictional regimes involving particle shearing, abrasion and rubbing (dense flowing avalanches, wet snow avalanches); disperse particle ensembles are associated with low-friction regimes dominated by intermittent particle interactions (powder avalanches, saltation fronts). The expansion and contraction of the particle ensemble defines the bulk flow density of the avalanche core which can vary significantly from front to tail of the flow. Volume changes cause the in-take and blow-out of air and therefore disperse structures are intimately linked to the formation of mixed flowing/powder snow avalanches (Bartelt et al., 2016). Snowcover entrainment therefore plays an important role in defining the avalanche flow regime and subsequently the possible avalanche inundation area.

The most elementary physical model describing of the interaction of a flowing avalanche with a mountain snowcover is as an elastic-plastic collision: A moving body (the avalanche Φ) collides with a non-moving body (the snowcover Σ), see Fig. 1. At the end of the interaction the avalanche body is no longer moving at the same speed. There are many different possible outcomes for the snowcover mass. When the speed of the snow originally at rest is moving with the post-collisional speed of the avalanche it is considered to be *entrained* (by definition). Physically, it is impossible to tell the difference between the moving snow and entrained snow. In reality, however, some of the snow involved in the interaction could be moving *faster* than the avalanche. (This we term *splashing*). Other snow could remain motionless. For example, snow could simply be *compacted* by the avalanche and remain in place, left behind by the avalanche. The many different possible outcomes (entrainment, splashing, compaction) of the avalanche interaction with the snowcover depend on the mechanical properties of the snow, which in turn depend its bonding, temperature, water content and micro-structure (Gauer and Issler, 2004).

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Whatever the outcome of the interaction, however, it is dissipative: flow energy is removed from the avalanche. It is therefore a paradox that entrainment should *enhance* avalanche runout. In this paper we derive the mechanical energy fluxes that induce flow regime transitions to understand how snowcover entrainment influences avalanche flow, specifically how entrainment changes avalanche flow structure.

2. SNOWCOVER COMPACTION AND ERODIBILITY

The mass of the snow substrate *eroded* by the avalanche as it moves from time t_0 to t_1 is given by M_Σ. A simple model would be to assume the snow mass disturbed (or disarranged) by the avalanche is eroded and therefore directly proportional to the speed of the avalanche u_Φ. In this case the faster the avalanche, the larger the volume of material eroded by the core,

$$\dot{M}_\Sigma = \kappa \rho_\Sigma \left\| \mathbf{u_\Phi} \right\|. \tag{1}$$

The dimensionless parameter κ we define as the *erodibility* coefficient. Again, we emphasize that the erodibility does not define the amount of snow taken-in by the avalanche, only the mass of snow per unit area that is affected by the avalanche core. The erodibility can be modified to include the bonding strength of the snow μ_b. Letting g_s be the gravitational acceleration in the direction of the avalanche and g_z the slope normal acceleration to the layer, we have

$$\kappa = \frac{\kappa'}{\|g\|}\left[g_s - \mu_b g_z\right] \text{ with } \kappa \geq 0 \text{ always.} \tag{2}$$

Basically, this model implies that it is easier to entrain and accelerate snow on a steeper slopes (if there is snow), than on a flatter slope. It accounts for the fact that the downward pull of gravity is stronger on a steeper slope, requiring smaller collisional forces to set the snow in motion. The Coulomb-type bonding parameter defines a slope angle θ_b at which the avalanche no longer entrains snow,

$$\tan\theta_b = \mu_b. \tag{3}$$

The bonding strength model is motivated by observations of eroded segments in avalanche tracks. On track segments where there are no depositions, the eroded snowcover layer can be observed. In this case the parameter μ_b must be smaller than the tangent of the slope angle. The model is valid for both frontal entrainment processes (high erodibility κ) or abrasive processes (low erodibility κ).

3. ENTRAINED AND SPLASHING MASS

We divide the mass eroded by the avalanche M_Σ into two parts,

$$M_\Sigma = M_{\Sigma\to\Phi} + M_{\Sigma\to\Gamma}. \tag{4}$$

The mass $M_{\Sigma\to\Phi}$ is entrained by the avalanche (moves with the speed of the avalanche, by definition). The mass $M_{\Sigma\to\Gamma}$ moves with a speed greater than the avalanche Φ-front and therefore forms the avalanche pre-front or splashing front, Γ-front. We apply a partitioning coefficient γ to divide M_Σ into the entrained and splashing parts,

$$M_{\Sigma\to\Phi} = (1-\gamma)M_\Sigma \tag{5}$$

and

$$M_{\Sigma\to\Gamma} = \gamma M_\Sigma. \tag{6}$$

The parameter γ is termed the *splashing coefficient* as it defines how much of the mass M_Σ produces the avalanche splashing Γ-front. For now γ is a time independent parameter and thus the total erosion rate is given by the sum of the entrainment and splashing rates,

$$\dot{M}_\Sigma = \dot{M}_{\Sigma\to\Phi} + \dot{M}_{\Sigma\to\Gamma} = (1-\gamma)\dot{M}_\Sigma + \gamma\dot{M}_\Sigma. \tag{7}$$

The entrained mass $M_{\Sigma\to\Phi}$ corresponds to a perfect inelastic (plastic) collision. The avalanche and entrained snow are moving at the same speed. The splashing part requires some elasticity because this part of the eroded mass is reflected (rejected) by the avalanche. In this case the mass of the avalanche does not increase and energy conservation demands that all the transferred kinetic energy of the splashed snow is taken from the avalanche. The splashing parameter γ partitions the erosion process into plastic (entrainment) and elastic (splashing) parts. Moreover,

$$\dot{M}_{\Sigma\to\Phi} = (1-\gamma)\dot{M}_\Sigma \tag{8}$$

and

$$\dot{M}_{\Sigma\to\Gamma} = \gamma\dot{M}_\Sigma. \tag{9}$$

The higher the speed of the splashed particles, the larger the decrease in avalanche velocity. This result suggests that significant avalanche damage can be caused by ”splashing particles” or avalanche ”saltation” fronts. Some physical constraints can be placed on the speed of the pre-front u_Γ. Letting Δu_Γ be the speed of the splashing front seen from an observer moving with the speed of the avalanche,

$$\Delta u_\Gamma = u_\Gamma - u_\Phi, \tag{10}$$

conservation of momentum during the avalanche-snowcover interaction demands

$$\Delta u_\Phi = -\left[\frac{M_\Sigma}{M_\Phi + M_{\Sigma\to\Phi}}\right](1+\gamma r)u_\Phi \tag{11}$$

where the parameter r >0 represents the bulk restitution coefficient between the avalanche and snowcover

$$r = \frac{\Delta u_\Gamma}{u_\Phi}. \tag{12}$$

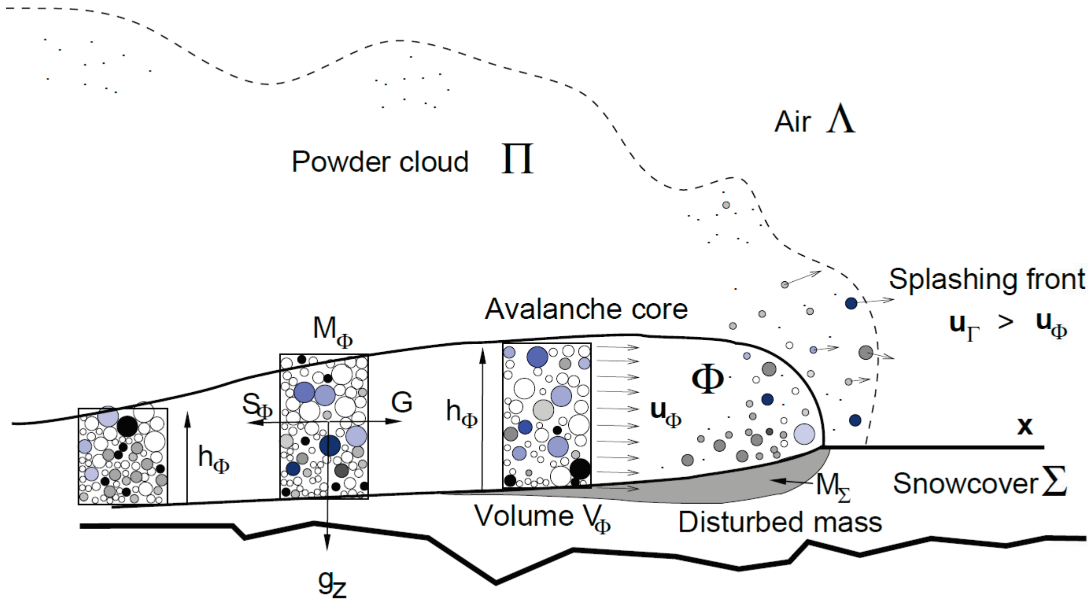

Figure 1: The avalanche core Φ collides with the snowcover Σ. During the collision mass M_Σ in the snowcover is disturbed by the avalanche. Some of the mass may be entrained $M_{\Sigma\to\Phi}$; some of the mass $M_{\Sigma\to\Gamma}$ may be accelerated to a velocity u_Γ higher than the velocity of the avalanche $u_\Gamma > u_\Phi$, forming a splashing front. The mass M_Σ may be compacted and remain in position. Mass is entrained directly by the core Φ. Entrainment can enhance the formation of the powder cloud Π. The avalanche body consists of flow volumes V_Φ consisting of mass M_Φ. The density ρ_Φ of the avalanche varies in the streamwise direction.

Conservation of energy provides the following balance equation,

$$\frac{1}{2}M_\Phi u_\Phi^2 = \frac{1}{2}(M_\Phi + M_{\Sigma\to\Phi})(u_\Phi + \Delta u_\Phi)^2 + \frac{1}{2}M_{\Sigma\to\Gamma}u_\Gamma^2 + L_\Sigma \quad (13)$$

where L_Σ contains all energy losses created during the collision. Eq. 13 balances the kinetic energy before and after the collision. The loss of kinetic energy is, of course, associated with the change in velocity Δu_Φ. Combining the momentum equation Eq. 11 and the energy equation Eq. 13, the energy loss L_Σ can be quantified,

$$L_\Sigma = \frac{1}{2}\left[(1-\gamma) + \gamma(1-r^2)\right] M_\Sigma u_\Phi^2. \quad (14)$$

The rate of heating during the entrainment process is therefore

$$\dot{L}_\Sigma = \frac{1}{2}\left[(1-\gamma) + \gamma(1-r^2)\right] \dot{M}_\Sigma u_\Phi^2. \quad (15)$$

This equation is necessary in thermomechanical avalanche models with entrainment, see for example Vera Valero et al. (2015, 2018). Physical insight into the equation can be gained by noting that it is composed of two parts,

$$\dot{L}_\Sigma = \dot{L}_{\Sigma\to\Phi} + \dot{L}_{\Sigma\to\Gamma}. \quad (16)$$

The first part contains the irreversible energy losses during the entrainment process,

$$\dot{L}_{\Sigma\to\Phi} = \frac{1}{2}\left[(1-\gamma)\right] \dot{M}_\Sigma u_\Phi^2. \quad (17)$$

When γ=1 (no plastic entrainment, all splashing) then $\dot{L}_{\Sigma\to\Phi} = 0$, i.e. no heat is generated during the entrainment process. The second part is the heat generated during the splashing,

$$\dot{L}_{\Sigma\to\Gamma} = \frac{1}{2}\left[\gamma(1-r^2)\right] \dot{M}_\Sigma u_\Phi^2. \quad (18)$$

When r = 1 then we have a completely elastic collision at the front, subsequently $\dot{L}_{\Sigma\to\Gamma}$=0, i.e. no heat production. The splashed particles are not part of the avalanche core (they are outside the core) and therefore they do not raise the temperature of the core (but they are heated during a partially elastic collision ($0 < r <$ 1).

4. PRODUCTION RANDOM KINETIC ENERGY

The entrainment part of the snowcover erosion is considered to be fully plastic. In such a case, all energy loses are dissipated entirely to heat,

$$\dot{Q}_{\Sigma\to\Phi} = \dot{L}_{\Sigma\to\Phi} = \frac{1}{2}\left[(1-\gamma)\right] \dot{M}_\Sigma u_\Phi^2. \quad (19)$$

where $\dot{Q}_{\Sigma\to\Phi}$ is the rise in internal energy in the core caused by entraining mass at the rate $\dot{M}_{\Sigma\to\Phi}$. Energy loses can, however, take another form. Instead of producing only microscopic fluctuations (heat) $\dot{Q}_{\Sigma\to\Phi}$, we postulate that the losses take an additional macroscopic form; that is, the production of random kinetic energy $\dot{P}_{\Sigma\to\Phi}$ (Bartelt et al., 2006; Buser and Bartelt, 2009),

$$\dot{L}_{\Sigma\to\Phi} = \dot{Q}_{\Sigma\to\Phi} + \dot{P}_{\Sigma\to\Phi}. \tag{20}$$

Again we apply the method of linear partitioning to separate the different energies,

$$\dot{P}_{\Sigma\to\Phi} = \epsilon \dot{L}_{\Sigma\to\Phi} = \frac{1}{2}\epsilon(1-\gamma)\dot{M}_{\Sigma} u_{\Phi}^2. \tag{21}$$

and

$$\dot{Q}_{\Sigma\to\Phi} = (1-\epsilon)\dot{L}_{\Sigma\to\Phi} = \frac{1}{2}(1-\epsilon)(1-\gamma)\dot{M}_{\Sigma} u_{\Phi}^2. \tag{22}$$

The partitioning parameter ϵ defines how much of the dissipated energy during the entrainment process is converted directly to heat and how much is converted to non-directional random kinetic energy (which will eventually be dissipated to heat).

5. THERMAL AND RANDOM KINETIC ENERGY BALANCES

We have shown that the avalanche interaction with the snowcover will produce two energy fluxes: thermal energy $\dot{Q}_{\Sigma\to\Phi}$ and random mechanical energy fluxes $\dot{P}_{\Sigma\to\Phi}$. The inclusion of entrainment in avalanche models therefore requires balance equations for internal (heat) energy E_Φ and random mechanical energy R_Φ (Vera Valero et al., 2015). These are

$$\frac{\partial(E_\Phi h_\Phi)}{\partial t} + \frac{\partial(E_\Phi h_\Phi u_\Phi)}{\partial x} + \frac{\partial(E_\Phi h_\Phi v_\Phi)}{\partial y} = \\ = \dot{Q}_\Phi + \dot{Q}_{\Sigma\to\Phi} + c_\Sigma \dot{M}_{\Sigma\to\Phi} T_\Sigma. \tag{23}$$

and (Buser and Bartelt, 2009; Bartelt and Buser, 2018)

$$\frac{\partial(R_\Phi h_\Phi)}{\partial t} + \frac{\partial(R_\Phi h_\Phi u_\Phi)}{\partial x} + \frac{\partial(R_\Phi h_\Phi v_\Phi)}{\partial y} = \dot{P}_\Phi + \dot{P}_{\Sigma\to\Phi} \tag{24}$$

The quantities E_Φ and R_Φ represent the specific thermal and random energy densities (J m^{-3}) of the avalanche core Φ. Because we apply a depth-average approach, these energy densities will vary in the streamwise directions (x, y), but not in the z-direction, that is, the avalanche flow height h_Φ. Both the internal and random energies are governed by the dissipation of kinetic energy by shearing $\dot{Q}_\Phi$,

$$\dot{Q}_\Phi = (1-\alpha)\left[\mathbf{S_\Phi}\cdot \mathbf{u_\Phi}\right] + \beta R_\Phi h_\Phi \tag{25}$$

and

$$\dot{P}_\Phi = \alpha\left[\mathbf{S_\Phi}\cdot \mathbf{u_\Phi}\right] - \beta R_\Phi h_\Phi \tag{26}$$

where $\dot{P}_\Phi$ is the total production of random kinetic energy in the avalanche core. The parameters α and β control the shearing and collisional dissipation rates, see Bartelt et al. (2006). The frictional resistance $\mathbf{S}_\Phi = (S_{\Phi x}, S_{\Phi y})$ consists of both a Coulomb friction S_μ (coefficient μ) and a velocity dependent stress S_ξ (coefficient ξ),

$$\mathbf{S}_\Phi = \frac{\mathbf{u}_\Phi}{\|\mathbf{u}_\Phi\|}\left[S_\mu + S_\xi\right]. \tag{27}$$

The internal energy E_Φ is related directly to the mean core temperature T_Φ by the specific heat capacity of the flowing snow c_Φ

$$E_\Phi = \rho_\Phi c_\Phi T_\Phi. \tag{28}$$

as well as the thermal energy input from entrained snow, which is composed of two parts, the energy dissipated by the plastic collision $\dot{Q}_{\Sigma\to\Phi}$ and internal energy of the entrained snow, which depends on the snow temperature T_Σ and density ρ_Σ (specific heat capacity c_Σ), see (Vera Valero et al., 2015, 2018). The initial temperature of the avalanche is given by the temperature of the released snow T_0. The snowcover substrate is defined in the same coordinate system by defining a layer heights $h_\Sigma(x,y)$ with densities $\rho_\Sigma(x,y)$ and temperatures $T_\Sigma(x,y)$.

6. ENTRAINMENT AND AVALANCHE FLOW REGIME

The most important result of our analysis is that mass entrainment produces fluxes of thermal and mechanical energy. The thermal energy fluxes are responsible for dry to wet flow transitions:

$$\underbrace{\dot{M}_\Sigma}_{\text{Interaction}} \rightarrow \underbrace{\dot{M}_{\Sigma\to\Phi}}_{\text{Entrainment}} \rightarrow \underbrace{\dot{Q}_{\Sigma\to\Phi}}_{\text{Heating/Melting}} \rightarrow \underbrace{\mathbf{S_\Phi}\downarrow}_{\text{Lubrication}} \tag{29}$$

The mechanical energy fluxes are responsible for avalanche fluidization (Buser and Bartelt, 2015) and the formation of powder avalanches (Bartelt et al., 2016):

$$\underbrace{\dot{M}_\Sigma}_{\text{Interaction}} \rightarrow \underbrace{\dot{M}_{\Sigma\to\Phi}}_{\text{Entrainment}} \rightarrow \underbrace{\dot{P}_{\Sigma\to\Phi}}_{\text{Core expansion}} \rightarrow \underbrace{\mathbf{S_\Phi}\downarrow}_{\text{Fluidization}} \tag{30}$$

Thus, avalanche interaction with an erodible substrate sets a chain of physical processes in motion which enhance runout, *for both dry and cold snowcovers*. However, the form and speed of the avalanche will differ for each flow regime. Flows dominated by thermal energy fluxes (wet snow avalanches) are dense because the larger the thermal energy fluxes, the less mechanical energy is available to fluidize the avalanche core. Flows controlled by mechanical energy fluxes (mixed flowing/powder avalanches) are less dense because more of the dissipated shear work is used to fluidize the avalanche. Thermal heating is postponed to the runout zone. Thus, there is a physical tendency for avalanches to adopt one of the extreme avalanche forms, depending on the temperature of the snow (Naaim et al., 2013).

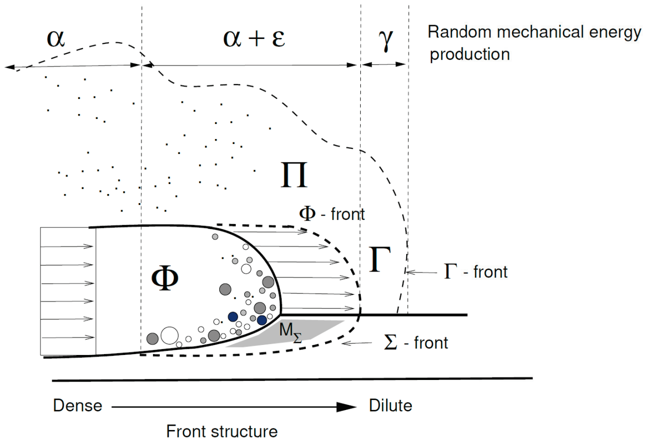

Fig. 2: The structure of a dry mixed flowing/powder avalanche front is controlled by the mechanical energy production. Particle splashing creates an avalanche pre-front, parameter γ, (splashing). Parameter ϵ controls the random mechanical energy production during entrainment. Different entrainment mechanisms are associated with different partitioning coefficients ϵ.

An important result of our analysis concerns the structure of dry mixed flowing/powder avalanches (Fig. 2). The front of a dry avalanche will contain several fronts formed by different physical processes defined by the interaction with the snowcover. A dilute pre-front of splashed particles can exist (parameter γ). Theoretically, this splashed mass Γ does not belong to the avalanche core Φ, because it is moving at a speed faster the avalanche Φ-front. The region M_Σ defines where mechanical energy is being produced by the collision between the avalanche and the snowcover (parameter ϵ). This region could be concentrated at the avalanche front, or extended over the entire length of the avalanche. Back calculations of documented case studies suggest that there exist entrainment mechanisms with high mechanical energy production, for example frontal entrainment of new snow ($\epsilon \approx 0.5$), or entrainment mechanisms with low mechanical energy production, e.g. basal erosion ($\epsilon < 0.1$). When entrainment ends in the streamwise direction of the avalanche there exists only the internal shearing (parameter α) to produce random mechanical energy. In many case studies we are finding that $\alpha \leq \epsilon$. This indicates that the collisional interaction with the snowcover is a more efficient producer of random mechanical energy.

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