MICROMECHANICAL ANALYSIS OF ENERGY RELEASE IN SNOW FRACTURE

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ABSTRACT: Fracture initiation and propagation in snow is paramount to understanding the avalanche process. There is ongoing discussion and debate regarding the mode of fracture and fracture properties of weak layers. Typically, snow fracture is modeled using layers of continuum materials. In actuality, snow is a highly porous, discontinuous medium with a granular microstructure that dictates where and how fracture propagates. Therefore, a microstructure-based approach was taken. A discrete element model (DEM) has been developed to analyze energy release in both layered and homogeneous snow samples undergoing shear fracture. Real snow microstructure obtained from a micro-CT scanner is modeled as rigid unbreakable grains connected by brittle elastic bonds. However, this does not mean that snow fracture is brittle in the macroscopic sense. The model shows strain softening prior to failure. A significant portion of the mechanical energy driving the fracture process goes into microcracking and grain rearrangement. These results suggest that plasticity is prominent in the fracture process, even at very high strain rates. Results from the model have been validated using laboratory shear experiments.

KEYWORDS: fracture, energy release rate

1. INTRODUCTION

Fracture in snow is paramount to the avalanche initiation process. However, there is no universally accepted model or consensus on the fracture mode of snow. Typically, snow fracture is assumed to be a mixed mode shear/anticrack (Heierli et al, 2008) or pure shear (McClung, 2011). However, both these models assume a fracture mode and make various assumptions about the slab and weak layer behavior and properties. As a polycrystalline material with relatively large grains, the macroscopic approach may oversimplify the complex grain interactions encountered in snow fracture.

Here, a microstructural discrete element model (DEM) is introduced. By focusing on the microscopic interactions between ice crystals, assumptions about macroscopic snow behavior are not used. Simply, it is assumed that snow sample failure consists of many failed bonds between ice grains.

2. THEORY

A fracture is propagated when mechanical energy forms two new surfaces within a material. Surface formation energy, γ (J/m²), is a well-defined material property for most materials. The energy required to form two new surfaces (i.e. fracture) of a material is then

$$G_c=2\gamma$$

(1)

$G_c$ is termed the critical energy release rate and it is a fundamental fracture property of a material. The coefficient 2 denotes that when a fracture propagates two new surfaces are created.

Most current fracture models for snow take a macroscopic approach (e.g. Heierli et al, 2008; McClung, 2011). However, fracture in snow actually occurs through the bonded ice network (McClung, 2009). Here, the microstructural details of snow are used to predict the fracture path through individual ice bonds rather than treating the snow as a continuum.

As snow is subjected to loading, various inter-grain bonds break throughout the sample. As these bonds break, they may coalesce into a main fracture path or they may remain as isolated “microcracks” within the sample. Although these microcracks require energy to create, they may not be part of the main fracture path. Therefore, microcracking may represent a significant portion of energy dissipated within the fracture process. By using a DEM approach, local load states at every bond are considered and the model should give a realistic representation of bond breakage through the loading process.
X-ray computed tomography (CT) scans were used to ascertain snow sample 3D microstructural geometry. A SkyScan 1173 Micro-CT scanner modified to work in a cold lab was used.

By using x-ray computed tomography (CT) images of snow, interconnecting bonds can be identified and the virtual snow sample subjected to loading.

3. DISCRETE ELEMENT MODEL

A discrete element model (DEM) was used to simulate snow fracture experiments and explicitly track energy sources and sinks during the fracture process. DEM tests were performed on models representing CT scans of snow. The model was built in the free open-source DEM software YADE (Yet Another Dynamic Engine). Forces acting on the elements are dictated by inter-elemental forces defined by constitutive laws. While undergoing mechanical testing, the energy into the system is simply the work done by a load actuator. During the mechanical loading process, it was assumed that mechanical work done on the sample was converted into either elastic strain energy or kinetic energy. As the sample was loaded, energy was dissipated through strain energy release from bond breakage, frictional loss, and damping.

A complex interaction of these processes dictates the energy release observed during the fracture process. It is the objective of this model to quantify the amount of energy stored in the system and dissipated at all times during fracture.

The model was run on a 1920-core Linux cluster with 4 GB of memory per core. YADE was run using four cores per model, although generally several models were run at once. Models were allowed to run for 24 hours and then forced to quit.

3.1 Pre-processing

A binary 3D CT image is segmented into bonds and grains using the watershed segmentation technique previously described by LeBaron and Miller (2014).

The 3D image must then be processed in a way that preserves the actual grain geometry but is usable in the DEM. In DEM, any sharp corners present a mathematical singularity and are computationally undesirable. Spheres are commonly used in 3D DEM simulations. However, simple spherical elements do not represent the complex geometry of ice grains found in snow. Therefore, grains were represented using rigid assemblies of spheres (Figure 1). The multisphere approximation, as described by Amberger et al (2012), these rigid assemblies of overlapping spheres have mass identical to the original grains and moments of inertia very similar to the original grains. They are therefore suitable for use in dynamic simulations.

Figure 1: Segmented grains (above) represented as multispheres (below)

3.2 DEM Constitutive Relations

The model was designed so that all material failure and deformation occurs in bonds. Bonds may resist normal and shear forces as well as bending and twisting moments. Stiffness of bonds in all six DOF is dictated by a combination of grain and contact properties. Contact properties were manually computed based on real contact and grain geometry derived from the CT scan. Contact properties between grains are calculated by assuming that interactions between bonded grains can be approximated by a linear elastic term.
In the normal direction, that is, a line directed from grain center to grain center,

\[ F_n = k_n u_n \]

Where \( F_n \) is normal force across the contact, \( k_n \) is normal stiffness (N/m), and \( u_n \) is normal displacement between grain centers. Similar relations exist for shear force \( F_s \), bending moment \( M_r \) and twisting moment \( M_{tw} \):

\[ F_s = k_s u_s \]
\[ M_r = k_r \phi_r \]
\[ M_{tw} = k_{tw} \phi_{tw} \]

Where \( \phi_r \) and \( \phi_{tw} \) are relative rotations. Values for all \( k \) are defined by Jauffres et al (2012) using a fracture mechanics deformation and failure criterion:

\[ k_n = E' r_i \]
\[ k_s = E'' r_i \]
\[ k_r = \frac{1}{4} E' r_i^3 \]
\[ k_{tw} = \frac{1}{2} E'' r_i^3 \]

Where \( r_i \) is bond radius and

\[ E' = \frac{E}{1 - \nu^2}; E'' = \frac{2E}{(2 - \nu)(1 + \nu)} \]

3.3 Model Parameters

Material properties of ice used in the DEM are summarized in Table 1:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho ) (g/cm(^3))</td>
<td>0.9167</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.2</td>
</tr>
<tr>
<td>( E ) (GPa)</td>
<td>1.4</td>
</tr>
<tr>
<td>( \gamma ) (J/m(^2))</td>
<td>0.0757</td>
</tr>
</tbody>
</table>

Modeled samples are deformed in shear at a rate of 1 mm/s. A sample image of a cubic snow specimen being sheared is seen in Figure 3. Separate multisphere clumps (representing individual grains) have different colors.

![Figure 2](image-url)  
Figure 2: Grain-grain interactions modeled using integranular forces and moments.

Bonds may also fail if normal or shear stress on the bond exceeds its capacity. The strength of each bond is (Jauffres et al 2012):

\[ \sigma_n = 2 \sqrt{\frac{2\gamma}{\pi r_i}} E' \]

![Figure 3](image-url)  
Figure 3: Shear model in progress. The base is fixed and the top of the sample moves right to left at 1 mm/s.

4. EXPERIMENTAL VALIDATION

As validation of the model, three laboratory fracture tests were performed on the same snow that was modeled. Homogeneous, rounded grains from a snow breeder were sifted into a box to create an isotropic homogeneous sample. A shear frame
was placed on the snow surface, and more snow was sifted in and around it. After allowing the grains to sinter for 14-20 hours, the snow inside the shear frame was isolated from surrounding snow to create a rectangular test specimen. Adjacent snow was put into the CT scanner to ascertain microstructure for modeling. Three arbitrary subsamples from within the CT specimen were modeled for comparison to the experimental results.

Constant velocity shear tests were performed using a Geo-Jac load actuator at a rate of 0.847 mm/s. The applied load P was monitored with a load cell at a rate of 1000 Hz. results

The algorithm was run on three samples of homogeneous, rounded snow. CT scan Resolution varied from 12.01 to 14.88 µm. Three arbitrarily located subsamples 3x3x3 mm from each sample were modeled. Larger subsamples were not considered due to computational expense. Results for elastic modulus, peak strength and energy release are given in Figures 4-6.

Figure 4: Experimental energy release rate compared to modeled energy release rate

Figure 5: Experimental shear modulus compared to modeled specimen strength

Figure 6: Experimental specimen strength compared to modeled specimen strength

Test 1 showed the best agreement between model and experimental results. Stress/strain curves for the experiment and the three model runs are shown in Figure 7.
5. CONCLUSIONS AND DISCUSSION

Model results for strength and shear modulus are generally within an order of magnitude of the experimental results. However, energy release rate may vary by as much as two orders of magnitude. This may be due to sample size of the model. It has been estimated before that the fracture process zone (FPZ) is large (Bazant et al 2003), perhaps bigger than our 3x3x3mm samples here. Unfortunately, even the small computational samples used here contained several thousand bonds each, making the process quite computationally expensive.

It is worth noting that our experimental energy release rate is much higher than published values for notched samples (e.g. Sigrist et al, 2006; Kirchner et al, 2002). To our knowledge, these are the first fracture mechanics tests on un-notched snow. Notched samples provide a stress concentration that may limit the size of the fracture process zone and not realistically simulate the microcracking and damage accumulation that occur in our tests (and in naturally occurring snow) to produce very high values of $G_c$. The difference between experimental $G_c$ and modeled $G_c$ may also be a function of numerical sample size. Computational speedup could address this shortcoming.

Pre-failure stiffness in the model was reasonable, although consistently too high by a factor of two or three. However, these values were reached by using published values of elastic modulus and surface energy for ice. Previous microstructure-based DEM efforts have relied on heavily adjusted material properties to achieve good results (Hagenmuller 2014), so our efforts represent a step forward in accurate microstructural modeling.

Furthermore, the Jauffres strength/stiffness model assumes spherical particles connected by circular bonds. While these tests were performed on rounded snow grains, no grain was a perfect sphere and no bond was a perfect circle. It may be necessary in the future to use shape correction factors to more accurately represent ice grain microstructural geometry.

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REFERENCES


