AN ASSESSMENT OF RUN-OUT MODELS APPLIED TO EXTREME NORWEGIAN SNOW AVALANCHES

INTERNATIONAL SNOW SCIENCE WORKSHOP 2016 IN BRECKENRIDGE, CO

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ABSTRACT: The aim of the present project was to evaluate four different models by using objective criteria and constant parameters, and to consider the models’ applicability to Norwegian avalanches. RAMMS, Elba+, the Alpha-Beta model and the Energy Line model are the assessed models. The models are applied to 15 well-documented Norwegian avalanches with return periods from 100 to 300 years. RAMMS and Elba+ represent numerical dynamical models, whereas the Alpha-Beta model and the Energy Line model are empirical models based on topographic parameters. A main concern of the project was to compare the accuracy of the numerical and empirical models in terms of run-out distance, maximum velocity and velocity distribution in the run-out zone.

The results showed that the two numerical models, RAMMS and Elba+, consistently modelled shorter run-out distances than recorded ones, but they had the best correlation coefficients in the statistical analysis. On the other hand, the two topographic models, developed for Norwegian conditions, had the least deviation in average run-out distance. All the models calculate maximum velocities within a realistic range. However, RAMMS and Elba+ probably calculate too small velocity gradients in the run-out zone compared to recordings from full-scale experiments. The Energy Line model, however, provided values that are more realistic. An accurately estimated distribution is extremely important when dimensioning avalanche protection measures. The statistical analysis shows that RAMMS and the Energy Line model overall provide the best results.

KEYWORDS: dynamical model, topographic model, run-out distance, velocity-distribution

1. INTRODUCTION

Models to estimate the flow of snow avalanches are important tools when defining safe residential areas and dimensioning avalanche protection measures. Operators in Norway have used simpler empirical models for several years. However, there exist few statistical analyses on the application of dynamical models to Norwegian avalanches. The objective of the present project is therefore to compare the accuracy and availability of four different models: two dynamical models (RAMMS and Elba+) developed in Switzerland and Austria, and two empirical models (the Alpha-Beta and the Energy Line model) based and developed on topographic conditions in Norway. 15 well-documented Norwegian avalanches, with an assumed return period of 100-300 years, are selected for this purpose. The model results are assessed in terms of run-out distance, maximum velocity and retardation in the run-out zone. Additionally, the results of the complicated dynamical models are compared to the simple empirical models.

This research is a part of the multi-disciplinary research project entitled “Natural Hazards” (NIFS), a joint enterprise involving the Norwegian Public Roads Administration (NPRA), the Norwegian Water Resources and Energy Directorate (NVE) and the Norwegian National Rail Administration (JVB). The paper is based on a more detailed report presented by Håland et al. (2015).

2. MODEL INTRODUCTION AND PARAMETER SELECTION

2.1 Types of models

- Dynamical models are based on flow theories of non-Newtonian fluids. The models calculate the flow from the starting position to the stopping position. RAMMS and

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Elba+ are the assessed models. They divide the avalanche area into several cells, and calculate velocity and flow height over a 3-dimensional terrain.

- **Empirical models** are based on topographic parameters. The Alpha-Beta model (Lied and Bakkehøi 1980) and the Energy Line model (Norem 2014) are considered in this research.

**Selection of objective input criteria:**

To compare the models optimally, the input parameters in dynamical models were selected based on objective criteria. The criteria derived from general knowledge of the physics in avalanche dynamics. All comparisons are made with the original criteria selected, and no adjustments are subsequently carried out to improve the results.

2.2 The dynamical models

Both RAMMS and Elba+ are based on the Voellmy rheology (Voellmy 1955), which divides the friction term into; a Coulomb friction and a velocity-dependent friction term. If the avalanche flows down an infinite plane with gradient $\alpha$, it will obtain a terminal velocity given by the equation:

$$v_{\text{term}} = \sqrt{h \xi (\sin \alpha - \mu \cos \alpha)} \text{ (m/s)}$$  \hspace{1cm} (1)

Where, $v_{\text{term}}$ = the terminal velocity (m/s), $h$=slab thickness, $\xi$= velocity-dependent friction coefficient (m/s²), $\mu$=Coulomb friction coefficient and $\alpha$=slope angle.

To run a calculation with the dynamical models some inputs are needed:

- A digital terrain model.
- Size of starting zone (length, thickness, width)
- Friction coefficient values

**Digital Terrain model**

Digital terrain models with a cell size of 10 m were created in ArcGIS. ArcGIS was additionally used to analyse the final model results.

**Size of the starting zone:**

Both RAMMS and Elba+ recommend to define the starting zone area manually based on the terrain. The starting zone is generally defined as the area where the slope exceeds 30°. There are, however, limitations to that criterion. According to Bakkehøi and Norem (1994), the length of the starting zone is assumed not to exceed 1/6 of the total length of the avalanche path, or a height difference exceeding 100 m (Fig 1). The assumption is based on experiences in which wind transported snow deposits close to mountain ridges. These objective criteria were applied to decide the length of the starting zone.

**Fig.1:** Criteria for selecting the length of the starting zone

Avalanches are generally released when the shear stress acting on a weak layer exceeds the shear strength of the layer. According to Bakkehøi and Norem (1994), it could be assumed that the strength of critical weak layers in extreme avalanches is the same for all avalanches.

The shear strength of a snow layer is usually expressed by the Mohr-Coulomb law for cohesive materials:

$$\tau_{\text{strength}} = c + \rho g z \tan \phi \text{ (Pa)}$$  \hspace{1cm} (2)

Where $\tau_{\text{strength}}$=shear strength (Pa), $c$=cohesion (Pa), $\rho$=density (kg/m³), $g$=coefficient of gravity (m/s²), $z$=depth (m) and $\phi$=friction angle (°).

The shear stress acting on a snow layer at depth, z, is:

$$\tau = \rho g z \sin \alpha \text{ (Pa)}$$  \hspace{1cm} (3)

Where $\tau$=shear stress and $\alpha$=slope angle, (Fig 2)
The critical depth, $z_{kr}$, where the shear stress equals the shear strength, is found by combining equations 2 and 3:

$$z_{kr} = \frac{c}{\rho g (\sin \alpha - \tan \varphi \cos \alpha)} \quad (m) \quad (4)$$

The critical thickness of the slab in the starting zone is defined by eq. 4, where $c = 2$ kPa and $\tan \varphi = 0.2$, according to Swiss guidelines. The slope angle $\alpha$ is equal to the average slope angle in the starting zone, $\theta$. The slab thickness will increase substantially as the slope of the starting zone decreases. These objective criteria were applied to decide the slab thickness.

The width of the starting zone was manually defined based on terrain analysis and experiences from observed starting zones.

Friction coefficients

Dynamical Coulomb friction:

Both RAMMS and Elba+ recommend standard values for the dynamical friction coefficient, $\mu$. The values were found by back calculation of recorded dry avalanches in Switzerland and Austria, respectively. RAMMS is able to automatically generate $\mu$ values based on the height level of the avalanche path, terrain classification, return period and volume. The height level was adjusted based on the tree line in each avalanche path. This adjustment was made to account for the lower tree line in Norway, due to a colder climate compared to Switzerland. Recommended $\mu$ values in RAMMS varies between 0.14 and 0.47.

Elba+ has possibilities for manual variation of the friction coefficient. In this research, standard values are used, which is 0.25 in the starting zone, 0.155 in the avalanche path and 0.25 in the run-out zone.

Velocity-dependent friction coefficient:

The velocity-dependent friction coefficient, $\xi$, includes the effect of surface roughness and energy dissipation caused by internal movements within the flowing snow. The values used in RAMMS are defined based on the same criteria as the Coulomb friction, $\mu$, and $\xi$ varies between 900 and 4000 m/s².

Elba+ operates with dynamically calculated $\xi$ values for each time step and raster cell. Manual adjustment of the $\xi$-value is not possible.

2.3 Empirical models

The Alpha-Beta model

The Alpha-Beta model was developed based on the theory that extreme climatic conditions occur at least once in a period of 100-300 years. However, it is thus assumed that extreme avalanches are more influenced by topographic conditions than climatic conditions. The Alpha-Beta model was developed by analysing the best-fit relationship between the run-out and the different topographic parameters. The $\beta$ angle, which is the angle from the starting point to the point where the slope angle is 10°, was found to be superior to the other parameters. The regression analysis gave the following simple relationship between the run-out angle, $\alpha$, and the $\beta$ angle:

$$\alpha = 0.96 \beta - 14° \quad (º) \quad (5)$$

The standard deviation was found to be 2.3° based on 250 avalanche sites in Norway. In practical zoning, standard deviation is subtracted from eq. 5 if a long return period is required. The Alpha-Beta model only gives an estimate of the run-out distance (Fig 3). Consequently, to obtain information regarding velocities, other models should be applied.
The Energy Line model

The Energy Line model was presented by Norem in 2014, and it estimates the velocity and the run-out distance of snow avalanches along a pre-defined avalanche path. The model is mainly developed based on experiences from the Ryggfonn project (Norem et al. 1983-1995) (Gauer et al. 2007) and radar recordings of avalanches, presented by Gubler in 1986. The model derives from basic knowledge of avalanche dynamics and on the influence of topographic conditions. When considering snow avalanches as a shallow flow in an open channel, hydrodynamics laws are used to describe the flow. The energy height for shallow flow is represented by the sum of potential energy and the kinematic energy (Fig. 4):

\[ H_e = H_z + H_k = H_z + \frac{v^2}{2g} \]  

Where \( H_e \) = Energy height, \( H_z \) = Height of the avalanche path, \( H_k \) = kinematic energy height and \( v \) = avalanche velocity. The height of the avalanche path is the elevation difference between the starting and the stopping position, where the velocity equals zero in both positions.

The height of the energy line decreases with distance from the starting position. Decrease in energy height represents the energy dissipation from one point to the next. The gradient of the energy line represents the total friction, i.e. both Coulomb friction and the velocity-dependent friction.

Figure 4 shows the avalanche path, energy line and calculated velocities of Nakkefonna, one of the analysed sites. The energy line shows a fairly straight line, with a gradient of 0.4:1 (21.8°) in the run-out zone. This gradient is very similar to recorded avalanches in Ryggfonn, and to the radar recording published by Gubler (1986). It is assumed that most extreme avalanches provide gradients close to 0.4:1. In certain cases, the energy line may be as gentle as 0.35:1 (19.3°), and exceptionally as low as 0.3:1 (16.7°). A gradient of 0.35:1 is applied in this research.

Estimating run-out distance and maximum velocity:

The maximum velocity is found where the avalanche path and the energy line have the same gradient. This is usually close to the 20°-point (tan 20°=0.36) of the avalanche path. The kinematic energy height, \( H_k \) in the 20°-point is estimated by using the lowest value of the following two criteria, (Fig. 5):

\[ v_{term} = \frac{3000 \cdot h_{g}^{1.5} \cdot (\sin \gamma - 0.31 \cdot \cos \gamma)}{16} \]  

Where, \( v_{term} \) = terminal velocity, \( h_{g} \) = slab thickness calculated by eq. 4, and \( \gamma \) = the angle from the starting point to the 20°-point (Fig. 5).

1. Avalanches with considerable height differences will probably obtain a terminal velocity. This velocity is estimated by eq.7:

2. Avalanches with limited height differences or a gentle path will never obtain a terminal velocity. For such avalanches, the maximum kinematic energy height is defined as the
difference between the line with gradient 0.3:1, drawn from the front of the avalanche, and the height of the path in the 20°-point (Fig. 5).

Velocity distribution in the run-out zone:

The velocity at any point in the run-out zone is found by making a line from the top of the energy height in the 20°-point towards the run-out zone. The line has a gradient of 0.35:1, and $H_{kx}$, the kinematic energy height, could be found at any point along this line. The velocity is then calculated by eq. 8, (Fig. 5):

$$v_x = \sqrt{2gh_{kx}} \quad (m/s)$$ (8)

Where, $H_{kx}$ and $v_x$ are the kinematic energy height and velocity at distance $x$ from the 20°-point.

The energy height at any point is also possible to calculate for the dynamical models by adding the height of the avalanche path and the kinematic energy height, $v^2/2g$. For the present analysis the gradient of the energy line is estimated from the stopping position and 200 m uphill in the run-out zone.

Run-out distance:

The run-out distance is found where the energy line with gradient 0.35:1 intersects the profile of the avalanche path.

3. CHARACTERISTICS OF THE ANALYSED AVALANCHE PATHS

The models are applied to 15 avalanches. One aim was to select well-documented avalanches with a return period exceeding 100 years, as well as to cover different climatic zones and height levels.

The main characteristics of the selected avalanche paths are shown in Tbl 1, where $\theta =$slope angle in the starting zone, $\beta =$beta angle and $\gamma =$gamma angle:

<table>
<thead>
<tr>
<th>Name</th>
<th>Climate</th>
<th>Height diff. (m)</th>
<th>$\theta$ (°)</th>
<th>$\beta$ (°)</th>
<th>$\gamma$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Haukelin</td>
<td>Above tree line</td>
<td>303</td>
<td>37</td>
<td>29</td>
<td>29</td>
</tr>
<tr>
<td>Vindalsfonna</td>
<td>Above tree line</td>
<td>713</td>
<td>35</td>
<td>28</td>
<td>30</td>
</tr>
<tr>
<td>Nakkefonna</td>
<td>Maritime. Run-out below tree line</td>
<td>843</td>
<td>36</td>
<td>26</td>
<td>27</td>
</tr>
<tr>
<td>Tverbotnflylet</td>
<td>Run-out below tree line</td>
<td>1074</td>
<td>35.5</td>
<td>31</td>
<td>32</td>
</tr>
<tr>
<td>Joengfonna</td>
<td>Maritime</td>
<td>1117</td>
<td>39</td>
<td>38</td>
<td>40</td>
</tr>
<tr>
<td>Ryggfonna</td>
<td>Maritime. Run-out below tree line</td>
<td>797</td>
<td>37.5</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>Tyinstolen</td>
<td>Above tree line</td>
<td>152</td>
<td>35</td>
<td>29</td>
<td>33</td>
</tr>
<tr>
<td>Vassfonna</td>
<td>Maritime. Run-out below tree line</td>
<td>960</td>
<td>33</td>
<td>30</td>
<td>32</td>
</tr>
<tr>
<td>Hopsedet</td>
<td>Maritime. Above tree line</td>
<td>196</td>
<td>39</td>
<td>36</td>
<td>35</td>
</tr>
<tr>
<td>Grøndalen</td>
<td>Continental. Run-out below tree line</td>
<td>505</td>
<td>35.5</td>
<td>25</td>
<td>31</td>
</tr>
<tr>
<td>Sørдалen</td>
<td>Maritime. Run-out below tree line</td>
<td>300</td>
<td>36.5</td>
<td>30</td>
<td>31</td>
</tr>
<tr>
<td>Knutstugugrove</td>
<td>Continental. Run-out below tree line</td>
<td>780</td>
<td>44</td>
<td>29</td>
<td>34</td>
</tr>
<tr>
<td>Kyrfonna</td>
<td>Maritime. Run-out below tree line</td>
<td>815</td>
<td>35.5</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>Sogndalsdalen</td>
<td>Continental. Run-out below tree line</td>
<td>618</td>
<td>36</td>
<td>31</td>
<td>33</td>
</tr>
<tr>
<td>Heggtveitjuvet</td>
<td>Continental. Run-out below tree line</td>
<td>595</td>
<td>38</td>
<td>23</td>
<td>28</td>
</tr>
</tbody>
</table>

4. RESULTS AND DISCUSSION

4.1 Run-out distance

Assessed results:

- RAMMS with standard parameter values, but with adjustments of the height level according to the tree line at each site.
- Elba+ with standard parameter values.
- The Alpha-Beta model minus one standard deviation.
- The Energy Line model with 0.35:1 gradient.
Tbl 2. Shows the model results in terms of calculated average run-out differences, correlation coefficients and standard deviations.

| Model         | Average run-out difference ($X_{\text{comp}} - X_{\text{rec}}$) | Av. absolute run-out difference ($|X_{\text{comp}} - X_{\text{rec}}|$) | Coefficient of correlation | Standard deviation (m) |
|---------------|---------------------------------------------------------------|-------------------------------------------------|---------------------------|------------------------|
| RAMMS         | -63                                                           | 78.6                                            | 0.961                     | 122.8                  |
| Elba+         | -125                                                          | 131                                             | 0.958                     | 165.8                  |
| Alpha-Beta    | 17                                                            | 133                                             | 0.944                     | 153.8                  |
| Energy Line   | -47                                                           | 92                                              | 0.947                     | 128.9                  |

The results for each avalanche, sorted by the three climatic groups (above tree line, continental, maritime), are shown in Fig. 6. There is a slight tendency of underestimated run-out distances for the avalanches above tree line and in the continental climates. This is most evident in the Alpha-Beta model.

In general, the dynamical models calculate shorter run-out than the recorded avalanches. The deviation is between 100-400 m in five of the cases. One reason for this may be the objective criteria for starting zone selection, which probably provide too small volumes. In addition, none of the dynamical models account for entrainment during the flow, which highly affects the total flow volume and standard deviation compared to the other models. None of the calculated run-out distances corresponds to the recorded distances, and only four results deviate with less than 60 m. This indicates that the model should be carefully applied. In comparison, the Energy Line model results equalled the recorded run-out distance in two of the cases.

The correlation coefficient represents the deviation from the trend line. Both the absolute deviation and the standard deviation are lower for RAMMS than for Elba+. This indicates that RAMMS may be more accurate than Elba+, even when accounting for the shorter run-outs modelled by Elba+.

The Alpha-beta model and the Energy Line model had the best fit to the average run-out distance. The correlation coefficients were, however, slightly lower for these models than for the dynamical models. RAMMS and the Energy Line model provided the lowest absolute deviation and standard deviation. Elba+ and the Alfa-Beta model got a significantly higher value.

The Alpha-Beta model got the lowest average run-out difference. However, the model got a high run-out distance. However, RAMMS and Elba+ got the best correlation coefficients.

Fig. 6: Recorded run-out distance compared to estimated run-out distances. Sorted by climatic conditions.
and deviate with less than 90 m in ten of the cases.

RAMMS estimated a run-out distance equal to the recorded run-out distance in two of the cases, and a deviation less than 60 m in nine of the cases. On the other hand, Elba+ got an average run-out difference that significantly deviates from zero. This is probably due to the systematically short run-out calculations. As the correlation coefficient is relatively high, it is likely that the model will obtain better results in Norwegian conditions by adjusting the friction parameters.

Based on the results in Tbl. 2, it seems like RAMMS and the Energy Line model are the most appropriate models for analysing Norwegian avalanches. Despite that the Alpha-Beta model is developed based on Norwegian conditions, it significantly deviates from recorded run-out distances in several cases.

In terms of the Energy Line model, the largest deviations occur when the beta-point and the gamma-point are close. This phenomenon is called “hockey stick-avalanches”, which probably indicates that the model is better suited for avalanche paths of a parabolic shape. This phenomenon also applies to the Alpha-Beta model.

4.2 Maximum velocity

Fig. 7 shows the calculated maximum velocities sorted by height differences. The highest velocities are generally calculated by RAMMS. On average, Elba+ estimates velocities 6 m/s lower than RAMMS. However, the velocities are all within realistic values compared to recordings of full-scale and radar experiments.

The velocities calculated by the Energy Line model show very few variations, and estimate terminal velocities around 40 m/s. The model is based on the assumption that all avalanches reach a terminal velocity where the slope of the avalanche path is similar to the gradient of the energy line. The estimated terminal velocity is most dependent of the gradient in the starting zone. Gradients of the analysed starting zones is within a small range, from 35° to 39°. This may be a possible explanation to the small deviations in calculated maximum velocities.

The Energy Line model calculates the largest velocity differences in the cases where the height difference of the avalanche path is small, 152 m and 196 m, respectively. Due to a short acceleration length, these avalanches are not able to reach their terminal velocity.

4.3 Velocity distribution in the run-out zone

The calculated gradient of the energy line is an important characteristic when assessing the quality of avalanche models. It shows how fast the velocity is reduced, and incorrect estimates may lead to under-dimensional protection measures. Experiences from full-scale experiments, both in Norway and Switzerland, indicate that the front velocity is rapidly reduced in the run-out zone. In the recorded avalanches, the gradient of the energy line varies between 0.35:1-0.4:1 (m/m).

The average gradient of the RAMMS and Elba+ is 0.26 and 0.30, respectively (Fig 8). These values are probably too small compared to the recorded events. Consequently, velocities calculated by the dynamical models should be applied with caution. When dimensioning protection measures, a gradient of 0.35 should be a minimum, and probably 0.4 would be more realistic for obtaining an even better higher safety.

The dynamical model results lead to gradients steeper than 0.35 in a few of the cases. These were avalanche paths more similar to the “hockey sticks” than the parabolic paths, which is probably the reason for the divergent results.
5. CONCLUSIONS

In terms of the dynamical models, both RAMMS and Elba+ calculate too short average run-out distance and too small velocity gradients compared to recorded avalanches. However, RAMMS with adjustments based on climatic conditions provide the best run-out results. The model got a high correlation coefficient and a lower standard deviation value, although the average run-out distance is too short. Elba+ provides more divergent results. However, if the friction parameter values are adjusted to Norwegian conditions, Elba+ might become appropriate to apply in risk-based zoning. Currently, the velocity gradient of both models should be carefully applied when dimensioning avalanche protection measures.

The statistical analyses indicate that the Energy Line model is more accurate than the Alpha-Beta model. Considering their simplicity, the empirical models are surprisingly accurate. The Energy Line model, for instance, seems to yield better results than the Elba+ model.

A disadvantage of applying empirical models is the limited results. Maximum run-out distance along a selected profile is often the only output provided. Admittedly, the Energy Line model allows for velocity estimation. However, the dynamical models provide significantly more output information, such as width, flow height, run-out and velocity in a 3-dimensional terrain. These features are great advantages when it comes to dimensioning protection measurements.

Defining reliable objective criteria in the dynamical models is a demanding task. Nevertheless, it is crucial when comparing different models. This analysis is only one of many approaches for such comparison. However, the results indicate that several of the models are suitable for Norwegian conditions. No one model is superior to other models. Overall, the statistical analysis shows that RAMMS and the Energy Line model provide the best results. To obtain the most realistic results, at least one dynamical and one empirical model should be applied. Further efforts should be made to establish methods for objective criteria selection, especially in terms of starting zone and friction parameters.

REFERENCES


