MEASURING SNOW WATER EQUIVALENT IN WET SNOW WITH ULTRAWIDEBAND GPR

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ABSTRACT: Accurate estimates of snow water equivalent (SWE) are important for mountainous areas where snow accounts for the majority of the annual precipitation; however, measurements are generally limited to the SNOw TELemetry (SNOTEL) network and detailed snow study plots at ski areas, avalanche forecasting programs, and research stations. Radar has been proven to be an effective tool for estimating SWE in dry snow but has not been developed as a wide-scale sensor for estimating SWE because the liquid water in wet snow packs causes frequency-dependent attenuation and decreases the velocity, leading to large uncertainties. A recently available, low cost 1-6 GHz ground penetrating radar is tested for measuring SWE in wet snow by using the spectral shift method to account for the attenuation and liquid water content (LWC). The difference in frequency of the source and ground surface reflection is used to estimate the attenuation factor in the snow pack. The attenuation factor is a function of the complex dielectric permittivity of the snow. Established empirical relationships relate the complex permittivity to LWC and SWE. Radar mounted on a fixed post and operating autonomously and inversion methods were used to produce SWE estimates that were within 8-18% of SWE values taken in a nearby snow pit.

KEYWORDS: Radar, snow, snow water equivalent, liquid water, frequency, permittivity.

1. INTRODUCTION

Snow accounts for 40 to 70% of the annual precipitation in mountainous areas of the western United States (Serreze et al., 1999), making accurate estimates of SWE important for water resource managers, avalanche forecasters, recreationists and others. Spatially dense measurements of SWE remain elusive, however. The SNOTEL network is the primary data source for SWE, but has only roughly 800 point measurements spread across the western United States, and the high cost and complex installation inhibit expansion of this valuable long-term network. Radar has been proven to be an effective tool for estimating SWE in dry snow (e.g. Ellerbruch et al, 1980; Marshall et al, 2005) but has not been developed as a wide-scale tool for SWE measurement because of the cost of commercially available GPR instrumentation and difficulties in wet snow packs, although recent efforts to characterize the liquid water content have been made (e.g., Okorn et al, 2014; Mitterer et al, 2011). Liquid water in wet snow rapidly attenuates the radar signal and decreases the velocity, leading to errors in SWE estimates of up to 20% (Lundberg, 2000).

Liquid water in wet snow also causes frequency-dependent attenuation of the radar signal. The spectral shift method estimates the attenuation of a signal using the difference in frequency content of the source and ground reflections, from which the complex electrical permittivity of the snow is known (Bradford, 2009). Established empirical petrophysical models relate the complex permittivity to liquid water content and snow density.

A 1-6 GHz ground penetrating radar (GPR) has recently become commercially available, which combines the increased resolution of higher frequencies while still being low enough to penetrate a moderately wet snow pack, and significantly lower cost than previously available GPRs. The GPR was installed on a fixed post at the Dry Creek Experimental Research Site just north of Boise, Idaho, operating autonomously from on-site solar power and collecting traces every 15 minutes. Traces were collected from March 7th through melt-out in mid-May over snow conditions ranging from 0 – 2 m snow with 2 – 8% LWC.

2. THEORY

A bulk velocity estimate for the snow, obtained from the radar mounted on a fixed post and known snow depth, gives the real permittivity of the snow ε_s' by

$$v = \frac{2z}{twt} = \frac{c}{\sqrt{\varepsilon_s'}} \tag{1}$$

where z is the snow depth and twt is the two way travel time. Throughout, subscript s refers to wet snow and d to dry snow. The complex electrical permittivity of snow $\varepsilon_s' + i\varepsilon_s''$ can be used to estimate SWE using radar data through well-established empirical relationships (Tiuri et al, 1984)

$$\varepsilon_d' = (1 + 1.7\rho_d + 0.7\rho_d^2) \tag{2}$$

$$\varepsilon_s' = (0.1W + 0.8W^2)\varepsilon_w' + \varepsilon_d' \tag{3}$$

$$\varepsilon_s'' = (0.1W + 0.8W^2)\varepsilon_w'' \tag{4}$$

where W is the snow wetness by volume, ρ_d is the dry snow density and $\varepsilon_w' + i\varepsilon_w''$ is the complex permittivity of water. The real component of permittivity is mainly a function of snow density in dry snow and is approximately independent of frequency in the 200 MHz to 2 GHz frequency range, and the imaginary component of permittivity, which describes the attenuation of the signal, is a function of liquid water in a wet snow pack (Fig. 1).

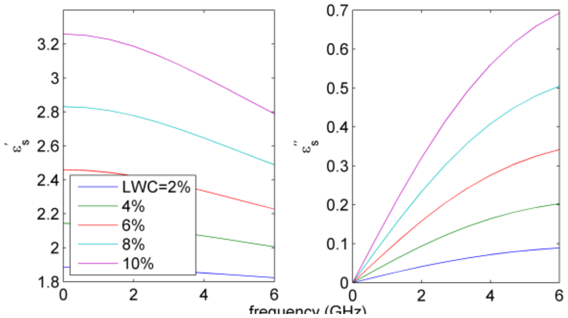

Fig. 1: Real and imaginary components of the dielectric permittivity of wet snow for $\rho_s = 350\ kg/m^3$as a function of frequency for a range of liquid water content values are shown. The approximation that the permittivity is independent of frequency does not hold for the full frequency range.

A Cole-Cole relaxation model describes the frequency-dependent complex permittivity of water (Bradford, 2007). The full derivation of the expression for the attenuation of wet snow in terms of the frequency band of the signal and the complex permittivity of the snow is given by Bradford (2007), the key points of which are outlined here. If the approximation is made that the real permittivity of wet snow is independent of frequency and the imaginary permittivity is linear in the 200 MHz to 2 GHz frequency range the attenuation coefficient can be written as

$$\alpha \approx \alpha_o + \frac{\sqrt{\mu_o \varepsilon_{\omega_o}'}}{2Q^*}(\omega_t - \omega_o) \tag{5}$$

where α_o and ε_{ω_o}' are the attenuation and real permittivity at the reference frequency ω_o, respectively, Q^* is the empirical constant that describes the relationship, and ω_t is the angular frequency of the signal after traveling through the snow for some time. The frequency after propagation ω_t could be any point in the snow pack and is used as the ground surface reflection here. The difference in frequency of the source and ground surface reflections can be used to estimate Q^* by

$$\frac{1}{Q^*} = \frac{4}{t}\frac{(\omega_o^2 - \omega_t^2)}{\omega_o^2 \omega_t} \tag{6}$$

If the real permittivity is approximated as independent of frequency the attenuation coefficient can be written as

$$Q_s^* = \frac{\varepsilon_s'}{2\varepsilon_s''}. \tag{7}$$

The GPR used in this study transmits in the 1-6 GHz range with a 2 GHz center frequency, where ε_s' begins to depend on frequency and the relationship is no longer linear. Some error is introduced by this approximation. The reference frequency is the maximum source frequency rather than the center frequency. The instantaneous frequency of the signal is used to convert the time domain trace to the frequency domain (Taner et al., 1979). If $x(t)$ is the real time domain signal the complex trace is given by

$$S(t) = x(t) + iy(t) \tag{8}$$

where $y(t)$ is Hilbert transform of the trace. The instantaneous phase $\theta(t)$ can be computed from the complex signal and the instantaneous frequency is defined as the time derivative of the phase

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t). \tag{9}$$

3. RESULTS

The GPR collected traces every 15 minutes from March 7 through May 6, 2014 (Fig. 2). Visits to the site were made approximately weekly, and measurements in a nearby snow pit included snow depth, density and wetness measurements with a Finnish snow fork (Sihvola et al, 1986). As a result of the relatively warm winter and mid-season installation there was liquid water present for all measurements, and a subsequently small dynamic range of LWC and SWE values.

Four days of measurements are compared to snow pit values of SWE (Fig. 3). The GPR-derived SWE values are within 8-18% of the actual SWE, with a mean difference across all traces and days of 8%. We could not measure SWE directly below the radar since that area had to remain undisturbed, and we expect some true spatial variability driven differences between the in-situ ground truth snowpit and the radar. For the observed snow densities and liquid water content of this study the range resolution of the radar is ~5 cm, resulting in ±5 cm SWE values. Single traces for each day showed a large range of LWC and snow density values driven mainly by the variation in the instantaneous frequency of the ground surface reflection picks. Without accounting for liquid water, the same measurements gave 11-35% error in SWE, with a mean SWE error across all measurements of 26%.

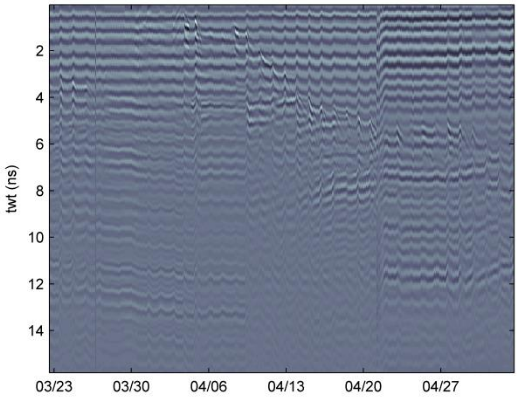

Fig. 2: Section of radar profile from the fixed post is shown. The ground surface reflection is at ~11–13 ns, and gaps are also visible. Some internal layers appear and the method could be used to determine individual layer properties.

Specific LWC values from the snow pit are available for comparison to radar-derived LWC values for two dates (Fig. 4). As with the SWE values, there was a large range of LWC values for individual traces caused by variation in the ground surface frequency picks.

For lower LWC values on March 18 the radar-derived liquid water content of 3.4% was in good agreement with the snow fork value of 2.6%. For higher LWC values seen on April 9th, for example, the radar-derived LWC of 2% was significantly less than the snow fork value of 6.8%, which contributed to the SWE error for that day.

For periods after the April 9th the ground surface reflection was no longer distinguishable in the radar image for a ~2 m snow pack with LWC values approaching 10%.

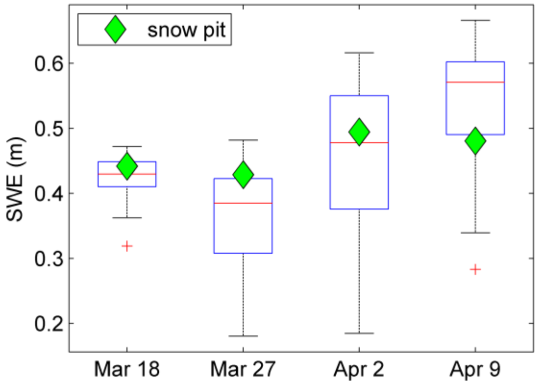

Fig. 3: Radar and snow pit-derived SWE for four dates. Each day's values are taken from ~50 traces from approximately 7am to 7pm. Although mean radar-derived SWE values agree well with snow pit measurements, individual traces show a large range.

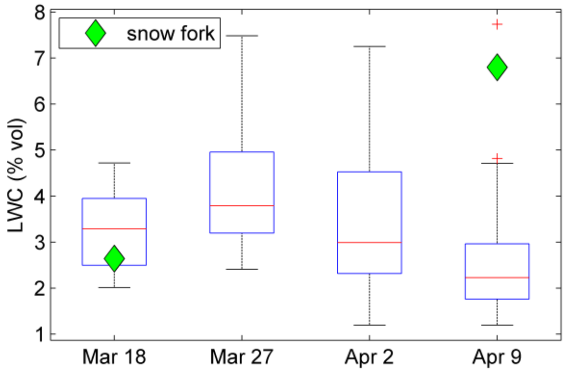

Fig. 4: Radar and snow fork-derived LWC values for available dates are shown. As with the bulk SWE, radar-derived LWC values show a large range over individual traces for each day. For the one day of higher LWC comparison available here, at 7%, the radar-derived value showed high error.

4. CONCLUSIONS

The spectral shift method applied to 1-6 GHz GPR is an effective tool for estimating the liquid water and subsequently measuring SWE in wet snow for moderate snow depths and LWC values. Estimated values were within 8 - 18% of measured values, or ±5 cm SWE, in snow below 7% LWC. The spectral shift method is very sensitive to the specific ground surface picks for each trace, as that determines the total travel time and instantaneous frequency that are used in subsequent calculations. The simplifying assumption that the real permittivity is independent of frequency above 2 GHz, and subsequently that the attenuation parameter Q^* is linearly related to the complex permittivity, introduces some error in the measurements.

The inability to distinguish the ground surface reflection at ~2 m depth with ~10% LWC, which are common conditions in seasonal snow packs at peak SWE, indicates that additional improvements, possibly including directional antennas, amplification and a metal plate on the ground surface to enhance that reflection, could be helpful.

These are encouraging first results for adapting 1-6 GHz GPR as an inexpensive, easy to maintain tool for measuring SWE, which could be deployed much more easily than traditional sensors.

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