THE METHOD FOR ESTIMATING OUTFLOW FROM THE SNOWPACK USING THE SIMPLIFIED HEATBALANCE MODEL AND THE PERCOLATION MODEL

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ABSTRACT: To ensure the safety of train operations, it is important to estimate the outflow from the bottom of snowpack which might incur a full-depth avalanche. Therefore, the authors examined a simplified method for estimating the outflow from the bottom of snowpack, which can be applied to railway disaster prevention, on the basis of heat balance observations, snowmelt observations and previous studies. Consequently, the simplified method, which combines a snowmelt (heat balance) model and a percolation model, yields a good estimate of the outflow from the bottom of snowpack at 1 hour intervals using four input data available from the nearest AMeDAS: i.e., air temperature, precipitation, wind speed and duration of sunshine.

KEYWORDS: snowmelt, outflow from snowpack, heat balance method, full-depth avalanche

1. INTRODUCTION

In snow-covered areas, at the beginning of spring, a large amount of snow melts on the snow surface and the water infiltrates the accumulated snow. In recent years, even in winter, temporary temperature rises have been observed and large volumes of rain and snowmelt have been recorded (Kawashima et al., 2002). For example, in Sapporo on January 23, 2009, the temperature rose to 7.6 °C in winter, and 18.5 mm of precipitation was observed. In such situations, it is known from a qualitative point of view, that water between snow and soil reduces the bearing capacity of the snowpack, creating the danger of a full-depth avalanche increase. However, it is still unknown the extent to which rain or snowmelt affects the danger of a full-depth avalanche. Therefore, in railway companies, when the increase of the temperature or rain was observed in winter, rounds and watches were carried out based on empirical rules. In recent years, the number of employees with abundant experience however, is decreasing. In order to carry out suitable rounds and watches, it is important to clarify the relationship between snowmelt and full depth avalanche occurrence, which should then be taken into account in setting guidelines for rounds and watches in winter.

2. ESTIMATION OF THE OUTFLOW FROM THE BOTTOM OF THE SNOWPACK

To estimate the outflow from the bottom of the snow pack, a method can be used to estimate the outflow directly (water equivalent method, lysimeter method\textsuperscript{*}) or indirectly (degree-day method\textsuperscript{*})(The Japanese Society of Snow and Ice Hokkaido-branch, 1991)(Kondo, 2000). The water equivalent method calculates outflow from the weight of the snowpack using snow pillows. The lysimeter method observes percolation through the snow pack and is set on the ground surface. Both methods have the advantage of offering direct observation of the outflow, but they also need large spaces (10 m\textsuperscript{2}) to set the observation equipment and careful maintenance management to continue exact measurement. Therefore, it is not reasonable to install this observation equipment along tracks from the view point of cost and maintenance. On the other hand, the degree-day method calculates the outflow in relation to temperature simply using empirical rules based on the relationship between outflow and temperature. Because this method is easy, and is the same time cost efficient compared with other methods, it was fur-
ther investigated with a view to application as a trackside disaster prevention measure. For example, we use this method to predict avalanche along railroad, and when the avalanche is predicted we will make official announcement to the station etc. The precision of the degree-day method is a daily moving average. As such it was difficult to compute outflow to reach a 1 hour time resolution required for railroad disaster prevention. The present research aimed to satisfy this requirement by examining techniques to estimate outflow per hour. In order for this method to be applied to railway disaster prevention it is preferable that input data be easily obtained. It was therefore determined that input values for this estimation method should be the observed data collected from the nearest AMeDAS (Automated Meteorological Data Acquisition System). Figure 1 shows the process for estimating outflow from the snowpack. It is necessary to estimate the outflow accurately every hour. Two phenomena found in snowpack, i.e. “snowmelt at snow-surface” and “infiltration in snowpack” where therefore considered separately. A model was then designed to estimate each phenomenon, based on weather/snowmelt observations and past research results. The following sections describe how each model was created and present the estimations subsequently obtained.

3. OBSERVATION

Snowmelt and weather data from the Shiozawa Snow Testing Station (Minami-Uonuma, Niigata, Japan) during the winter of 2011 were observed to obtain the data needed to develop a method for estimating outflow from the bottom of the snowpack. Observations were made of temperature, humidity, precipitation, snow depth, wind direction, wind speed, air pressure and solar radiation (hours of sunshine) (see Section 4.1). Outflow was measured with a lysimeter. Measurements were collected every ten minutes for 110 days from January 6 to April 25 (date when no snow was left) in 2012. Below is a sample of snowmelt observation data.

4. BUILDING A MODEL FOR ESTIMATING THE SNOWMELT AT THE SNOW SURFACE

Snowmelt \((M_s)\) amount of snowmelt at the snow surface) was calculated using the heat-balance method that has been widely used in snowmelt estimation analysis (Fig. 1). This study first presents an outline of the heat-balance method, followed by the description of a method for calculating \(M_s\) from the results of observations (four types of meteorological data: temperature, precipitation, wind speed, hours of sunshine).

4.1 Estimation of the snowmelt using the heat balance method

The heat balance method is a method for calculating the amount of heat in snowmelt estimation \((Q_M)\) from the heat balance at the melting snow surface (Fig. 2). \(Q_M\) at each time can be found from a heat budget (Equation 1).

\[
Q_M = Q_R + Q_H + Q_L + Q_P + Q_C
\]

(1)

where \(Q_R\) is net radiation, \(Q_H\) is detectable heat exchange, \(Q_L\) is latent heat exchange, \(Q_P\) is heat content of liquid precipitation, \(Q_C\) is heat exchange at snow surface. In equation 1, the snow surface was defined as the reference plane, the amount of heat into the reference plane is assumed to be positive, and the amount of heat out of the reference plane negative, and the unit of heat is represented as W/m². \(M_s\) is calculated by dividing \(Q_M\) (W/m²) obtained in equation 1 by latent heat of ice \((=0.334\times106\) J/kg). The methods for calculating each heat quantity shown in equation 1 are shown.
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Heat exchange into the snowpack

\[ Q_F = \rho \cdot C_w \cdot T_w \cdot P_r \]  

where \( \rho \) is the density of water (1000 kg/m\(^3\)), \( C_w \) is the specific heat of water (4.186 J/kg/K), \( T_w \) is wet-bulb temperature (\(^\circ\)C), \( P_r \) (kg) is precipitation.

(4) Heat exchange at the snow surface \( Q_C \)

Heat exchange at the snow surface \( Q_C \) is generated by the temperature gradient in the snowpack. If the snow is 0 \(^\circ\)C, \( Q_C \) is almost 0 (Maeno and Fukuda, 1994). And if there is a temperature gradient in the snow, \( Q_C \) is smaller than the other elements. For this reason, in this study, we gave \( Q_C \) as 0 W/m\(^2\).

### 4.2 Estimation of the snowmelt using four types of meteorological data

As shown in Section 4.1, if only weather observation data have been obtained, the heat balance method could be used to calculate snowmelt at the snow surface per hour regardless of the region. However, there remains a problem in that many sorts of input data are necessary for applying the conventional heat balance method to railway disaster prevention. Therefore, in this section, a method was examined to see how to incorporate the weather/outflow observation data and the result of previous studies into the estimation process of each element required for the heat balance method, and to estimate the \( M_s \) from only four weather observation data. In the following, the method for estimating each element is described.

(1)Net Radiation \( Q_R \)

The net shortwave radiation \( K_d \) (that is given by the solar radiation onto the snow surface) was calculated from the solar radiation in the upper atmosphere (Kun and Tosho, 2005). The amount to be reflected at the snow surface \( K_d \) (the reflective shortwave radiation) of \( K_d \) reaching the snow surface could be represented by the product of reflectance (albedo) \( \alpha \) and \( K_d \). Here, it is known that \( \alpha \) is variable according to the amounts of moisture and

<table>
<thead>
<tr>
<th>Meteorological Condition</th>
<th>( T_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ( T_s \geq 0 )</td>
<td>( T_s = 0 )(^\circ)C</td>
</tr>
<tr>
<td>(b) Night time(Solar Radiation at the top of atmosphere =0), and ( T_a ) is decreasing (Radiative Cooling arising)</td>
<td>( T_s = T_a - 3 )(^\circ)C</td>
</tr>
<tr>
<td>Except the above (a) and (b)</td>
<td>( T_s = T_a )(^\circ)C</td>
</tr>
</tbody>
</table>
relative humidity presumed from the clouds and water vapor of the air was obtained using the logarithm rule for perpendicular distribution of wind velocity (Kondo, 2000) and setting \( z \) at 2 m as indicated in the previous study (Pellicciotti et al., 2008). Although snow depth \( D_s \) at an observation point is needed at this time, \( D_s \) is computable from four observation data (refer to Section 5.1). Moreover, local atmospheric pressure \( AP \) was computed from temperature \( T_a \) and the altitude of the observation point based on the sea-level atmospheric pressure reported by weather government offices (Japan Meteorological Agency, 2002). Water vapor pressure \( e_r \) was computed from \( T_a \) and \( r_h \) (Meteorological Society of Japan, 1996). The saturation water vapor pressure at snow surface temperature \( e_{st} \) was estimated from \( T_{st} \) of the observation point using the approximate Teten’s expression (Kondo, 1994).

\[
\text{(3) Heat content of liquid precipitation } Q_r
\]

The wet-bulb temperature used in equation 5 was computed from temperature \( T_a \), relative humidity \( r_h \), and atmospheric pressure of the observation point \( AP \) using the expression of Sprung (Japan Meteorological Agency, 2002). Moreover, as for precipitation \( P_r \), rain/snow distribution was performed from the temperature \( T_a \) measured when the rainfall was observed. In this research, the precipitation observed when \( T_a \) was below 0 °C was categorized as snowfall and that observed when \( T_a \) was over 0 °C was categorized as rain.

As mentioned above, using the method proposed this time, each of the elements shown in equation 1 can be determined from four observation data (temperature, precipitation, wind velocity and day light hours) which are obtained by AMeDAS etc. (Tbl. 3).

### 4.3 Result of estimations of snowmelt

In order to investigate the degree of difference between the quantity of heat for snowmelt \( Q_M \) calculated from the heat balance method shown in Section 4.1 considered to be a true value and \( Q_M \) obtained by the presumed model (refer to Section 4.2), both of the \( Q_M \) s were computed from the observational data at Shiozawa Snow Testing Station (the heat balance budget method needs seven types of observation data shown in Chapter 3 and the presumed model does four observation data shown in Section 4.2.) and were compared (Fig. 3). A total of 1689 hours were available for analysis (from January 6 to April 25 in 2011) which were the data without snow accretion to the measurement apparatus Fig. 3 shows snowmelt at the snow surface \( M \) (mm) converted from \( Q_M \) using the results from the above calculation.

<table>
<thead>
<tr>
<th>Tbl. 2: Relative Humidity ( r_h )</th>
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<tbody>
<tr>
<td><strong>Meteorological Condition</strong></td>
</tr>
<tr>
<td>( P_r &gt; 0 )</td>
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<tr>
<td>( P_r = 0 ) and ( T_a &lt; 0 )</td>
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<tr>
<td>( P_r = 0 ) and ( T_a \geq 0 )</td>
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<tr>
<th>Tbl. 3: Estimating Method of Snowmelt</th>
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<tr>
<td><strong>Quantity of Heat</strong></td>
</tr>
<tr>
<td>Net radiation ( Q_R )</td>
</tr>
<tr>
<td>Sensible heat exchange ( Q_L )</td>
</tr>
<tr>
<td>Latent heat exchange ( Q_H )</td>
</tr>
<tr>
<td>Heat content of precipitation ( Q_P )</td>
</tr>
<tr>
<td>Heat exchange into snowpack ( Q_c )</td>
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</table>
Although in both cases there was a maximum difference in $M_s$ of 3.3 mm/h, there was such a strong correlation that the coefficient of determination was 0.87, and the root-mean-square error (RMSE) over the true value was 0.5 mm/h. Moreover, comparing the integrated value of $M$ during the observation, the true value was 1202 mm and the value acquired from the presumed model was 1266 mm, and this result shows both were in agreement in general. As mentioned above, it turned out that the presumed model created by this research can estimate $M$ without the use of the heat balance method which needs many observation data.

5. DEVELOPMENT OF A MODEL FOR ESTIMATING SNOWMELT INFILTRATION

In order to reproduce the phenomenon of snowmelt infiltration, it is necessary to consider the influences of heat loss (refer to Section 5.1.) and time delay (refer to Section 5.2.). This chapter describes the snowmelt infiltration model created on the basis of weather/outflow observations and the results of previous studies.

5.1 Development of a model for estimating the characteristics of accumulated snow

Even if snow melt at the snow surface, when there is a snow layer below 0 °C in the snowpack, a part or all of the quantity of heat is used for rising the temperature of the snow pack, and melting water could not reach the bottom of snowpack. As a result snowmelt at the snow surface $M_s$ and outflow from the bottom of snowpack $M_b$ are not necessarily in agreement. Consequently, in order to estimate $M_b$ with sufficient accuracy, it is necessary to compute the loss of heat for fusion (called heat loss). In addition, in case all layers of snowpack are 0 °C, the quantity of heat loss is 0 W/m².

In this research, in order to compute heat loss temperature, “the snow coverage quality model” which estimates physical-property values of a snow layer (one layer of snow is formed in 1 hour) such as temperature of each snow layer, snow depth, density, weight etc. from four observation data. This model can estimate snow temperature $T_s$ in consideration of heat loss, and outflow from snowpack $M_b$ by adding $Q_M$ to the input value of the past viscous compression model (Endo et al.2004)(Suizu,2002). The calculation method of $T_s$ and $M_b$ examined in this research is as follows. First, we calculate snow temperature of each layer from the surface to the bottom using temperature $T_a$ (Suizu,2002). Next, in order to compute heat loss, the quantity of heat required for raising the temperature of each layer to 0 °C is calculated from equation 6.

$$Q_A = C_i w T_s / 3.6 \quad (6)$$

where $C_i$ is the specific heat of ice ($2.1 \times 10^3 \text{ J/kg \degree C}$), and $w$ is the mass of each snow layer (kg/m²). The difference in heat loss of each snow layer $Q_A$ was calculated by the heat of fusion $Q_M$ which the infiltrating snowmelt contained from the surface to the bottom of the snowpack. Then the heat loss of
each snow layer was calculated in consideration of the heat of fusion (Fig. 4). The heat of fusion in consideration of heat loss \( Q_{\text{h}} \) was obtained by deducting the amount of heat of fusion \( Q_{\text{m}} \) from the integrated value of the heat loss of each snow layer computed by this series of calculations. In this model, even if snow melted at the snow surface when \( Q_{\text{h}} \) was smaller than, \( M_b \) was computed at 0mm since the heat of fusion is used to obtain the rise in \( T_s \). In this research, \( M_b \) was continuously estimated by repeating the abovementioned calculation every hour over a whole winter. In addition, in this calculation process, \( T_s' \) in consideration of heat loss can be found by substituting \( Q_s \) for equation 6.

5.2 Development of a model for estimating accumulated snow

When rain and snowmelt infiltrate thorough snowpack, we must consider the influence of the storage effect i.e., the problem of “delay time” process. Delay time is the time lag between snow melting on the snow surface until it reach the bottom of the snowpack. This influence cannot be disregarded in the estimation of the outflow from the bottom of snowpack in units of hours. In a recent study (Nakatsugawa et al.2004)(Matsumoto et al.2010), assuming the infiltration of snowmelt through saturated snow layer, the outflow from the snowpack \( M(t) \) is estimated with the delay time \( (t) \) according to the Darcy’s law (equation 7). The research in question also used this technique.

\[
M_b'(t) = M_b'(t-1) \exp\left(-\frac{1}{k_0}\right) + M_b(t) + p_r(t) - \{M_b(t) + p_r(t)\} \exp\left(-\frac{1}{k_0}\right) + M_b(t)
\]  

(7)

Where \( M_b'(t-1) \) is the outflow from the snowpack 1 hour before the time \( t \) (delay time is taken into consideration), \( k_0 \) is the coefficient of storage which expresses the delay time, \( k_0 \) and \( D_s \) are outflow and precipitation in time \( t \) (without considering delay time), respectively. \( M_b \) is the amount of snowmelt at the bottom of the snowpack (snowmelt generated by ground warmth), referred to as 0.075 mm/h (Izumi,1983) throughout this research. It is known that delay time is dependent on snow depth or the structure of the snow layer. And in this research, \( k_0 \) was given as a function of snow depth \( D_s \) from the result of the weather/outflow observation in the Shiozawa Snow Testing Station. In addition, at the present stage, it is difficult to presume the structure of snow layer with sufficient accuracy, and there are many unknown variables in the relationship between the structure of the snow layer and the delay time. For this reason, the influence of snow layer structure on delay time was denoted by functions of \( k_0 \) and \( D_s \). When obtaining the functions of \( k_0 \) and \( D_s \) it is considered that 24 hours are one snow melting event. And the period of the event is ten days during which the estimated outflow of 24 hours \( k_0 \) and \( D_s \) the observed outflow of 24 hours \( k_0 \) and \( D_s \) are mostly in agreement and daily variation is clear. Next, \( k_0 \) and \( D_s \), which are the input values for each time were substituted in equation 7 to obtain.

After this, \( k_0 \) was sought for every snowmelt event by trial and error until the smallest integrated value of the square error of the calculated \( k_0 \) and \( D_s \) the measured, obtained from a lysimeter, was found. The relation of \( k_0 \) and \( D_s \) calculated by the above method is shown in Fig. 5. In this Figure, it can be observed that the longer the penetration time is, the larger \( k_0 \) is, and that when \( D_s \) exceeds 1.5 m, \( k_0 \) becomes remarkably large. Therefore, in this research, it was presupposed that the relation of \( k_0 \) and \( D_s \) is expressed with an exponential function as shown in the equation 8 and equation 9 (Nakatsugawa et al.,2004.)

\[
D_s > 0.5 \text{ (m)}, k_0 = 1.654 \exp(1.143 D_s) \\
0.5 \leq D_s \leq 0 \text{ (m)}, k_0 = 1.654
\]  

(8)

(9)

In addition, since \( k_0 \) may be subject to not only \( D_s \), but the snow layer structure etc., it is also assumed that \( k_0 \) is changing depending on the area or years. Therefore, continuous examination about how to give \( k_0 \) and the applicable condition of equation 8 is required.
6. RESULT OF ESTIMATION OF THE OUTFLOW FROM UNDER ACCUMULATED SNOW

According to the estimation procedure shown in Fig. 1, the outflow $M_b'$ computed from a series of models shown in Chapter 4 and Chapter 5 was compared with actual measurements of $M_b'$ obtained from observation data from the Shiozawa Snow Testing Station. As a result, the estimated peak value of $M_b$ and the appearance time of the peak showed the proximity to the observed value actually (Fig. 6). Fig. 7 shows the result of the comparison between the estimated values and the observed values during the whole observation period. Although at peak level, both values have a difference of 4.3 mm/h, the root-mean-square errors (RMSE) over the actual measurement are 0.5 mm/h, and both values correlate well. Estimating $M_b$ with not only the estimated model of snowmelt at the snow surface but also the infiltration model of the snowmelt have been created in this research, the RMSE of $M_b$ (estimated value) was decreased by about 45%, compared with the presumed model which infiltration process is not taken into consideration.

7. EXAMING APPLICABILITY OF THIS ESTIMATION METHOD FOR ANOTHER AREA

To examine applicability of this estimation method for another area, observation was made at Iwamizawa in 2013. Iwamizawa is about 800km north from Shiozawa Testing Station. And we estimate the outflow from the snowpack by these models. Fig. 8 shows the result of the comparison between the estimated values and the observed values in Iwamizawa and Shiozawa. As compare with Shiozawa, in Iwamizawa, it is cold and precipitation is small, and there is little outflow. Although at peak level, both values have a difference of 2.2 mm/h, the root-mean-square errors (RMSE) over the actual measurement are 0.4 mm/h, and both values are as well as the values of Shiozawa.

![Fig. 6: Outflow from under the snowpack at Shiozawa](image1)

![Fig. 7: The observed and estimated outflow from under the snowpack at Shiozawa](image2)

![Fig. 8: The observed and estimated outflow from under the snowpack at Iwamizawa and Shiozawa](image3)
The estimation method proposed in this paper makes it possible to estimate the outflow from the bottom of the snowpack per hour from only four types of observation data (temperature precipitation, wind velocity, and daylight) collected from AMeDAS observation points etc. which can be obtained easily, and the time resolution of the estimate is better compared to the past degree day method. In addition, by including the snow coverage quality model in the estimation process, the influence of snow temperature could be taken into consideration and the rain and thaw phenomenon accompanied by a temporary temperature rise in the middle of winter could also be evaluated. However, since many underestimated or overestimated points are observed in the actual measurement as shown in Fig. 7 and Fig.8, the continuous examination of the estimation of snow melt by the simplified heat balance model we made or the method of setting the coefficient of storage is required.

8. CONCLUSION
This research examined a method for estimating outflow from under a snowpack which is closely related to the generation of a full depth avalanche. The method proposed in this paper makes it possible to estimate the outflow from under the snowpack per hour from four types of observation data (temperature, precipitation, wind velocity, daylight) collected from nearby AMeDAS points. For this reason, its application to railroad disaster prevention (for example, a point where the observation of the outflow from the snowpack is difficult because of problems such as inaccessibility due to surrounding environment) is expected.

From now on, while aiming at the improvement and extension of the estimated method, we advance analysis about the relation between the outflow and the stability of the snowpack that is the danger index of a full depth avalanche. And we would like to establish a stability assessment method of snowpack on the slope. In addition, further research aims to explore the application of these methods to earth-and-sand collapse, etc.

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