AN ENERGY-BASED MICROSTRUCTURAL CONSTITUTIVE MODEL FOR FRACTURE IN SNOW

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ABSTRACT: Understanding fracture initiation and propagation in snow is paramount to avalanche science. There is ongoing discussion and debate regarding the mode of fracture and fracture properties of weak layers. Most approaches consider snow fracture by approximating the snow as a continuum. In actuality, snow is a highly porous, discontinuous medium with a complex microstructure that dictates when and how fracture propagates. A microstructure-based model has been developed to predict the critical energy release rate by considering the minimum energy fracture path through the ice network. The approach considers the intimate details of the snow grains, interconnecting bonds and pore spaces to predict the critical energy release rate of the snow sample. By only considering energy release, the model functions independently of fracture mode. A preliminary experiment, using high speed optical strain measurements, is used to determine material properties and derive experimental critical energy release rate. Experimental and analytical comparisons are presented. This approach provides an alternative to predicting snow fracture energy with no assumptions on macroscopic failure modes or mechanisms.

KEYWORDS: fracture, energy release rate

1. INTRODUCTION

Fracture in snow is paramount to the avalanche initiation process. However, there is no universally accepted model or consensus on the fracture mode of snow. Typically, snow fracture is assumed to be a mixed mode shear/anticrack (Heierli et al, 2008) or pure shear (McClung, 2011). However, both these models assume a fracture mode and make various assumptions about the slab and initiation. As a polycrystalline material with relatively large grains, the macroscopic approach may oversimplify the complex grain interactions encountered in snow fracture.

Here, a microstructural, mode-independent fracture model is introduced. By focusing on the microscopic interactions between ice crystals, assumptions about macroscopic snow behavior are not used.

2. THEORY

A brittle elastic fracture is propagated when mechanical energy forms two new surfaces of a material. Surface formation energy, $\gamma$ (J/m$^2$), is a well-defined material property for most materials. The energy required to form two new surfaces (i.e., fracture) of a material is then

$$G_c = 2\gamma$$

(1)

$G_c$ is termed the critical energy release rate and it is a fundamental fracture property of a material. The coefficient 2 denotes that when a fracture propagates two new surfaces are created. Most current fracture models for snow take a macroscopic approach (e.g. Heierli et al, 2008; McClung, 2011). However, fracture in snow actually occurs through the bonded ice network (McClung, 2009). Here, the microstructural details of snow are used to predict the fracture path through individual ice bonds rather than treating the snow as a continuum.

It has been shown that fracture in brittle polycrystalline materials tends to follow the path requiring the least energy (e.g. Berlyand et al, 1998; Holm, 1998; Spychalski et al, 2002). As a material made of bonded ice crystals, a snow sample has a large but finite number of possible fracture paths through the bonds. Therefore, it is possible to compute the path requiring the least energy. This minimum path has previously been identified as a relevant property of snow (Hagenmuller et al, 2013).

X-ray computed tomography (CT) scans were used to ascertain snow sample 3D microstructural geometry. A SkyScan 1173 Micro-CT scanner modified to work in a cold lab was used.

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By using x-ray computed tomography (CT) images of snow, interconnecting bonds can be identified and the minimum energy path subsequently computed. Then, the energy cost over a sample can be normalized to derive the macroscopic material property \( G_c \) for snow.

To mathematically represent the network of bonds (potential fracture surfaces), we start with a hypothetical microstructure as illustrated in Figure 1. Then, a node-line graph network corresponding to the geometry of the snow sample is created. The node-line graph is a mathematical abstraction of snow microstructure. After bonds (red) are identified, lines representing those bonds (green) are drawn between grains as shown in Figure 1. The graph thus represents the snow ice grain network.

In order to accurately represent the snow microstructure, lines must be weighted according to the energy required to break the associated bond. Hence, lines in the graph are weighted according to the surface area of the bond. Then, the minimum amount of energy to split the graph into two pieces is calculated using the Ford-Fulkerson algorithm (Ford and Fulkerson, 1962).

3. ALGORITHM

The algorithm to calculate the minimum energy cut in snow was written in MATLAB. It is run on a 64-bit Linux machine with 32 GB of memory. Run times are on the order of a few seconds with a 50x50x50 voxel cube, up to several hours with a 400x400x400 cube. The algorithm consists of five subroutines:

3.1 Import Image Data
A binary 3D CT image is imported.

3.2 Segment image into bonds and grains
The 3D binary image is segmented using watershed segmentation (Meyer, 1994). Essentially, this technique finds necks in the ice crystal microstructure that are significantly smaller than the grains around them. Bonds and grains are now identified.

3.3 Create graph of microstructure
The algorithm identifies which bonds are connected to which grains and a node-line graph network is created. Lines (bonds) in the graph are weighted according to their area. To determine the area of each bond, the projection method of Flin et al (2005) is used.

3.4 Determine Minimum Cut
Once the graph has been created, the minimum cut to separate the sample and the surface area through the bonds is calculated. This step identifies the actual bonds along the minimum energy path that must be broken to break the sample into two pieces (Figure 2).

3.5 Calculate \( G_c \)
Once the minimum cut area (\( C_{\text{min}} \)) is calculated from the CT image, critical energy release rate is calculated by determining the energy cost of
breaking these bonds and normalizing over surface area:

\[ G_c = \frac{c_{\min} S^2 y_{\text{Ice}}}{A_s} \]  

(2)

Where \( A_s \) is the macroscopic cross-sectional area of the sample and \( S \) is scale (meters/voxel). The surface formation energy of ice, \( y_{\text{Ice}} \), is 75.7 mJ/m\(^2\) (Petrenko and Whitworth, 1999).

4. MODEL RESULTS

The algorithm was run on a sample of homogeneous, rounded snow at a resolution of 14.88 \( \mu \)m. All samples modeled were \( n \times n \times n \) voxel cubes selected from arbitrary locations within an 800x800x800 voxel (1.2x1.2x1.2 cm) region of interest.

4.1 Sample Size and Representative Volume Element

Fracture was modeled in cubic samples ranging from 50x50x50 voxels up to 400x400x400 voxels. Modeled critical energy release rate tended to increase with volume. \( G_c \) varied from about 0.005 to 0.035 J/m\(^2\), depending on sample volume (Figure 3). Although the results may approach an asymptotic value, larger samples were not considered due to the computational time required.

![Figure 3: Calculated energy release rate varies with sample size](image)

5. EXPERIMENTAL VALIDATION

As preliminary validation of the model, a laboratory fracture test was performed on the same snow that was modeled. Homogeneous, rounded grains from a snow breeder were sifted into a box to create an isotropic homogeneous sample. An open-front shear frame was placed on the snow surface, and more snow was sifted in and around it. After allowing the grains to sinter for two hours, the snow inside the shear frame was isolated from surrounding snow and the sample notched to create a rectangular test specimen (Figure 4). Test properties are summarized in Table 1.

A shear (Mode II) test was performed. The applied load \( P \) and displacement were monitored with a load cell and LVDT at a rate of 1000 Hz. The load was applied using a Geo-Jac load actuator at a constant displacement rate of 0.847 mm/s. Strain in the entire sample was monitored optically via high speed video at 500 frames per second (fps) using a Fastec TS3-100 high speed camera. High speed video frames were analyzed for shear strain using the ARAMIS particle image velocimetry (PIV) software package.

![Figure 4: Experimental test configuration. The shear frame is open in front to allow optical strain monitoring of the entire sample.](image)

5.1 Material Properties

To ascertain shear modulus, the specimen was loaded in shear and monitored prior to failure. A plot of shear stress vs average shear strain in the sample showed a shear modulus \( G = 2.180 \) MPa. Since the test was in shear, Poisson’s ratio was not measured directly, but was approximated as 0.2 based on prior tests on similar snow in the lab (Walters and Adams, 2014) and similar results from field tests (Schweizer, 1999)

Elastic modulus \( E \) was calculated for the sample using the isotropic relation (Shames and Cozzarelli, 1997):

\[ E = 2G(1 + \nu) \]  

(3)
5.2 Critical Energy Release Rate

For this test geometry, and assuming plane strain due to the large relative sample depth (B), the critical energy release rate can be computed as (Tada et al, 2000)

\[ G_c = \frac{1+\nu^2}{2E}\left(\frac{p^2}{h} + \frac{p^2}{H}\right) \] (4)

Where \( E \) is elastic modulus, \( \nu \) is Poisson’s ratio, and \( P \) is the load (N) at failure. It was found that \( G_c = 0.062 \text{ J/m}^2 \).

Tbl. 1: Mechanical Test Summary

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho ) (g/cm(^3))</td>
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<td>H (cm)</td>
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<td>h (cm)</td>
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<tr>
<td>B (cm)</td>
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<tr>
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</tr>
<tr>
<td>P (N)</td>
<td>13.8</td>
</tr>
<tr>
<td>( G_c ) (J/m(^2))</td>
<td>0.062</td>
</tr>
</tbody>
</table>

6. CONCLUSIONS AND DISCUSSION

Both the model and the experiment give results that are comparable to published values for energy release rate in certain snow samples (Sigrist et al, 2006; Kirchner et al, 2002).

The values of \( G_c \) calculated by the model show a dependence on sample size. The results may approach an asymptotic value around 0.035 as sample size increases. The goal of this research is to use a representative volume element (RVE) to calculate \( G_c \), and further work is needed to confirm that sample size is adequate.

More specifically, the experiment gives a \( G_c \) that is higher than the values calculated from the model. The model calculates the lowest possible energy required to fracture a sample, implying brittle fracture through some bonds and elastic recovery of strained regions. In this model, then, it is assumed that no bonds break outside the minimum energy fracture path. However, preliminary experimental work indicates that the actual energy release rate could be significantly higher than the minimum energy release rate calculated by the model. Therefore, microcracking outside the minimum energy path likely plays a role in the fracture process.

Future model work should account for microcracking outside the minimum energy path. A “fracture energy sink” term could be used to describe this additional energy loss and has been suggested previously for work with polycrystalline ice (Nixon and Schulson, 1987):

\[ G_c^* = G_c + G_{\text{dam}} \] (5)

Where \( G_c^* \) is a corrected energy release rate and \( G_{\text{dam}} \) accounts for damage energy dissipated through microcracking.

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