Modelling Wet Snow Avalanche Flow with a Temperature Dependent Coulomb Friction Function

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ABSTRACT: We present a model to describe the motion of wet snow avalanches. This model is used to simulate a well-documented wet snow avalanche event in Switzerland. We begin by identifying the physical processes which are most affected by snow temperature and moisture, including meltwater lubrication at the sliding surface and the cohesive, plastic interactions between wet snow granules. We then introduce two conservation equations for internal heat energy and advective water transport into the governing differential equations describing avalanche motion. Different physical mechanisms (dissipative energy fluxes) contribute to the rise in temperature within the avalanche. Meltwater can be produced depending on the temperature of the flowing snow, which is strongly related to the temperature of the entrained snowcover. We then identify how temperature and moisture affect different constitutive relationships: (1) we modify the basal friction to account for water at the sliding surface, (2) we increase the dissipation of turbulent kinetic energy to account for the moisture controlled, plastic interactions between granules and (3) to model wet snow surges and levee formation we include a cohesion term in the basal shear stress. Because the temperature varies in the stream-wise and lateral flow directions different deposition structures result. Melting can occur in different flow regions, including the tail of the avalanche. This helps explain wet snow avalanche pile-ups. Even though the flow remains laminar and slug-like, we find that wet snow avalanche runout distances can be large due to the meltwater lubrication observed in the study cases.

KEYWORDS: wet snow, avalanche dynamics, ice friction, meltwater lubrication, numerical modelling.

1 INTRODUCTION

Snow avalanches are typically classified into one of two wide-ranging categories: (1) flowing and (2) powder snow avalanches (McClung and Schaerer, 2006). Wet snow avalanches belong to the first category, but exhibit such distinctive – *and dangerous* -- flow behaviour that they are usually considered as a separate avalanche class. In many regions of the world they represent the primary avalanche danger, especially in maritime climates where warm, moist snow is common (e.g. Chile, Pacific coast of North America, Western Himalayans,Northern Russia). Understanding how wet snow avalanches reach long runout distances and can deliver such extreme destructive forces remains one of the principle problems in avalanche dynamics (Fig. 1).

In this short contribution we briefly present a wet snow avalanche dynamics model that accounts for several important physical processes that are unique to wet snow avalanches.

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At the core of the problem is the constitutive behaviour of warm-moist snow: When a warm snowcover releases the snowcover fragments into blocks. Typically the upper surface of the snowcover is at $T=0°C$ with some liquid water already in the free pore space of the snow. During the flow the blocks are sculptured into well-rounded granules (Bozhinskiy and Losev, 1998). Wet snow avalanche granules are larger and exhibit uneven size distributions in comparison to dry (cold) flowing avalanche granules (Bartelt and McArdell, 2009). Large granules in the deposition zone are often particle conglomerates consisting of smaller, less rounded particle aggregates, suggesting strong cohesive bonding (capillary) forces between the granules.

Wet snow avalanche deposits can be similar to dry flowing snow deposits, but they can also differ significantly. For example, we have observed wet snow avalanches that exhibit wide spreading on open slopes with homogenous height distributions (Fig. 1). However, we have also observed highly irregular deposits consisting of flow arms, levees and en-enchelon shear planes (Bartelt and others, 2012). Granular pile-ups are produced, indicating strong variations in basal friction. In these deposits internal shear planes with refrozen melt layers are common. The shear planes are located both at the basal surface as well as at levee sidewalls. In both cases, whether regular or highly irregular flow behaviour in the deposition zone, the runout distances of wet snow avalanches can be large. Long runout distances are often associated with large release volumes, full-depth entrainment of the snowcover, producing immense deposition heights with a dirty, muddy exterior (Fig. 1).

Existing avalanches dynamics models employ simple mass and momentum conservation equations to simulate avalanche flow.(Perla and others, 1980; Salm, 1993; Savage and Hutter, 1989; Pitman and others, 2005; Christen and others, 2010). Although theses approaches accurately model the bulk, plug-like flow behaviour of wet snow avalanches (Kern and others,2009), they do not account for the internal energy fluxes (thermal temperature) of the flowing snow. This restricts the constitutive modelling to finding friction parameters that are valid for specific temperature ranges. It does not allow us to study the role of meltwater lubrication or how snowcover temperature modifies entrainment rates, flow granulometry and therefore the avalanche flow regime.

In this paper we will first introduce two additional depth-averaged differential equations into the **RAMMS** model (Christen and others, 2010): The first is the internal energy equation; the second is the advective transport of meltwater, produced by dissipative heating of the flowing snow. Therefore, the equations are coupled. We modify the Coulomb shearing according to the water-film theory of Colbeck (1992; 1995) using additional experimental results from Evans and others 1976; Baürle and others, 1996). Using a simple case-study, we demonstrate how meltwater lubrication decreases both internal and basal shear resistance leading to long runout avalanches.

Wet snow granulometry is taken into account by increasing the granule cohesion as well as increasing the decay of collisional fluctuation energy (Buser and Bartelt, 2009; Bartelt and others, 2012; Bartelt and McArdell, 2009). This produces dense, heavy flows with modest fluctuation energy where the primary flow regime contains enduring granular contacts. By including the rapid decay of fluctuation energy we ensure plug-like flow behaviour (Kern and others, 2009) Thus, although the runout distances are enhanced, flow velocities remain small, leading to many of the observed depositional features of wet snow avalanches.

Fig 1. Wet snow avalanche at Drusatcha 2013 (Davos)

2 MODEL EQUATIONS

We modify the existing mass and momentum fluxes of **RAMMS** model (Christen, 2010; Bartelt, 2012;) to include the transport of thermal energy and meltwater. The mathematical description of the mountain terrain is defined in a horizontal coordinate system *(X,Y):* The elevation *Z(X,Y)* is specified for each coordinate pair. A local coordinate system *(x,y,z)* is introduced with directions *x* and *y* parallel to the geographic coordinates *X* and *Y*.

We employ depth-averaged velocity, shear force and gravity vectors in the plane parallel *(x,y)* coordinate system:

$$\mathbf{V} = u\mathbf{i} + v\mathbf{j} \tag{1}$$

$$\mathbf{S} = S_x\mathbf{i} + S_y\mathbf{j} \tag{2}$$

$$\mathbf{G} = mg_x\mathbf{i} + mg_y\mathbf{j} \tag{3}$$

where *(u,v)* are the mean avalanche velocities in the *x* and *y* directions; (S_x, S_y) are the *(x,y)* components of the depth-averaged shear stress $\mathbf{S}$; *m* is the avalanche mass per unit area and g_x and g_y are the gravity components in the *x*, *y* directions respectively.

Mass m and momentum (mV) equations are:

$$\frac{\partial m}{\partial t} + (\mathbf{V}\cdot\nabla)m = \dot{Q}_e \tag{4}$$

$$\frac{\partial (m\mathbf{V})}{\partial t} + (\mathbf{V}\cdot\nabla)(m\mathbf{V}) = \mathbf{G} - \mathbf{S} - \frac{1}{2}\nabla\left(\rho g_z h^2\right) \tag{5}$$

where $\dot{Q}_e$ designates the entrained snowcover mass. Two additional energy equations are employed, the first one governing the granular fluctuation *R* and the second one the internal heat energy *E*:

$$\frac{\partial(hR)}{\partial t}+\left(\mathbf{V}\cdot\nabla\right)(hR)=\alpha\left(\mathbf{S}\cdot\mathbf{V}\right)-\beta(hR) \qquad (6)$$

$$\frac{\partial(hE)}{\partial t}+\left(\mathbf{V}\cdot\nabla\right)(hE)= \qquad (7)$$
$$(1-\alpha)\left(\mathbf{S}\cdot\mathbf{V}\right)+\beta(hR)+c_sT_s\dot{Q}_e-c_s\rho h\dot{T}_m$$

The fluctuation parameters α and β (Buser and Bartelt, 2009) govern the increase of granular fluctuation (random fluctuation energy) from the mean shear work $\mathbf{S}\cdot\mathbf{V}$ (parameter α) and the dissipation of fluctuation energy to heat by plastic deformations (parameter β). For wet snow avalanches, the parameter β is large. The model equations accounts for the heat energy of the entrained snowcover $c_sT_s\dot{Q}_e$ (c_s is the specific heat of snow, T_s the temperature of the snowcover). We add a constraint equation to consider the phase changes from snow to melt water.. When the flow temperature reaches 0°C, the mean "excess" heat energy rise $c_s\rho h\dot{T}_m$ is converted to water (that is, the calculated temperature never exceeds 0°C). The volumetric generation of meltwater $\dot{Q}_w=\frac{c_s\rho}{L_f\rho_w}h\dot{T}_m$, providing the right hand side of the depth-averaged meltwater transport equation:

$$\frac{\partial(hW)}{\partial t}+\left(\mathbf{V}\cdot\nabla\right)(hW)=\dot{Q}_w \qquad (8)$$

where *W* is the volumetric meltwater content of the flow; L_f is the latent heat of fusion and ρ_w is the density of water.

Figure 2. Avalanche deposits Brämabüel (Davos)18th April, 2013

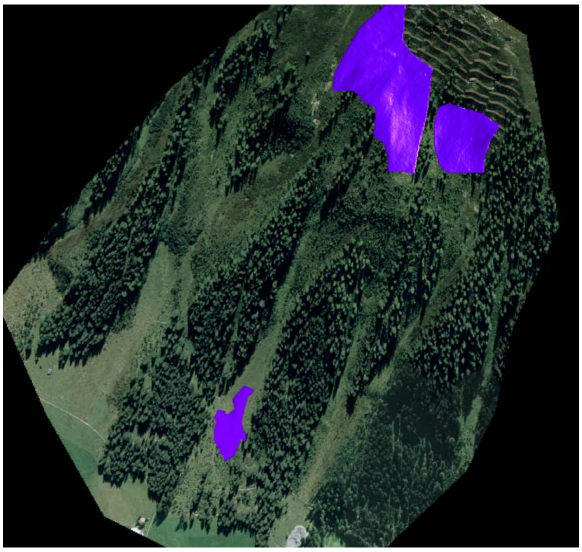

Figure 3. Contour line of the release area (upper) and deposits at the middle channel (lower)

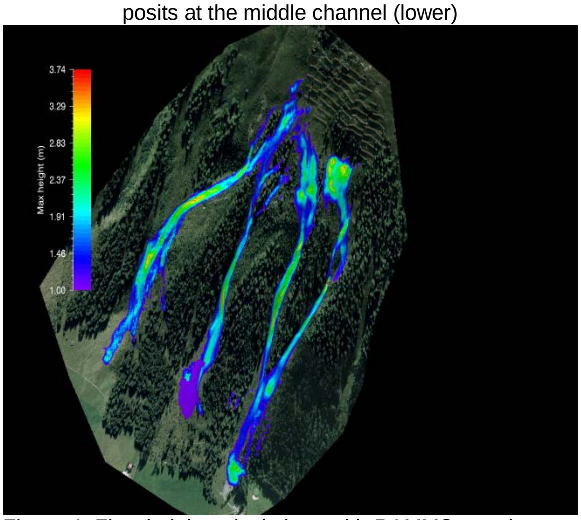

Figure 4. Flow height calculations with RAMMS over impose with the measure deposits at Brämabüel (Davos)

3 CONSTITUTIVE RELATIONS

The wet snow avalanche model requires three constitutive relations:

3.1 Shear

To calculate the avalanche shear *S*, a depth averaged Voelmy-type relation was employed (Voellmy, 1955;): The two frictional parameters depend on the (1) the collisional frictional regime (state variable R, and the volumetric water content W).

$$S=\mu(R,W,c)N+\frac{\rho g\left\|V\right\|^2}{\xi} \qquad (9)$$

where N is the normal stress and μ and ξ are the Voellmy parameters. We also increase the shear stress by a normal stress dependent cohesion, based on the flowing snow experiments of Platzer and others (2007).

3.2 Meltwater Lubrication

To include the lubrication process in the model, it is necessary to add a constitutive relation between the friction coefficient μ and the water content. Investigations by (Colbeck, 1992;) and (Evans and others, 1976;) reveals a decreasing exponential relationship between the water film thickness and the dry friction coefficient μ.

The first function we have postulated is:

$$\mu(R,W,c) = \mu_{wet} + \left[\mu(R) - \mu_{wet}\right]\exp(-\frac{W}{W_0}) \quad (10)$$

where μ_{wet} is the lowest sliding friction value possible for high (saturated) water contents. Observations reveal that this value corresponds to calibrated μ values of extreme avalanche events, $\mu_{wet} \approx 0.12 - 0.15$; the parameter W_0 steers the exponential decay and requires calibration. The reduction of Coulomb friction because of granular fluctuations $\mu(R)$ is discussed in (Bartelt and others, 2012;). However, because R is small in wet snow avalanches (extreme dissipation of granular fluctuations), $\mu(R \approx 0) = \mu_{dry}$ where μ_{dry} represents the static, dry friction coefficient of wet snow (large, say $\mu_{dry} \approx 0.40 - 0.60.$ (Platzer and others, 2007).

4 CASE STUDY

To demonstrate the applicability of the model, we simulate a wet snow avalanche that released spontaneously on the 18th of April, 2013 from the northeast flank of the Brämabüel mountain (Davos, Switzerland) (Fig. 2). The avalanche was observed directly from the SLF facilities. From a single release zone the avalanche split into three separate avalanches, each with a separate deposition pile.

The release area was estimated from terrain observations (Fig. 3). We approximated the release height (h = 0.30 m) and starting volume (V=45000 m^3) from weather stations and snowover observations near the release zone. GPS measurements were conducted in the middle channel. The runout distances of the avalanches in the two adjacent channels were mapped (Fig. 3). No information could be gathered concerning flow velocities.

The snow surface temperature was *T = 0°C* and with high free water content. The temperature profile was isothermal in the first 30 cm of snow cover (coinciding with the snow height released). The simulation was performed considering the lubrication effect on the *μ(R,W)* coefficient (Eq. 10). Fig 5 shows the coefficient *μ* varying with melt water content and *R* (Fig 7). The friction coefficient *μ(R,W)* values varied between *μ(R,W) = 0.20* with the highest water content in the deposition area and *μ(R,W) = 0.55* in the case of low water content and low granular fluctuations.

When *T > 0°C* meltwater was produced within the flow due to frictional work and dissipation of random fluctuation energy to heat (Fig 6). Calculations showed a maximum water production of 2 mm of water per square meter within the interior of the avalanche. Meltwater production was concentrated at points of high velocity. The highest production rates were at the center of the avalanche; much smaller production amounts (0.10 mm) were found at the flow edges of the avalanche. The water content is calculated by adding the melt water production from the current time-step to the melt water transported with the flow. (Fig 7). Maximum water contents of 6 mm per square meter in the deposit area were encountered.

5 DISCUSSION AND CONCLUSIONS

With the lubrication model it was possible to reproduce not only the measured runout distances, but also the shape of the avalanche deposits in the runout zone. The calculated flow heights showed a good fit with the deposition heights recorded along the central channel (Fig. 4). The simulated run-out distances and the spreading of the deposits in the adjacent channels coincides with the photographic documentation.

As the production of meltwater is directly related to the calculated shear work, meltwater production is higher in the core of the avalanche, where the velocity is highest, in comparison to the flow edges, where the flow velocities are low (Fig 5)..This leads to zones in the avalanche where meltwater is concentrated (centrelines) and regions where the meltwater is sparse (edges). Gradients in velocity coincide with water production gradients. Because the friction coefficient *μ* is lowest when the mean water content becomes relatively high, the avalanche core will flow longer and faster, in comparison to the flow edges, which will stop and begin to form shear planes and levees. Therefore, the spatial gradients across the flow width could help explain typical wet snow avalanche deposition patterns such as flow fingers and levees. These results are based on depth-averaged calculations. Dependencies on flow are presently not included in the model calculations.

In the study case the warm and moist snow entrained by the avalanche stimulated the melt water production. We assumed that the properties of the snowcover did not vary from release to deposition. This will not always be the case. It is entirely possible that snow releasing from a cold release zone will entrain warm moist snow at lower elevations. In fact, this source of heat energy (entrained mass) could be the dominant heating mechanism within the avalanche. The elevation drop in the avalanche case study was not large enough to produce considerable heating by frictional dissipation. A primary conclusion of our work is that energy fluxes (both thermal heat and random kinetic) should be included when considering the role of mass entrainment in avalanche flow.

We could not model the wet snow avalanche case study by considering only the decrease in friction due to the production of random kinetic energy R and the onset of a fluidized flow regime. In wet snow avalanches, granular pulsations are damped. Variations in flow density are small. This result stresses the importance of finding a function that lubricates the flow, dependent on the local water content. We have postulated such a function, based on the event of the presented case study and other wet snow avalanche events. Whether this function is general enough to replicate a wide-range of wet snow avalanche behaviour is the subject of ongoing investigations.

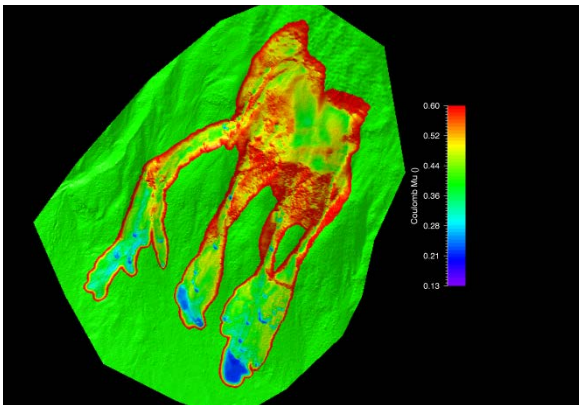

Fig 5. μ coefficient calculated with expression (9)

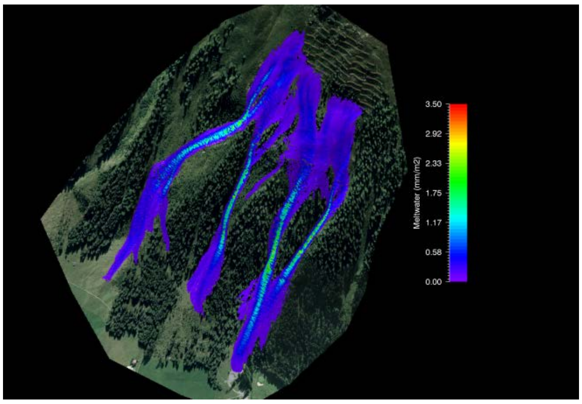

Fig 6. Melt water production calculation

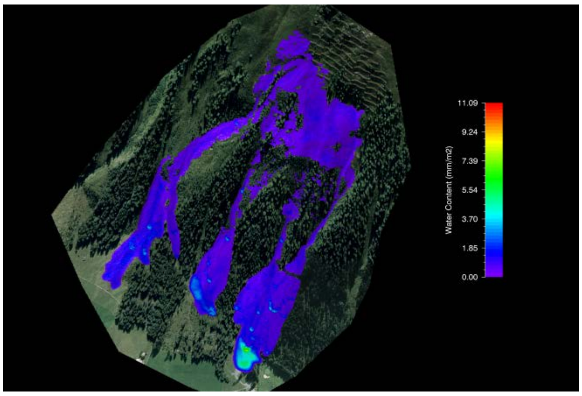

Fig 7. Melt water content calculation

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